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# On the impact of exponential functors on some types of continuous mappings

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**Abstract**

The work is devoted to behaviors and some applied fields in a discrete dynamical system in a $S^3$ simplex and the study of the motion of the trajectory using simulation, instead of four-dimensional space drawing in a three-dimensional space.

**Key words and phrases:** Exponential space, permutation degree, almost open mapping, Vietoris topology, pseudo-open mapping.
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# 1 Introduction

Let $X$ be a $T_1$ topological space. The set of all non-empty closed subsets of $X$ is denoted by $\exp X$. The topology on $\exp X$ generated by the basis

$$O\langle U_1, \ldots, U_n\rangle = \{F \in \exp X : F \subset \bigcup_{i=1}^{n} U_i, F \cap U_i \neq \emptyset, \forall i\}$$

is called the Vietoris topology. The space $(\exp X, \text{Vietoris topology})$ is called the hyperspace of $X$.

A mapping $f\colon X \to Y$ is called almost-open if, for each $y \in Y$, there exists $x \in f^{-1}(y)$ such that for every neighborhood $U$ of $x$, the image $f(U)$ is a neighborhood of $y$.

The purpose of this paper is to investigate how the functor $exp_n$ influences almost-open and pseudo-open mappings. Key results presented here extend previous work by Arhangel'skii [1], Kaygorodov [6], Ljuisa [7], Fedorchuk and Filippov [2], Nagata [3], and Koussaour [8] on general mappings, as well as Lin's studies on point-countable covers [4]. For example, we show that:

For the exponential functor $exp_n$, the mapping $exp_n f\colon SP^n X \to SP^n Y$ is almost-open and pseudo-open whenever $f\colon X \to Y$ is almost-open or pseudo-open, respectively.

By establishing these results, we contribute to a broader understanding of functorial transformations in topology and their implications for continuous mappings. This research opens avenues for further exploration of functorial interactions in generalized topological settings.

# 2 Main results

**Theorem 2.1.** *The mapping $exp_n f\colon exp_n X \to exp_n Y$ is almost-open for every almost-open mapping $f\colon X \to Y$ .*

*Proof.* Let $f\colon X \to Y$ be an almost-open and surjective mapping. We take an arbitrary element $F \in \exp_n Y$. Let us say $F = \{y_1, y_2, \ldots, y_n\}$. By the normality of the functor $\exp_n\colon Comp \to Comp$ the mapping $\exp_n f\colon \exp_n X \to \exp_n Y$ is also surjective. We show that this mapping is almost-open.

Since $f\colon X \to Y$ is almost-open, for every $y_i \in F$ there exists $x_i \in X$ such that the image $f(W)$ of an arbitrary neighborhood $W$ of $x_i$ is a neighborhood of $y_i$, as well. Put $C = \{x_1, x_2, \ldots, x_n\}$. Then clearly $(\exp_n f)(C) = f(C) = F$, or equivalently $C \in (\exp_n f)^{-1}(F)$. Now, consider an arbitrary basic neighborhood $\langle U_1, U_2, \ldots, U_n\rangle$ of $C$ in $\exp_n X$, where $U_1, U_2, \ldots, U_n$

are open neighborhoods of $x_1, x_2, \dots, x_n$, respectively. We must show that $F \in Int((\exp_n f)(\langle U_1, U_2, \dots, U_n \rangle))$.

First of all, note that

$$(\exp_n f)(\langle U_1, U_2, \dots, U_n \rangle) = \langle f(U_1), f(U_2), \dots, f(U_n) \rangle.$$

By Lemma 2.3 in [5], we have

$$F \in \langle Int(f(U_1)), Int(f(U_2)), \dots, Int(f(U_n)) \rangle \subset$$

$$\subset Int(\langle f(U_1), f(U_2), \dots, f(U_n) \rangle) = Int((\exp_n f)(\langle U_1, U_2, \dots, U_n \rangle)).$$

Theorem 2.1 follows. □

**Theorem 2.2.** *The mapping $exp_n f \colon exp_n X \to exp_n Y$ is pseudo-open for every pseudo-open mapping $f \colon X \to Y$.*

*Proof.* Let $f \colon X \to Y$ be a pseudo-open and surjective mapping. We take an arbitrary element $F \in \exp_n Y$. Let us say $F = \{y_1, y_2, \dots, y_n\}$. By the normality of the functor $\exp_n \colon Comp \to Comp$, the mapping $\exp_n f \colon \exp_n X \to \exp_n Y$ is also surjective. We show that this mapping is pseudo-open.

Since $f \colon X \to Y$ is pseudo-open, for every $y_i \in F$ there exists $x_i^j \in X$, $j \in A$, such that the image $f(W)$ of an arbitrary neighborhood $W$ of $x_i^j$ is a neighborhood of $y_i$ as well. Put $C^j = \{x_1^j, x_2^j, \dots, x_n^j\}$. Then clearly $(\exp_n f)(C^j) = f(C^j) = F$, or equivalently $C^j \in (\exp_n f)^{-1}(F)$.

Now, consider an arbitrary basic neighborhood $\langle U_1, U_2, \dots, U_n \rangle$ of $C^j$ in $\exp_n X$, where $U_1, U_2, \dots, U_n$ are open neighborhoods of $x_1^j, x_2^j, \dots, x_n^j$, respectively. We must show that $F \in Int((\exp_n f)(\langle U_1, U_2, \dots, U_n \rangle))$.

First of all, note that $(\exp_n f)(\langle U_1, U_2, \dots, U_n \rangle) = \langle f(U_1), f(U_2), \dots, f(U_n) \rangle$.

Consequently,

$$\langle Int(f(U_1)), Int(f(U_2)), \dots, Int(f(U_n)) \rangle \subset Int(\langle f(U_1), f(U_2), \dots, f(U_n) \rangle).$$

Since $y_i \in Int(f(U_i))$ for $i = 1, 2, \dots, n$, we obtain

$$F \in \langle Int(f(U_1)), Int(f(U_2)), \dots, Int(f(U_n)) \rangle \subset Int(\langle f(U_1), f(U_2), \dots, f(U_n) \rangle) =$$

$$= Int((\exp_n f)(\langle U_1, U_2, \dots, U_n \rangle)).$$

Theorem 2.2 follows. □

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