International Journal of Mathematics and Computer Science
Volume **20,** Issue no. 2, (2025), 705–708
DOI: https://doi.org/10.69793/ijmcs/02.2025/kavitha

Derivations on seminearrings

Kedukodi Babushri Srinivas,
Kavitha Koppula, Kuncham Syam Prasad

Department of Mathematics
Manipal Institute of Technology
Manipal Academy of Higher Education
Manipal-576104, Karnataka, India

email: kavitha.koppula@manipal.edu

(Received March 2, 2025, Accepted April 12, 2025,
Published April 13, 2025)

Abstract

In this paper, we define a derivation on an algebraic structure seminearring S. Moreover, we prove that the partial distributive law holds on seminearrings using 3-prime strong ideals of S. Furthermore, we prove that the intersection of 3-prime strong ideals contains the constant elements of any non-empty subset of S.

1 Introduction

A right seminearring S is a generalized algebraic structure of nearring and semiring; i.e., S is a semigroup with respect to both the binary operations '+' and '$\cdot$' and satisfies the right distributive law. The concept of seminearring was introduced by Hoorn and Rootsellar [6] and the kernel of a seminearring homomorphism is considered as the ideal of a seminearring. Subsequently, various authors discussed the results on seminearrings using the generalization of this ideal definition. Later, Koppula, Kedukodi and Kuncham [5] provided an explicit definition of strong ideal of a seminearring. Then they proved the standard isomorphism theorems on seminearrings.

Key words and phrases: Seminearring, Derivation, Strong ideal.
AMS (MOS) Subject Classifications: 16W25, 16Y30, 16Y60.
The corresponding author is Kavitha Koppula.
ISSN 1814-0432, 2025, https://future-in-tech.net

Derivation D on a ring R is an additive endomorphism of R satisfying the rule $D(ab) = aD(b) + D(a)b,\ \forall\ a, b \in R$. Derivations on rings play a significant role in different branches of mathematics and has been extensively studied in the areas of algebraic structures including rings, nearrings, and semirings. In this paper, we prove results of derivations in seminearirngs.

2 Preliminaries

Definition 2.1. A non-empty set S is a right seminearring, if S is a semigroup with respect to addition and multiplication and

1. $(s_2 + s_3)s_1 = s_2s_1 + s_3s_1, \forall s_1, s_2, s_3 \in S$.
2. $0 + s_1 = s_1 + 0 = s_1,\ \forall\ s_1 \in S$.
3. For all $s_1 \in S, 0s_1 = 0$.

Definition 2.2. Let B be any non-empty subset of S. Then, for $s_1, s_2 \in S$, $s_1 \equiv_B s_2$ if and only if there exist $b_1, b_2 \in B$ such that $b_1 + s_1 = b_2 + s_2$.

Throughout this paper, S is considered as a right seminearring.

Definition 2.3. [5] A non-empty subset B of S is a strong ideal of S if

1. $b_1 + b_2 \in B$, for all $b_1, b_2 \in B$.
2. $s_1 + B \subseteq B + s_1, \forall s_1 \in S$.
3. For $s_1, s_2 \in S$, if $s_1 \equiv_B s_2$, then $s_1 \in B + s_2$.
4. $s_1(B + s_2) \subseteq B + s_1s_2$ for all $s_1, s_2 \in S$.
5. $Bs_1 \subseteq B$ for all $s_1 \in S$.

Definition 2.4. A strong ideal B of S is known as a 3-prime strong, for $x_1, x_2 \in S$, if $x_1sx_2 \in B$ for all $s \in S$, then $x_1 \in B$ or $x_2 \in B$.

Definition 2.5. Let R be a ring and $r \in R$. If $rx = r,\ \forall\ x \in R$, then r is said to be a constant element.

For the basic definitions and results on derivations, we refer the reader to [1, 2, 3].

3 Derivations

Definition 3.1. Let S be a seminearring and $d : S \to S$. Then d is said to be a derivation on S, if d is an additive endomorphism and satisfying the condition $d(s_1s_2) = s_1d(s_2) + s_2d(s_1)$ for all $s_1, s_2 \in S$.

Remark 3.2. Let S be a seminearring and $a, b, s_1, s_2 \in S$. We assume that if $a + s_1 + b = a + s_2 + b$, then there exist 3-prime ideals $B_k,\ k = 1, 2, 3, ...$ of S such that $s_1 \equiv_{\cap B_k} s_2$.

The following example shows that the intersection of 3-prime ideals need not be a trivial ideal always.

Example 3.3. Consider $S = \{0, a, b, c\}$ with $+$ and $\cdot$ defined as:

$+$	0	s_1	s_2	s_3
0	0	s_1	s_2	s_3
s_1	s_1	0	s_3	s_2
s_2	s_2	s_3	0	s_1
s_3	s_3	s_2	s_1	0

.	0	s_1	s_2	s_3
0	0	0	0	0
s_1	0	0	0	0
s_2	s_2	s_2	s_2	s_2
s_3	s_3	s_3	s_3	s_3

Here $(S, +, \cdot)$ is a right seminearring and $\{0, s_1\}, \{0, s_1, s_2, s_3\}$ are 3-prime strong ideals of S. Hence the intersection of 3-prime strong ideals is $\{0, s_1\}$. Observe that $B = \{0\}$ is a strong ideal of S, and not a 3-prime as $s_1ss_1 \in B,\ \forall\ s \in S$, but $s_1 \notin B$.

