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A New Softmax Method Performance for Solving Chaffee-Infante Equation

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Abstract

In this paper, we present a novel application of the softmax method to solve the Chafee-Infante equation, providing various analytical solutions such as hyperbolic, trigonometric, rational, and polynomial forms. This approach simplifies solving nonlinear differential equations to obtain exact analytic solutions, particularly hyperbolic and trigonometry wave solutions by constructing the softmax function's computational model. In this study, we compare this technique to traditional methods and demonstrate its efficacy in producing more explicit and diverse solutions. The results have potential applications in fluid dynamics, nonlinear optics, and plasma physics.

1 Introduction

The normalized exponential, also known as the softmax function, is a smoothed version of the "max" function. The sigmoid function represents a one-class exponent while the softmax functions conduct multi-class exponent. Activation functions are crucial to neural networks' ability to provide complex judgments and predictions in deep learning and machine learning. For a

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recurrent neural network model, the softmax functions in the hidden layer are just as crucial as the sigmoid or tanh functions. Among these, the softmax activation function stands out, especially for classification tasks with mutually exclusive outcomes. The functioning of the softmax function, its applications, and its significance in the field of artificial intelligence (AI) are examined in [1]. The softmax function plays an active role in the hidden layer of several models, including Transformer, Recurrent Neural Network (RNN), Gated Recurrent Units (GRU), Long Short-Term Memory (LSTM), and Attention. Activation functions are crucial to neural networks' ability to provide complex judgments and predictions in deep learning and machine learning. Among these, the softmax activation function stands out, especially for classification tasks with mutually exclusive outcomes. In this article, we examine the softmax function along with information on its applications and significance to AI and how it works.
Often used in the final layer of a neural network model for classification tasks, the softmax function normalizes the values by taking the exponential of each output and dividing it by the sum of all the exponentials. Logits, or raw output scores, are converted into probabilities by this technique. This process makes the output numbers understandable as probabilities by guaranteeing that they fall between 0 and 1 and add up to 1. The softmax function can be expressed mathematically as follows: [1]:

$$f(z_i) = \frac{e^{z_i}}{\sum_{j=1}^{k} e^{z_j}}, \tag{1.1}$$

where z_i represents the input to the softmax function for class i and the denominator is the sum of the exponentials of all the raw class scores in the output layer. Think about a neural network that must classify images of handwritten numerals ranging from 0 to 9. A vector of ten values, each representing a digit, might be the final layer's output. However, options are not accurately depicted in these figures. The softmax function [2] converts this vector into a probability distribution for every digit class. The softmax function's form is derived from the exponent expression used in the Exp-function technique of mathematics [3, 4].
Using the extended sinh-Gordon equation expansion technique, the solitary wave behavior of the (2+1)-dimensional Chaffee-Infante equation is explored. When the parameters are chosen appropriately, patterns of traveling waves that are bright, dark, periodic, kink, anti-kink, solitary wave solutions, and single are noticed. This study will utilize the construction of the softmax

method for solving the Chafee–Infante equation given as follows: [5]:

$$u_{xt} + \left(-u_{xx} + nu^3 - nu\right)_x + mu_{yy} = 0. \tag{1.2}$$

2 Methodology

In this section, we construct the softmax method for Chafee-Intante equations. The phases are performed as follows:
Phase 1: Consider a nonlinear differential equation

$$F(u, u_x, u_t, u_{xx}, u_{xt}, \cdots) = 0. \tag{2.3}$$

where F is an aggregation function containing the members of linear and nonlinear terms $u, u_x, u_t, u_{xx}, u_{xt}, \cdots$. Using the traveling transform

$$u(x, y, t) = u(\xi), \xi = kx + sy + wt, \tag{2.4}$$

where k, s, t are the constants, Eq. (2.3) becomes an ordinary differential equation (ODE)

$$H(u, u', u'', \cdots, u^2, u^3, \cdots) = 0. \tag{2.5}$$

where $u = u(\xi)$, and $u' = \frac{du}{d\xi}$.
Phase 2: The softmax function is constructed as follows

$$f(\xi_k) = \frac{\exp(\xi_k)}{a_0 + \sum_{j=0}^{m} b_j \exp(\xi_j)}, k = 1, \cdots, m. \tag{2.6}$$

