International Journal of Mathematics and Computer Science
Volume **20,** Issue no. 2, (2025), 667–670
DOI: https://doi.org/10.69793/ijmcs/02.2025/bokhodir

A note on isolated points of the compact superkernel of compact spaces

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(Received February 24, 2025, Accepted March 24, 2025,
Published April 3, 2025)

Abstract

In this paper, we study some properties of the set of isolated points of the compact superkernel of compact spaces. For a T_1-space, we prove that $I(\lambda X) \subset \lambda_c X$; i.e., each isolated point in λX must be a compact maximal linked system. Moreover, for an infinite compact space X with $I(X) = \emptyset$, we prove that $I(\lambda X) = \emptyset$; i.e., if X is a pointless space, then its superextension λX is also pointless.

1 Introduction

A system $\xi = \{F_\alpha : \alpha \in A\}$ of closed subsets of a space X is called linked if any two elements of ξ intersect. Any linked system can be upgraded to a maximum linked system (MLS). However, as a rule, such upgrade is not

Key words and phrases: Hewitt-Nachbin number, superextension, isolated point, τ-placedness, compact superkernel.
AMS (MOS) Subject Classifications: 54B20, 54A25.
ISSN 1814-0432, 2025, https://future-in-tech.net

one valued. A linked system of space is MLS if and only if it possesses the following completeness property [1]:
If a closed set $A \subset X$ intersects with every element of ξ, then $A \in \xi$.
We denote by λX the set of all MLS of the space X. For the closed set $A \subset X$, we consider $A^+ = \{\xi \in \lambda X : A \in \xi\}$. For the open set $U \subset X$, we consider $O(U) = \{\xi \in \lambda X : \text{there exists } F \in \xi \text{ such that } F \subset U\}$.

The family of sets of the form $O(U)$ covers the set λX $(O(X) = \lambda X)$. So, it forms an open prebase of the topology on λX. The set λX, equipped with this topology, is called as the superextension of the space X. Let X be topological space and λX be its superextension. MLS $\xi \in \lambda X$ is called compact if it contains at least one compact element and is denoted by CMLS. The space $\lambda_c X = \{\xi \in \lambda X : \xi \; is \; CMLS\}$ is called as compact superkernel (or compact superextension) of the topological space X [2].

The notion of linked systems wins an essential part in the branches of the theory of functors and the theory of cardinal invariants (see for example [3]). Several properties of the theory of functors and the theory of cardinal invariants have been studied in the view of spaces of the linked systems and their cardinal invariants.

At the same time, the importance of this research issue and the meaning of linked systems have led to define new types of linked systems; namely compact maximal linked systems and compact complete linked systems for compact spaces; $\mathcal{N}-$compact nucleus, $\mathcal{N}-$subtle nucleus, $\mathcal{N}_\tau^\varphi-$ nucleus of a topological space X (see for example [4]).

On the other hand, since the spaces of maximal and complete linked systems have their own significant role in the field of Topology, it is absolutely natural to have new related realms of spaces. In Particular, Yuldashev and Mukhamadiev [4, 5], studied the notion of space of complete linked systems as a generalization of the space of maximal linked systems. Note that any maximal linked system is complete, but the converse is not always true. In these works, we can also find an initial study of cardinal and hereditary cardinal invariants of the space of compact complete linked systems. A related study on the space of complete linked systems can be found in [2, 5, 6, 7, 8].

By establishing these results, we contribute to a broader understanding of the kernels of topological spaces in topology and their implications for isolated points. This research opens avenues for further exploration of functorial interactions in generalized topological settings.

2 Main results

Denote by $I(\lambda X)$ the set of all isolated points of the superextension λX. Let's see what connection exists between $I(\lambda X)$ and $\lambda_c X$.

