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Independence and clique numbers of the complement of the complementary prism

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Abstract

In this paper, we find the independence number of the complement $H = \overline{G\overline{G}}$ of the complementary prism $G\overline{G}$ of a graph G in terms of G. We obtain sharp upper and lower bounds of the clique number of H in terms of G. In particular, the chromatic and clique numbers of H when G is a complete graph or a star are computed.

1 Introduction

The concept of the complementary prism was first introduced in [7]. The complementary prism of a graph G, denoted by $G\overline{G}$, is the graph obtained by taking a copy of G together with a disjoint copy of $\overline{G}$ and adding the perfect matching between corresponding vertices of G and $\overline{G}$. The Petersen graph is just the complementary prism of the 5-cycle.

The independence saturation in complementary prisms was considered in [4], where complementary prisms having small independence saturation

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numbers were characterized. The Roman domination nmber of the complementary prism $G\overline{G}$ was studied in [2], where some inequalities comparing it with the Roman domination numbers of G and $\overline{G}$ were obtained. The automorphism group of the complementary prism $G\overline{G}$ was described in [10] and its cardinality was compared with the cardinality of the automorphism group of G. Indeed, complementary prisms have been studied by many authors, see also [3], , [6] and [9]. In [8], the pointwise clique-safe domination number was studied for the complement and the complementary prism of some special classes of graphs, and some characterizations of the minimum pointwise clique-safe dominating sets were obtained.

Complements of graphs provide information about the structure and properties of the graphs themselves. Any isomorphism between two graphs is also an isomorphism between their complements. So, complements are used in graph isomorphism testing. The cliques in a graph correspond bijectively to the independent sets in their complements, a color class in a colored graph is an independent set. Thus complements are useful in the problems involving independent sets, cliques and graph coloring.

The complement $\overline{G}$ of a graph G is the graph having the same vertex set as G where two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. Therefore, the complement $\overline{G\overline{G}}$ of the complementary prism $G\overline{G}$ consists of a copy of $\overline{G}$ with a disjoint copy of G together with all edges of the form xy, where x is a vertex from $\overline{G}$ and y is a vertex from G that is not the corresponding vertex of x.

Throughout this paper, we will use H to refer to $\overline{G\overline{G}}$. Hamiltonicity and some other properties of H were studied in [1]. Characterizations of those graphs G for which H satisfies some critria (as being Eulerian, being regular) were also given in [1].

An *independent set* in a graph G is a set of pairwise nonadjacent vertices. The *independence number* $\beta(G)$ of G is the maximum cardinality of an independent set in G. A *clique* in G is a complete subgraph of G. The order of a largest clique in G is its *clique number* $\omega(G)$. The *chromatic number* of G, denoted by $\chi(G)$, is the minimum number of colors needed to label the vertices of G such that adjacent vertices receive different colors. For undefined notions, the reader is referred to [5]. All graphs considered have no loops and no multiple edges.

In this paper, we will find the independence number of H, and obtain sharp upper and lower bounds of the clique number of H in terms of G. In particular, we compute the chromatic and clique numbers of H when G is a complete graph or a star.

2 Main results

Throughout this section, n denotes the order of G, and the vertices of H will be labeled as $V(H) = X \cup Y$ where $X = \{x_1, x_2, \cdots, x_n\}$ is the vertex set of the copy of $\overline{G}$ in H, while $Y = \{y_1, y_2, \cdots, y_n\}$ is the vertex set of the copy of G in H. For each $i \in \{1, 2, \cdots, n\}$, y_i is the vertex corresponding to x_i. Thus $E(H)$ is the union of the three sets $E(\overline{G})$, $E(G)$ and the set of all edges of the form $x_i y_j$ where $i \neq j$. More explicitly, the edge set of H equals

$$\{x_i x_j : y_i y_j \notin E(G)\} \cup \{y_i y_j : y_i y_j \in E(G)\} \cup \{x_i y_j : i \neq j\}.$$

When G is the trivial graph K_1, the independence number of H equals 2.

Theorem 2.1. *For any nontrivial graph G, $\beta(H) = \max\{\omega(G), \beta(G)\}$.*

Proof. Let $k = \max\{\omega(G), \beta(G)\}$. Since G is not the trivial graph, we have $k > 1$. Since the clique number of a graph equals the independence number of its complement and a graph and its complement produce isomorphic complementary prisms, we can assume that $k = \omega(G)$. Then $\overline{G}$ has an independent set B with cardinality k. Without loss of generality we can assume that $B = \{x_1, x_2, \cdots, x_k\}$. Clearly B is an independent set in H and hence $\beta(H) \geq k$. Assume to the contrary that H has an independent set D with cardinality greater than k. But since $k = \max\{\omega(G), \beta(G)\} = \max\{\beta(\overline{G}), \beta(G)\}$, the set D can be neither a subset of X nor a subset of Y. Thus $D \cap X \neq \phi$ and $D \cap Y \neq \phi$. Then $|D \cap X| = |D \cap Y| = 1$, for otherwise we would have either $\{x_i, x_j, y_k\} \subseteq D$ or $\{x_i, y_j, y_k\} \subseteq D$ for some $i, j, k \in \{1, 2, \cdots, n\}$, in contradiction with the assumption that D is an independent set. Then $|D| = 2$. But $|D| > k$, so we get that $k = 1$, a contradiction. Therefore $\beta(H) \leq k$ and hence $\beta(H) = k$. □

Before turning to the clique number of H in general, we consider the special case in which G or $\overline{G}$ is complete.

