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Semi-Analytical Solutions of Asset Flow Differential Equations Using an Enhanced DTM-Adomian Polynomials

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Abstract

In this paper, we solve an asset flow differential equations (AFDEs) system, which plays an important role in financial modeling and market dynamics, using an improved differential transform method (DTM). The improved method combines the conventional DTM with Adomian polynomials to enhance the accuracy and efficiency of solving complex nonlinear systems of equations. This hybrid approach overcomes some of the limitations found in traditional numerical methods when dealing with AFDEs. It is faster than numerical methods, easy to compute, and highly accurate. The solution obtained from this method can be expressed as a convergent infinite series, providing a semi-analytical representation of the system's behavior.

Key words and phrases: Asset flow differential equations, differential transform method, Adomian polynomial, approximate analytical solution.
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1 Introduction

It is well known that the financial system is one of the most important tools in the world that affects people worldwide. The financial system is a complicated system that is not easy to understand. One of the good ways to understand financial dynamics is a mathematical model that can use variables and parameters to describe the movements in economic systems. The asset flow differential equations (AFDEs) system is the mathematical model that considers demand, supply, market price, investor preferences, and transition rate [3]. This model had been used to explain the process behind the price change and market behavior [1].

Over the past several decades, researchers have employed various approaches to study asset flow differential equations. Duran and Gaginalp [7] proposed the algorithm that estimated the parameters in the AFDEs that are used to find the daily market price and net asset values by outperforming the random walk model. Prathumwan et al. [10] proposed the fractional-order asset flow differential equations and analyzed the stability of the proposed model.

This paper proposes a novel hybrid approach [8, 5] combining an improved differential transform method (DTM) [9] with Adomian polynomials [2]. The DTM, known for its efficiency in solving differential equations, is enhanced through integration with the Adomian decomposition method, which can handle nonlinear terms. This combination represents the strengths of both methods and reduces the limitations of each method.

2 Asset Flow Differential Equations

The AFDEs was proposed by Gaginalp et al. [3]. Then, it was modified by Duran [6] who considered the dynamics of four states as $x_1(t)$ is the market price of an asset at time t, $x_2(t)$ is the fraction of total asset invested in the asset at time t, $x_3(t), x_4(t)$ are the trend-based component and the value-based component of the investor preference at time t, respectively. The

AFDEs can be written in following form:

$$\begin{aligned}
\frac{dx_1}{dt} &= \delta x_1\Big(\frac{k}{1-k}\frac{1-x_2}{x_2} - 1\Big),\\
\frac{dx_2}{dt} &= k(t)(1-x_2) - (1-k)x_2 + \frac{x_2(1-x_2)}{x_1}\frac{dx_1}{dt},\\
\frac{dx_3}{dt} &= -c_1x_3 + c_1q_1\frac{1}{x_1}\frac{dx_1}{dt},\\
\frac{dx_4}{dt} &= c_2\Big(q_2(\frac{P_a - x_1}{P_a}) - x_4\Big),
\end{aligned} \tag{2.1}$$

with constrains
$x_1 > 0, 0 < x_2 < 1, -1 < x_3 + x_4 < 1, P_a > 0$,
where P_a is the fundamental price, $k(t) = 0.5 + 0.5\tanh(x_3 + x_4)$ is the transition rate. Throughout this paper, all parameters are assumed to be nonnegative.

3 DTM incorperated with Adomian polynomials method

3.1 DTM

For the convenience of readers, the definition and procedure of the DTM [9] are reviewed. The differential transform of the k^{th} differentiable function $f(x)$ at $x = 0$ can be defined by

$$F(k) = \frac{1}{k!}\left(\frac{d^k f(x)}{dx^k}\right)\Bigg|_{x=0}, \tag{3.2}$$

where $F(k)$ is the transformed function and $f(x)$ is the original function. The differential inverse transform of $F(k)$ can be written as

$$f(x) = \sum_{k=0}^{\infty} F(k)x^k, \tag{3.3}$$

which can be approximated by

$$f_N(x) = \sum_{k=0}^{N} F(k)x^k.$$

Using equation (3.2) and equation (3.3), we obtain

$$f(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!} \left(\frac{d^k f(x)}{dx^k} \right) \Big|_{x=0}.$$

