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On the Diophantine equation $456^x + 987^y = z^2$

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Abstract

In this paper, we show that the Diophantine equation $456^x + 987^y = z^2$ has no non-negative integer solutions.

1 Introduction

In 2023, N. Viriyapong and C. Viriyapong [1] showed that the Diophantine equation $255^x + 323^y = z^2$ has exactly two non-negative integer solutions (x, y, z); namely, $(1, 0, 16)$ and $(0, 1, 18)$. In 2024, N. Viriyapong and C. Viriyapong [2] showed that the Diophantine equation $147^x + 741^y = z^2$ has no non-negative integer solutions. In 2025, N. Viriyapong and Puiwong [3] proved that $(x, y, z) = (1, 0, 26)$ is the unique non-negative integer solution to the Diophantine equation $675^x + 896^y = z^2$.

In this paper, we study the Diophantine equation $456^x + 987^y = z^2$ where x, y and z are non-negative integers.

2 Preliminaries

Throughout this paper, we use the notation $a \equiv_m b$ to mean that a is congruent to b modulo m, where a, b and m are integers such that $m \geqslant 1$. Moreover,

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we write $a \equiv_m b, c$ to indicate that $a \equiv_m b$ or $a \equiv_m c$.

We now recall two lemmas from [4] and [5] for later use.

Lemma 2.1. [4] *If z is an integer, then $z^2 \equiv_{13} 0, 1, 3, 4, 9, 10, 12$.*

Lemma 2.2. [5] *If z is an integer, then $z^2 \equiv_{19} 0, 1, 4, 5, 6, 7, 9, 11, 16, 17$.*

Next, we recall Catalan's conjecture [6] in 1844. This conjecture was proved in 2004 by Mihailescu [7].

Theorem 2.3 (Catalan's conjecture). *The Diophantine equation $a^x - b^y = 1$ has a unique solution $(a, b, x, y) = (3, 2, 2, 3)$ where a, b, x and y are integers with $\min\{a, b, x, y\} > 1$.*

3 Main Results

We give a theorem which will be useful in our work.

Theorem 3.1. *Let a and b be two positive integers such that $a \equiv_{247} 209$ and $b \equiv_{247} 246$. Then the Diophantine equation $a^x + b^y = z^2$ has no positive integer solutions where x, y, z are positive integers.*

Proof. Assume that there exist positive integers x, y, z such that $a^x + b^y = z^2$. Since $a \equiv_{247} 209$ and $b \equiv_{247} 246$, we obtain that $a \equiv_{13} 1$, $b \equiv_{13} 12$ and $a \equiv_{19} 0$, $b \equiv_{19} 18$. If y is even, then $z^2 = a^x + b^y \equiv_{13} 2$, which contradicts Lemma 2.1. Then y must be odd. Since $b \equiv_{19} 18$, we have $b^y \equiv_{19} 18$. In addition, since $a \equiv_{19} 0$, it follows that $a^x \equiv_{19} 0$, and therefore $z^2 = a^x + b^y \equiv_{19} 18$, which contradicts Lemma 2.2. This completes the proof. □

We now state our main result, which follows from an application of Theorem 3.1.

Corollary 3.2. *The Diophantine equation $456^x + 987^y = z^2$ has no non-negative integer solutions where x, y, z are non-negative integers.*

Proof. Assume that there exist non-negative integers x, y, z such that $456^x + 987^y = z^2$. Since $456 \equiv_{247} 209$ and $987 \equiv_{247} 246$, Theorem 3.1 implies that $x = 0$ or $y = 0$. If $x = 0$, then by Theorem 2.3 we have $y = 1$, giving $z^2 = 988$, which is impossible. If $y = 0$, then Theorem 2.3 gives $x = 1$, yielding $z^2 = 988$, which is also impossible. Therefore, the equation $456^x + 987^y = z^2$ has no non-negative integer solutions. □

4 Conclusion

In this paper, we showed that the Diophantine equation $456^x + 987^y = z^2$ has no non-negative integer solutions where x, y and z are non-negative integers.

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