where m is collected by balancing the highest power of the terms.
Balance the term u''' and u^2u' to get the value m, and $\xi = \xi(x)$ satisfy the ODE

$$\xi_j'(x) = \exp(-\xi_j(x)) + a_j \exp(\xi_j(x)) + b_j, j = 1, \cdots, m. \tag{2.7}$$

where a_j, and $b_j, j = 1, \cdots, m$ are constants.
Phase 3: The solutions of Eq. (2.7) are obtained by considering the Riccati equation

$$Y' + Y^2 + aY + b = 0, \tag{2.8}$$

where a, b are constants. Using $Y' = \frac{\eta'}{\eta}$ leads to the ODE [3]:

$$\eta'' + a\eta' + b\eta = 0.$$

Using Maple, the solutions of Eq. (2.7) are simplified as follows:
a) When $a \neq 0, b^2 - 4a > 0$,

$$\xi(x) = \ln\left(\frac{-\sqrt{b^2-4a}}{2a}\tanh\left(\frac{\sqrt{b^2-4a}}{2}(x+C)\right) - \frac{b}{2a}\right). \tag{2.9}$$

b) When $a \neq 0, b^2 - 4a < 0$,

$$\xi(x) = \ln\left(\frac{\sqrt{-b^2+4a}}{2a}\tan\left(\frac{\sqrt{-b^2+4a}}{2}(x+C)\right) - \frac{b}{2a}\right). \tag{2.10}$$

c) When $a = 0, b^2 - 4a > 0$,

$$\xi(x) = -\ln\left(\frac{b}{\exp(b(x+C)) - 1}\right). \tag{2.11}$$

d) When $a \neq 0, b \neq 0, b^2 - 4a = 0$,

$$\xi(x) = -\ln\left(-\frac{2b(x+C)+4}{b^2(x+C)}\right). \tag{2.12}$$

e) When $a = 0, b = 0, b^2 - 4a = 0$,

$$\xi(x) = \ln(x+C). \tag{2.13}$$

where C is a constant.
Phase 4: Substitute Eq. (2.9)–Eq. (2.13) into Eq. (2.5) to collect the systems of algebraic equations of coefficients. Use mathematical tools to find the values and replace the values in the solution forms to gather the expressions.
Phase 5: Resubmitting the collected expressions to check the solutions satisfied Eq. (2.3) and making a conclusion.

3 Solution of Chafee-Infante equation

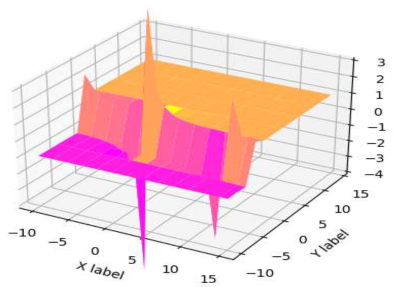

Figure 1: Lump solution graph of u_{12} when $k = 0.3; s = 1.7; w = 0.1; c = 3.5; a0 = 1.2; a1 = 1.5; t = 1.0; m = 0.2$.

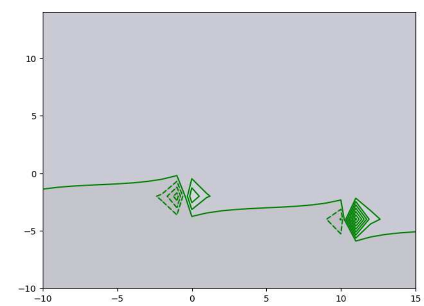

Figure 2: Contour graph of u_{12} when $k = 0.3; s = 1.7; w = 0.1; c = 3.5; a0 = 1.2; a1 = 1.5; t = 1.0; m = 0.2$

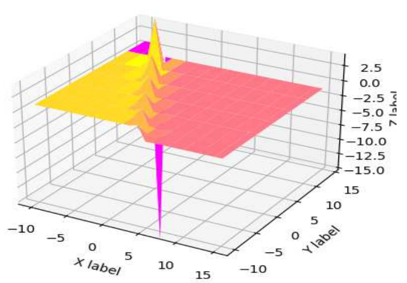

Figure 3: Rogue solution graph of u_{21} when $k = 0.3; s = 1.7; w = 0.1; c = 3.5; a0 = 1.2; a1 = 1.5; t = 1.0; m = 0.2$.