Proposition 3.4. Let d be an arbitrary additive endomorphism on S and $x_1, x_2 \in S$. If $d(x_1x_2) = x_1d(x_2) + d(x_1)x_2$ and $B_k,\ k = 1, 2, 3, ...$ are 3-prime strong ideals of S, then $d(x_1)x_2 + x_1d(x_2) \equiv_{\cap B_k} x_1d(x_2) + d(x_1)x_2$.

Proof. Suppose $d(x_1x_2) = x_1d(x_2) + d(x_1)x_2$. Now, $d((x_1 + x_1)x_2) = (x_1 + x_1)d(x_2) + d(x_1 + x_1)x_2 = x_1d(x_2) + x_1d(x_2) + d(x_1)x_2 + d(x_1)x_2$. Now, $d((x_1 + x_1)x_2) = d(x_1x_2) + d(x_1x_2) = x_1d(x_2) + d(x_1)x_2 + x_1d(x_2) + d(x_1)x_2$. By Remark 3.2, $d(x_1)x_2 + x_1d(x_2) \equiv_{\cap B_k} x_1d(x_2) + d(x_1)x_2$. □

Proposition 3.5. Let d be any arbitrary derivation on the seminearring S and let $B_k,\ k = 1, 2, 3, ...$ be 3-prime strong ideals of S. Then S satisfies the partial distributive law.

Proof. We have $d((s_1s_2)s_3) = s_1s_2d(s_3) + d(s_1s_2)s_3 = s_1s_2d(s_3) + [s_1d(s_2) + d(s_1)s_2]s_3 = s_1s_2d(s_3) + s_1d(s_2)s_3 + d(s_1)s_2s_3$. Now, $d(s_1(s_2s_3)) = s_1d(s_2s_3) + d(s_1)s_2s_3 = s_1(s_2d(s_3) + d(s_2)s_3) + d(s_1)s_2s_3$. By associativity, $d((s_1s_2)s_3) =$

$d(s_1(s_2s_3))$. Then $s_1s_2d(s_3) + s_1d(s_2)s_3 + d(s_1)s_2s_3 = s_1(s_2d(s_3) + d(s_2)s_3) + d(s_1)s_2s_3$. By Remark 3.2, $s_1(s_2d(s_3) + d(s_2)s_3) \equiv_{\cap B_k} s_1s_2d(s_3) + s_1d(s_2)s_3$. □

Proposition 3.6. Let X be any non-empty subset of S, d is any arbitrary derivation on S and $B_k,\ k = 1, 2, ...$ are 3-prime strong ideals of S. Then $\cap B_k$ has the constants of X.

Proof. Let $n \in X/\cap B_k$ be a constant element. Then $d(n) = d(nn) = nd(n) + d(n)n = n + d(n)n$. Let $x \in S$. Then $nxd(n) = nd(n)$. Consider $d(n)x = (n+d(n)n)x = nx+d(n)nx = n+d(n)n = d(n)$. Then $d(n)x = d(n)$ and so $d(n)$ is a constant. Now, $d(n) = n + d(n)n = n + d(n)$. By Remark 3.2, $n \equiv_{\cap B_k} 0$. So, $n \in \cap B_k$ which contradicts $n \notin \cap B_k$. □

Definition 3.7. A seminearirng S is said to be a 2 torsion-free if there exist 3-prime ideals $B_k,\ k = 1, 2, 3, ...$ of S such that $x + x \in \cap B_k$. Then $x \in \cap B_k$.

Example 3.8. The seminearring S from ([5], Example 3.5) is 2-torsion free.

Proposition 3.9. If S is a zero-symmetric and 2-torsion free seminearring, $B_k,\ k = 1, 2, 3, ...$ are 3-prime strong ideals of S and d is any arbitrary derivation on S such that $d^2 \subseteq \cap B_k$, then $d \subseteq \cap B_k$.

Acknowledgment. All authors thank Manipal Institute of Technology, MAHE, Manipal for the support and encouragement.

References

[1] S. Aishwarya, B. S. Kedukodi, S. P. Kuncham, Γ− Derivations in Rings, Palest. J. Math., **12,** no. 1, (2023).

[2] H. E. Bell, G. Mason, On derivations in near-rings, Proceedings of Near-rings and Near-Fields, North-Holland Math. Studies, (1987), 31–35.

[3] H. E. Bell, On derivations in near-rings, II. In Proceedings of Nearrings, Nearfields and K-Loops, Kluwer Acad. Publ., (1997), 191–197.

[4] K. Koppula, B. S. Kedukodi, S. P. Kuncham, On prime strong ideals of a seminearring, Mat. Vesn., **72,** no. 3, (2020), 243–256.

[5] K. Koppula, B. S. Kedukodi, S. P. Kuncham, Congruences in seminear-rings and their correspondence with strong ideals, Alg. Struct., **11,** no. 4, (2024), 255–272.

[6] W. G. Van Hoorn, B. Van Rootselaar, Fundamental notions in the theory of seminearrings, Composit. Math., **18,** (1967), 65–78.