3.2 Adomian Polynomials

The Adomian decomposition method (ADM) [2] is a useful tool to solve linear and nonlinear problems which can be represented as the solution in infinite series form. Briefly, ADM for solving a system of equations can be shown as follows:
The solution u_i can be written as

$$u_i = f_i + N_i, \qquad i = 1, 2, \ldots, p,$$

where $f_i(u_1, u_2, \ldots, u_p)$ are nonlinear terms and $N_i(u_1, u_2, \ldots, u_p)$ are other analytic terms.
The terms u_i and f_i can be written in series forms as

$$u_i = \sum_{n=0}^{\infty} u_{i(n)} \lambda^n$$

$$f_i(u_1, \ldots, u_p) = \sum_{n=0}^{\infty} A_{in}(u_{1(0)}, \ldots, u_{1(n)}, \ldots, u_{p(0)}, \ldots, u_{p(n)}) \lambda^n,$$

where A_{in} is the Adomian polynomial which can be represented by

$$A_{in} = \frac{1}{n!} \frac{d^n}{d\lambda^n} f_i \left(\sum_{j=0}^{\infty} u_{1(j)} \lambda^j, \ldots, \sum_{j=0}^{\infty} u_{p(j)} \lambda^j \right) \Big|_{\lambda=0}, \qquad i = 1, 2, \ldots, p.$$

4 Application to AFDEs

Rearranging AFDEs (2.1), we obtain

$$\begin{aligned} \frac{dx_1}{dt} + x_1 \delta \Big(\frac{k}{1-k} \Big) + x_1 \delta - \frac{x_1}{x_2} \delta \Big(\frac{k}{1-k} \Big) &= 0, \\ -x_2(1 - x_2) \frac{dx_1}{dt} + x_1 \frac{dx_2}{dt} - kx_1 + x_2 x_1 &= 0, \\ -c_1 q_1 \frac{dx_1}{dt} + x_1 \frac{dx_3}{dt} + c_1 x_1 x_3 &= 0, \\ \frac{dx_4}{dt} - c_2 \Big(q_2 (\frac{P_a - x_1}{P_a}) - x_4 \Big) &= 0. \end{aligned} \tag{4.4}$$

Define

$$
\begin{aligned}
f_1(x_1,x_2,x_3,x_4) &= -\frac{x_1}{x_2}\delta\left(\frac{k}{1-k}\right),\\
f_2(x_1,x_2,x_3,x_4) &= x_2x_1,\\
f_3(x_1,x_2,x_3,x_4) &= c_1x_1x_3.
\end{aligned} \tag{4.5}
$$

By applying Adomian polynomials, we have the following:
For $n=0$;

$$
\begin{aligned}
A_{1(0)} &= -\delta\left(\frac{k}{1-k}\right)\frac{x_{1(0)}}{x_{2(0)}},\\
A_{2(0)} &= x_{2(0)}x_{1(0)},\\
A_{3(0)} &= c_1x_{1(0)}x_{3(0)}.
\end{aligned} \tag{4.6}
$$

For $n=1$,

$$
\begin{aligned}
A_{1(1)} &= -\delta\left(\frac{k}{1-k}\right)\left(\frac{x_{1(1)}x_{2(0)}-x_{2(1)}x_{1(0)}}{x_{2(0)}^2}\right),\\
A_{2(1)} &= x_{2(0})x_{1(1)}+x_{1(0)}x_{2(1)},\\
A_{3(1)} &= c_1\left(x_{1(1)}x_{3(0)}+x_{3(1)}x_{1(0)}\right).
\end{aligned} \tag{4.7}
$$

For $n=2$,

$$
\begin{aligned}
A_{1(2)} &= -\delta\Big(\frac{k}{1-k}\Big)\Big(\frac{x_{1(2)}}{x_{2(0)}}-\frac{x_{2(2)}x_{1(0)}}{x_{2(0)}^2}+\frac{x_{2(1)}^2x_{1(0)}}{x_{2(0)}^3}-\frac{x_{1(1)}x_{2(1)}}{x_{2(0)}^2}\Big),\\
A_{2(2)} &= x_{1(1)}x_{2(1)}+x_{1(2)}x_{2(0)}+x_{2(2)}x_{1(0)},\\
A_{3(2)} &= c_1\big(x_{1(1)}x_{3(1)}+x_{1(2)}x_{3(0)}+x_{3(2)}x_{1(0)}\big).
\end{aligned} \tag{4.8}
$$