Theorem 2.2. *If G is the complete graph K_n, then $\omega(H) = \chi(H) = \beta(H) = n$.*

Proof. Assume that $G = K_n$. Obviously Y induces a complete subgraph in H. Thus $\omega(H) \geq n$. Assume to the contrary that B is a clique in H such that the order of B is greater than n. Then $V(B) \cap X \neq \phi$ and $V(B) \cap Y \neq \phi$. But there is no i such that $\{x_i, y_i\} \subseteq V(B)$ because $x_i y_i \notin E(H)$ for each i. Thus $V(B) = \{x_{i_1}, x_{i_2}, \cdots, x_{i_k}, y_{j_1}, y_{j_2}, \cdots, y_{j_s}\}$ for some distinct

$i_1, i_2, \cdots, i_k, j_1, j_2, \cdots, j_s$. Then the order of B is at most n, a contradiction. Therefore $\omega(H) \leq n$ and hence $\omega(H) = n$.

Clearly $\chi(H) \geq n$ since H contains K_n. Consider the n-coloring of H which assigns different colors to the vertices $y_1, y_2, \cdots, y_n$ and for each i it assigns the color of y_i to the vertex x_i. Therefore $\chi(H) \leq n$ and hence $\chi(H) = n$.

Finally, by Theorem 2.1, we get $\beta(H) = n$. □

Now we find upper and lower bounds of the clique number of H.

Theorem 2.3. *For any graph G of order n, we have $\omega(H) \leq n$. And if both G and $\overline{G}$ are not complete, then $\omega(H) \geq 1 + \max\{\omega(G), \beta(G)\}$.*

Proof. Let B be a clique in H. If $V(B) \subseteq X$ or $V(B) \subseteq Y$, then the order of B is at most n. So assume that $V(B) \cap X \neq \phi$ and $V(B) \cap Y \neq \phi$. But there is no i such that both x_i and y_i belong to $V(B)$ because $x_i y_i \notin E(H)$ for each i. Thus $V(B) = \{x_{i_1}, x_{i_2}, \cdots, x_{i_p}, y_{j_1}, y_{j_2}, \cdots, y_{j_s}\}$ for some distinct $i_1, i_2, \cdots, i_p, j_1, j_2, \cdots, j_s$. Then the order of B is at most n. Therefore $\omega(H) \leq n$.

Assume that both G and $\overline{G}$ are not complete and let $k = \max\{\omega(G), \beta(G)\}$. Then $k < n$. Since the clique number of a graph equals the independence number of its complement and a graph and its complement produce isomorphic complementary prisms, we can assume that $k = \omega(G)$. Without loss of generality we can assume that $\{y_1, y_2, \cdots, y_k\}$ induces a complete subgraph in H. Then the set $D = \{y_1, y_2, \cdots, y_k\} \cup \{x_n\}$ induces a complete subgraph in H. Therefore $\omega(H) \geq 1 + k$. □

The upper and lower bounds of $\omega(H)$ given in the previous theorem are sharp. Note that when $G = P_3$, we have $\omega(H) = 3 = 1 + \max\{\omega(P_3), \beta(P_3)\}$. And when $G = K_n$, we have $\omega(H) = n$, by Theorem 2.2.

Consider the star graph $K_{1,m}$ of order $n = m + 1$. For $m = 1$, the star $K_{1,1}$ is the complete graph K_2 and hence $\omega(H) = 2$ when $G = K_{1,1}$. So, assume that $m > 1$. Then $G = K_{1,m}$ is not complete, therefore by Theorem 2.3 we have $\omega(H) \leq n = m + 1$ and $\omega(H) \geq 1 + \max\{2, m\} = 1 + m$. Thus we have the following result.

Corollary 2.4. *If G is the star graph $K_{1,m}$, then $\omega(H) = m + 1$.*

We conclude this section by finding the chromatic number of H when G is a star.

Theorem 2.5. *If G is the star graph $K_{1,m}$, then $\chi(H) = m + 1$.*

Proof. Assume that $G = K_{1,m}$. Notice that $K_{1,m}$ has order $n = m + 1$. Let $y_n = y_{m+1}$ be the central vertex of $K_{1,m}$. Then $x_n = x_{m+1}$ is an isolated vertex in $\overline{K_{1,m}}$. The set $\{x_1, x_2, \cdots, x_m\} \cup \{y_{m+1}\}$ induces a clique B in H. Then $\chi(H) \geq |V(B)| = m + 1$. Now consider the following coloring of H: Assign different colors to the vertices of B. For $1 \leq i \leq m$, color y_i by the same color as x_i. And finally give x_{m+1} the same color as y_{m+1} to get an $(m+1)$-coloring of H. Therefore $\chi(H) \leq m+1$. and hence $\chi(H) = m+1$. $\square$

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