Theorem 2.1. *Let X be a T_1-space. Then, $I(\lambda X) \subset \lambda_c X$; i.e., each isolated point in λX must be a CMLS.*

Proof. Let $\xi \in I(\lambda X)$. We show that $\xi \in \lambda_c X$. Since ξ is an isolated point in λX, there exists a minimal neighborhood $O = O(U_1, U_2, \ldots, U_n)$ points ξ in λX; i.e., $O = \{\xi\}$. Let $S(O) = \{V_1, V_2, \ldots, V_s\}$ be a pairwise trace of the element O in X. In each set $V_i \in S(O)$, we fix one point x_i. Then, we get the set $\sigma = \{x_1, x_2, \ldots, x_s\}$. Let $\Phi_i = \{x_j \in \sigma : \ x_j \in U_i\}$, where $i = 1, 2, \ldots, n$. Obviously, the system $\mu = \{\Phi_1, \Phi_2, \ldots, \Phi_n\}$ is a linked system of closed sets in X. Since $\Phi_i \subset U_i$, where $i = 1, 2, \ldots, n$, we have $\mu^+ \subset O = \{\xi\} \Rightarrow \mu \subset \xi$. Therefore, ξ is a CMLS; i.e., $\xi \in \lambda_c X$. Since ξ is arbitrary in $I(\lambda X)$, we have $I(\lambda X) \subset \lambda_c X$. The proof of Theorem 2.1 is complete. □

Let us denote by $C(X, Y)$ the set of all continuous mappings of the space X into the space Y and by $C(X)$ the set $C(X, R)$ endowed with the topology of pointwise convergence.

It is known (see [2]) that if X is compact, then $q(X) \leq \omega$ ($q(X)$ is the Hewitt-Nachbin number of the space X). So, if X is a normal space, then $q(X) \leq \omega$.

For the Hewitt-Nachbin number of the space $\lambda_c X$, we have the following result

Theorem 2.2. *If X is compact, then $q(\lambda_c X) \leq d(X)$.*

Proof. It is known (see [2]) that $q(X) = t_R(C_p(X)) = t_\theta(C_p(X))$ and $t_\theta \leq t_c(X) \leq d(X)$ for any Tikhonoff space X. So, we have $q(\lambda_c X) = t_R(C_p(\lambda_c X)) = t_\theta(C_p(\lambda_c X)) \leq t_c(\lambda_c X) \leq d(C_p(\lambda_c X)) \leq d(\lambda_c X) = d(X)$. This completes the proof of Theorem 2.2. □

The proof of the following corollaries are clear.

Corollary 2.1. *If X is separable compact, then $q(\lambda_c X) \leq \omega$.*

Corollary 2.2. *If X is separable compact, then the space $\lambda_c X$ is a $Q_\omega-$space.*

References

[1] V. V. Fedorchuk, V. V. Filippov, General topology. Basic constructions. *Moscow: Fizmatlit,* 2006, 333 pages (in Russian).

[2] F. G. Mukhamadiev, On Certain Cardinal Properties of the N_τ^φ-Nucleus of a Space X, *Journal of Mathematical Sciences,* **245,** no. 3, (2020), 411–415.

[3] F. G. Mukhamadiev, Some topological and cardinal properties of the N_τ^φ-nucleus of a space X, *Appl. Gen. Topol.,* **24,** no. 2, (2023), 423–432.

[4] T. K. Yuldashev, F. G. Mukhamadiev, Caliber of space of subtle complete coupled systems, *Lobachevskii J. Math.,* **42,** no. 12, (2021), 3043–3047.

[5] T. K. Yuldashev, F. G. Mukhamadiev, Some Topological and Cardinal Properties of the $\lambda_\tau^\varphi-$Nucleus of a Topological Space X, *Lobachevskii J. Math.,* **44,** no. 4, (2023), 1513–1517.

[6] Ivan Kaygorodov, Abror Khudoyberdiyev, Zarina Shermatova, Transposed Poisson structures on Virasoro-type algebras, Journal of Geometry and Physics, 207, (2025), 105356.

[7] Shokhzod Jumaniyozov, Muhammad Zahid, Mirzabek Alloqulov, Inomjon Ibragimov, Javlon Rayimbaev, Sardor Murodov, Radiative properties and QPOs around charged black hole in Kalb–Ramond gravity, Eur. Phys. J. C., (2025).
https://doi.org/10.1140/epjc/s10052-025-13863-1

[8] S. Rakhmanov, K. Matchonov, H. Yusupov, K. Nasriddinov, D. Matrasulov, Optical high harmonic generation in Dirac materials, Eur. Phys. J. B., (2025).
https://doi.org/10.1140/epjb/s10051-025-00885-7