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Polynomial Representation of Cycles in the Edge Corona of Graphs

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Abstract

A graph G is said to be cyclic if it contains a cycle. A cycle generated by $S \subseteq V(G)$, which carries the adjacency property in G, is an induced cycle of G. In this study, we establish the explicit form of the polynomial representation of the number of cycles in the edge corona of two cyclic graphs.

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1 Introduction

The study of graph representations in terms of polynomials captured the interests of discrete mathematicians because of their contributions to Biology, Physics, and Chemistry [1]. Several mathematicians have worked on this area of research in recent years. Villarta, Eballe, and Artes [4] introduced the concept of induced path polynomials and established results for graphs under some binary operations. In 2023, Madalim et al. [3] pioneered a study on induced cycle polynomials of graphs and established some of the properties. Villarta et al. [5] recently established results on the cycle polynomial of the vertex corona of two connected graphs.

In this work, we extend the work in [3] to graphs resulting from the edge corona of two cyclic graphs. The *induced cycle polynomial* of a cyclic graph G is given by

$$\Gamma_{ic}(G;x) = \sum_{i=3}^{c(G)} c_i(G)x^i,$$

where $c_i(G)$ is the number of induced cycles in G of order i and $c(G)$ is the cardinality of a maximum cycle in G [3].

2 Main Results

Consider simple graphs G and H. We define the *edge corona* of G with H, denoted by $G \overset{e}{\circ} H$, as a graph obtained by taking one copy of G and $|E(G)|$ copies of H and adjoining each edge e of G with H^e, a copy of H [2]. The resulting graph has

$$V(G \overset{e}{\circ} H) = \bigcup_{e \in E(G)} V(H^e) \cup V(G),$$

and

$$E(G \overset{e}{\circ} H) = E(G) \cup \bigcup_{e \in E(G)} E(H^e) \cup \bigcup_{uv \in E(G)} \{uw, vw : w \in V(H^{uv})\}.$$

It is clear that $uv \in E(G)$ together with $w \in V(H^{uv})$ will generate a cycle $C_3 = \langle \{u, v, w\} \rangle$. Thus, $G \overset{e}{\circ} H$ is cyclic. Hence, the induced cycle polynomial is well-defined in this graph operation.

Lemma 2.1. *Any induced cycle in $G \overset{e}{\circ} H$ cannot intersect different copies of H.*

Proof. Consider a subgraph G_1 of $G \overset{e}{\circ} H$. Suppose G_1 is a cycle. Then $G_1 = \langle S \rangle$ for some $S \subseteq V(G \overset{e}{\circ} H)$. Suppose there exist $e_1 = u_1v_1$ and $e_2 = u_2v_2$ in $E(G)$ such that $S \cap V(H^{e_1}) \neq \varnothing$ and $S \cap V(H^{e_2}) \neq \varnothing$. Let $w \in V(H^{e_1})$. Then wu_1 and wv_1 are edges in $G \overset{e}{\circ} H$. Hence, either w will make a path via u_1 and proceed to e_2, or w will make a path via v_1 and proceed to e_2. In both cases, the induced subgraph $\langle S \rangle$ does not generate a cycle since different copies of H are disconnected in $G \overset{e}{\circ} H$. □

Now, let us examine the nature and properties of cycles.

Lemma 2.2. *Given cyclic graphs G and H. A subset S of the set of vertices of the edge corona of G with H generates a cycle in $G \overset{e}{\circ} H$ if and only if it meets one of the conditions below:*

- (i) *The subgraph induced by S is a cycle in G.*
- (ii) *The subgraph induced by S is a cycle in H^e for some $e \in E(G)$.*
- (iii) $S = \{u\} \cup S_{H^{e=uv}}$, *where $\langle S_{H^e} \rangle$ is a K_2 in H^e.*
- (iv) $S = \{v\} \cup S_{H^{e=uv}}$, *where $\langle S_{H^e} \rangle$ is a K_2 in H^e.*
- (v) $S = \{u, v, w : uv \in E(G), w \in V(H^{uv})\}$.

Proof. Consider $S \subseteq V(G \overset{e}{\circ} H)$ and suppose that the subgraph induced by S is a cycle. Then (i) is satisfied whenever S is entirely in G. If S and $V(G)$ do not share an element, then $\langle S \rangle$ must be in a copy H^e of H for some $e \in E(G)$, by Lemma 2.1. Hence, (ii) is satisfied. Suppose S and $V(G)$ intersect only at unique vertex u. Let $e \in E(G)$ such that u is incident with e. Then $e = vu$ for some v in the vertex-set of G. Now, for every edge $wz \in E(H^{uv})$, $\{w, u, z\}$ generates a triangle in $G \overset{e}{\circ} H$, and we have (iii). Similarly, $\{w, v, z\}$ generates a C_3 in $G \overset{e}{\circ} H$, which asserts (iv).

Now, suppose $|S \cap V(G)| = 2$. Assume S and $V(G)$ intersect at $\{u, v\}$. Suppose further that $uv \notin E(G)$. Then u is incident with e_1 for some edge e_1 in G. Moreover, v is incident with e_2 in G different from e_1. Hence, $H^{e_1} \neq H^{e_2}$. But $G \overset{e}{\circ} H$ has no edge joining H^{e_1} and H^{e_2}. Hence, S cannot induce a cycle in this case. Thus, $uv \in E(G)$. In this case, for every $w \in H^{uv}$, $\langle \{u, v, w\} \rangle$ is a C_3 in $G \overset{e}{\circ} H$. Moreover, $|S \cap V(H^{uv})| = 1$. Hence, (v) holds.

The converse can be seen easily. □

Finally, we will now present the main result.

Theorem 2.3. *For cyclic graphs G and H,*

$$\begin{aligned}\Gamma_{ic}(G \overset{e}{\circ} H; x) &= \Gamma_{ic}(G; x) + \Gamma_{ic}(H; x)|E(G)| \\ &\quad +(2|E(H)||E(G)| + |E(G)||V(H)|)x^3.\end{aligned}$$

Proof. Lemma 2.2 (i) gives us the first quantity, while the second part of Lemma 2.2 leads to the second quantity. From Lemma 2.2 (iii), we have $|E(G)||E(H)|x^3$. Similarly, Lemma 2.2 (iv) gives $|E(G)||E(H)|x^3$. Finally, Lemma 2.2 (v) contributes $E(G)||V(H)|x^3$. Combining, the desired result follows. □

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