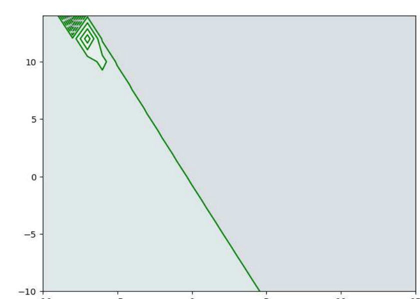

Figure 4: Contour graph of u_{21} when $k = 0.3; s = 1.7; w = 0.1; c = 3.5; a0 = 1.2; a1 = 1.5; t = 1.0; m = 0.2$

We deploy the Chaffee-Infante equation given as follows:

$$u_{xt} + \left(-u_{xx} + pu^3 - pu\right)_x + qu_{yy} = 0. \tag{3.14}$$

Using traveling transform $\xi = kx + sy + wt$, Eq.(3.14) becomes

$$\left(kw + qs^2\right) u' - k^3 u'' + pku^3 - pku = 0. \tag{3.15}$$

Balancing the highest order of ODE, $m = 1$. The solution is:

$$u = \frac{\exp(\varphi_1(\xi))}{a_0 + a_1 \exp(\varphi_1(\xi)) + b_1 \exp(\varphi_2(\xi))}. \tag{3.16}$$

where φ_1, φ_2 satisfy the equation $\varphi_1' = e^{-\varphi_1} + ae^{\varphi_1} + b$, $\varphi_2' = e^{-\varphi_2} + ce^{\varphi_2} + d$. Plugging Eq.(3.16) into Eq.(3.16), the algebraic system is gathered as follows:

$$\begin{aligned}
1 :& - bk^3a_0^2 + 2k^3a_0a_1 + 2k^3a_0b_1 + qs^2a_0^2 + kwa_0^2 \\
P(\xi) :& - 2bk^3a_0b_1 + 2dk^3a_0b_1 + 2k^3a_1b_1 + 2k^3b_1^2 + 2qs^2a_0b_1 + 2kwa_0b_1 \\
Q(\xi) :& - b^2k^3a_0^2 - 2ak^3a_0^2 + 3bk^3a_0a_1 + 2bk^3a_0b_1 + bqs^2a_0^2 + dk^3a_0b_1 \\
& + bkwa_0^2 - 2k^3a_1b_1 - 2k^3b_1^2 + qs^2a_0a_1 - qs^2a_0b_1 - kpa_0^2 + kwa_0a_1 - kwa_0b_1 \\
Q(\xi)^3 :& - 2a^2k^3a_0^2 + abk^3a_0a_1 - 2ak^3a_1b_1 + aqs^2a_0a_1 + aka_0a_1 - kpa_1^2 + kp. \\
Q(\xi)^2 :& - 3abk^3a_0^2 + b^2k^3a_0a_1 + 2ak^3a_0a_1 + 2ak^3a_0b_1 + aqs^2a_0^2 - 2bk^3a_1b_1 \\
& + bqs^2a_0a_1 + dk^3a_1b_1 + aka_0^2 + bka_0a_1 - qs^2a_1b_1 - 2kpa_0a_1 - ka_1b_1 \\
P(\xi)^2 :& - bk^3b_1^2 + 2k^3a_0b_1 + 2dk^3b_1^2 + qs^2b_1^2 + kb_1^2 \\
P(\xi)Q(\xi)^3 :& - 4a^2k^3a_0b_1 + abk^3a_1b_1 - 2adk^3a_1b_1 + aqs^2a_1b_1 + aka_1b_1 \\
P(\xi)^2Q(\xi)^2 :& - 3abk^3b_1^2 + 2ak^3a_0b_1 + 2adk^3b_1^2 - 2bk^3a_1b_1 \\
& + 3dk^3a_1b_1 + aqs^2b_1^2 - cqs^2a_1b_1 + akb_1^2 - cka_1b_1 \\
P(\xi)Q(\xi)^2 :& - 6abk^3a_0b_1 + 2adk^3a_0b_1 + b^2k^3a_1b_1 - 2bdk^3a_1b_1 + d^2k^3a_1b_1 \\
& + 2ak^3a_1b_1 + 2ak^3b_1^2 + 2aqs^2a_0b_1 + bqs^2a_1b_1 \\
& + 2k^3a_1b_1 - dqs^2a_1b_1 + 2aka_0b_1 + bka_1b_1 - dka_1b_1 - 2kpa_1b_1 \\
P(\xi)Q(\xi) :& - 2b^2k^3a_0b_1 + 2bdk^3a_0b_1 + d^2k^3a_0b_1 - 4ak^3a_0b_1 + 3bk^3a_1b_1 \\
& + 2bk^3b_1^2 + 2bqs^2a_0b_1 + 2k^3a_0b_1 - 2dk^3a_1b_1 - 3dk^3b_1^2 \\
& - dqs^2a_0b_1 + 2bka_0b_1 - dka_0b_1 + qs^2a_1b_1 - qs^2b_1^2 - 2kpa_0b_1 + ka_1b_1 - kb_1^2 \\
P(\xi)^2Q(\xi) :& - b^2k^3b_1^2 + 2bk^3a_0b_1 + 2bdk^3b_1^2 + 3dk^3a_0b_1 - d^2k^3b_1^2 \\
& - 2ak^3b_1^2 + bqs^2b_1^2 - 2k^3a_1b_1 - 2k^3b_1^2 - cqs^2a_0b_1 \\
& - dqs^2b_1^2 + bkb_1^2 - cka_0b_1 - dkb_1^2 - kpb_1^2 \\
P(\xi)^3Q(\xi) :& 2bk^3b_1^2 + 2^2k^3a_0b_1 - cdk^3b_1^2 - cqs^2b_1^2 - ckb_1^2 \\
P(\xi)^3Q(\xi)^2 :& 2ak^3b_1^2 + 2^2k^3a_1b_1 \\
P(\xi)^2Q(\xi)^3 :& - 2a^2k^3b_1^2 - 2ak^3a_1b_1 \\
P(\xi)^3 :& 2k^3b_1^2,
\end{aligned}$$

