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Solving the Stochastic Delay Differential Equation by using Taylor's and Modified Euler's Methods

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Abstract

In this paper, we solve the Stochastic Delay Differential Equation by using Taylor's and Modified Euler's Methods. Moreover, we derive numerical examples by using Python software.

1 Introduction

We consider the Stochastic Delay Differential Equation (SDDE)

$$dA(t) = f(A(t), A(t-r))dt + g(A(t))dW(t), t \geq 0, t \in [-r, 0] \qquad (1.1)$$

where $f(A(t), A(t-r))$, $g(A(t))$ is the drift and diffusion term, $dW(t)$ is a Weiner process and $r > 0$ is the delay term.

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2 Stochastic Delay Differential Equation

2.1 Taylor's Method

Expanding equation (1.1) by using Taylor's expansion of the first order $A(t_{n+1})$
$A(t_n + 1) = A(t_n) + f(A(t_n), A(t_n - r))h + g(A(t_n))\Delta W_n$,
where h is the step size ($t_{n+1} = t_n + h$) and ΔW_n is the Weiner process increment ($\Delta W_n = W(t_{n+1}) - W(t_n)$). For $t_n > 0$,
$x(t_n - r) \approx A(t_k) + \frac{t_n - r - t_k}{t_{k+1} - t_k}(A(t_{k+1}) - A(t_k))$, where $t_k \leq t_n - r < t_{k+1}$. Then, the discretized equation is $A_{n+1} = A_n + hf(A_n, A_{n-r/h}) + g(A_n)\Delta W_n$

3 Modified Euler Method (MEM)

Expand equation (1.1) by using the MEM and define the domain $t \in [t_0, T]$, step size h, and delay r.
Use the current value $A(t_n)$ and $A(t_n - r)$ to predict $A_p(t_{n+1})$.
$A_p(t_{n+1}) = A(t_n) + hf(A(t_n), A(t_n - r)) + g(A(t_n))\Delta W_n$
where $\Delta W_n = W(t_{n+1}) - W(t_n)$ and $\Delta W_n \approx \mathcal{N}(0, \sqrt{h})$
By using the MEM which averages the drift and stochastic terms, we get
$A(t_{n+1}) = A(t_n) = \frac{h}{2}[f(A(t_n), A(t_n - r)) + f(A_p(t_{n+1}, A(t_{n+1} - r))] + g(A(t_n))\Delta W_n$.
Increment the step size $t_{n+1} = t_n + h$ and repeat for all $n = 0, 1, 2, .., N - 1$ until the result reaches the final time T.

4 Numerical Results

Example 1: $dA(t) = (A(t) - A(t - r))dt + dW(t)$, where $r = 0.1, f(A(t) - A(t - r)) = (A(t) - A(t - r)), g(A(t)) = 1$ (Constant diffusion).
With initial condition $A(t) = 0$, for $t \in [-r, 0]$ and numerically approximating the solution for $t \in [0, T]$ using a small step size $h = 0.01$, time interval $[0, T] = [0, 2]$ and delay $r = 0.1$ by using Taylor's method.

Figure 1 shows the oscillating trajectory of $A(t)$ due to the interaction of drift and the randomness from the Weiner process.
Example 2: $dA(t) = (-A(t) + A(t - r))dt + \sigma A(t)dW(t)$, with $A(t) = e^{-}t$ for $t \in [-r, 0]$ and numerically approximate the solution for delay $r = 0.5$, step size $h = 0.01$, simulation time $T = 2$ and $\sigma = 0.1$ by using MEM.
The MEM efficiently balances accuracy by combining predictions and corrections. Figure 2 shows a smooth but realistic trajectory incorporating both deterministic delay effect and stochastic noise.

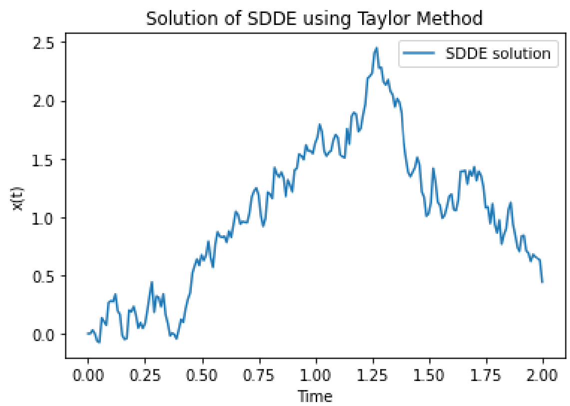

Figure 1: Indicating the numerical solution by using Taylor's method

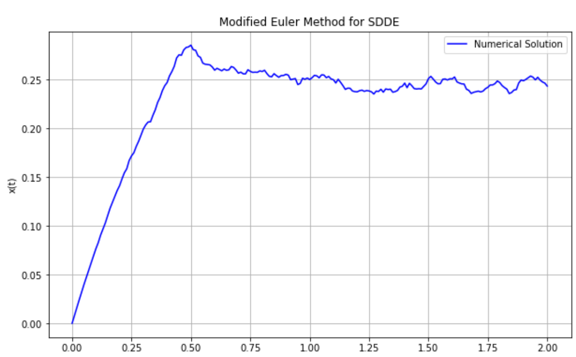

Figure 2: Indicating the numerical solution by using MEM

References

[1] Yang Li et al., "New Order 2.0 Simplified Weak Itô–Taylor Symmetrical Scheme for Stochastic Delay Differential Equations", Symmetry, **16,** no. 6, (2024), 685.

[2] Paweł Przybyłowicz, Yue Wu, Xinheng Xie, "On approximation of solutions of stochastic delay differential equations via randomized Euler scheme", Applied Numerical Mathematics, **197,** (2024), 143–163.

[3] Falah Hassan Sarhan, Hassan Khaleel Ismail, "Approximation Solutions of Backward Fuzzy Stochastic Differential Equations", International Journal of Mathematics and Computer Science, **18,** no. 4, (2023), 647–654.