where $Q(x) = e^{\varphi_1(\xi)}, P(x) = e^{\varphi_2(\xi)}$. Using Maple, we collect the twelve systems and select the general system satisfying Eq. (3.14) and Eq. (3.15) as (c, d, k, q, s, a_0, a_1 are arbitrary):

$$\{a = \frac{a_1^2 - 1}{a_0^2}, b = \frac{2a_1}{a_0}, p = \frac{2k^2}{a_0^2}, w = -\frac{qs^2}{k}, b_1 = 0\}.$$

Using $\xi = kx + sy + wt$, the exact solutions are obtained as follows:

$$u_{12} = \frac{\tanh\left(\frac{k^2x+(sy+c)k-qs^2t}{ka_0}\right)+a_1}{\tanh\left(\frac{k^2x+(sy+c)k-qs^2t}{ka_0}\right)a_1+1},\ u_{21} = \frac{\tan\left(\sqrt{-\frac{1}{a_0^2}}(\xi+C)\right)\sqrt{-\frac{1}{a_0^2}}a_0-a_1}{\tan\left(\sqrt{-\frac{1}{a_0^2}}(\xi+C)\right)\sqrt{-\frac{1}{a_0^2}}a_0a_1-1},$$

$$u_{22} = \frac{\sqrt{-\frac{1}{a_0^2}}\tan\left(\sqrt{-\frac{1}{a_0^2}}\frac{k^2x+(sy+C)k-qs^2t}{k}\right)a_0 - a_1}{\sqrt{-\frac{1}{a_0^2}}\tan\left(\sqrt{-\frac{1}{a_0^2}}\frac{k^2x+(sy+C)k-qs^2t}{k}\right)a_0a_1 - 1}.$$

4 Discussion and Conclusion

In conclusion, the softmax method has illustrated an effective and versatile approach to solve the Chafee-Infante equation, producing a variety of solutions including hyperbolic and trigonometric forms. This study provides the potential of softmax-based methods in solving nonlinear evolution equations with applications spanning multiple scientific and engineering fields. However, there are limitations regarding its extension to more complex problems and therefore more research is needed to overcome these challenges.

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