For $n=3$,

$$
\begin{aligned}
A_{1(3)} =& -\delta\big(\frac{k}{1-k}\big)\Bigg[-\frac{x_{1(1)}x_{2(2)}}{x_{2(0)}^2}-\frac{x_{1(2)}x_{2(1)}}{x_{2(0)}^2}+\frac{2x_{1(2)}x_{2(4)}}{x_{2(0)}^3}+\frac{x_{1(3)}}{x_{2(0)}}\\
&\qquad -\frac{x_{2(3)}x_{1(0)}}{x_{2(0)}^2}-\frac{x_{2(1)}^3x_{1(0)}}{x_{2(0)}^4}+\frac{x_{1(1)}x_{2(1)}^2}{x_{2(0)}^3}\Bigg],\\
A_{2(3)} =& x_{1(1)}x_{2(2)}+x_{2(1)}x_{1(2)}+x_{1(3)}x_{2(0)}+x_{2(3)}x_{1(0)},\\
A_{3(3)} =& c_1(x_{1(1)}x_{3(2)}+x_{3(1)}x_{1(2)}+x_{1(3)}x_{3(0)}+x_{3(3)}x_{1(0)}),
\end{aligned} \tag{4.9}
$$

and so on.

Next, we consider $f_i(x_1, x_2, x_3, x_4)$ for $i = 1, 2, 3$ by applying DTM as follows:

$$
\begin{aligned}
F_1(0) &= -\delta\Big(\frac{k}{1-k}\Big)\frac{X_1(0)}{X_2(0)},\\
F_2(0) &= X_2(0)X_1(0),\\
F_3(0) &= c_1X_1(0)X_3(0),\\
F_1(1) &= -\delta\Big(\frac{k}{1-k}\Big)\Big(\frac{X_1(1)X_2(0)-X_2(1)X_1(0)}{X_2(0)^2}\Big),\\
F_2(1) &= X_2(0)X_1(1)+X_1(0)X_2(1),\\
F_3(1) &= c_1\big(X_1(1)X_3(0)+X_3(1)X_1(0)\big),\\
F_1(2) &= -\delta\Big(\frac{k}{1-k}\Big)\Big(\frac{X_1(2)}{X_2(0)}-\frac{X_2(2)X_1(0)}{X_2(0)^2}+\frac{X_2(1)^2X_1(0)}{X_2(0)^3}-\frac{X_1(1)X_2(1)}{X_2(0)^2}\Big),
\end{aligned}
$$

$$
\begin{aligned}
F_2(2) &= X_1(1)X_2(1)+X_1(2)X_2(0)+X_2(2)X_1(0),\\
F_3(2) &= c_1\big(X_1(1)X_3(1)+X_1(2)X_3(0)+X_3(2)X_1(0)\big),\\
F_1(3) &= -\delta\big(\frac{k}{1-k}\big)\Bigg[-\frac{X_1(1)X_2(2)}{X_2(0)^2}-\frac{X_1(2)X_2(1)}{X_2(0)^2}+\frac{2X_2(1)X_2(2)X_1(0)}{X_2(0)^3}+\frac{X_1(3)}{X_2(0)},\\
&\qquad -\frac{X_2(3)X_1(0)}{X_2(0)^2}-\frac{X_2(1)^3X_1(0)}{X_2(0)^4}+\frac{X_1(1)X_2(1)^2}{X_2(0)^3}\Bigg],\\
F_2(3) &= X_1(1)X_2(2)+X_2(1)X_1(2)+X_1(3)X_2(0)+X_2(3)X_1(0),\\
F_3(3) &= c_1(X_1(1)X_3(2)+X_3(1)X_1(2)+X_1(3)X_3(0)+X_3(3)X_1(0)),
\end{aligned}
$$

and so on.

By using the DTM to system (4.4), we have

$$
\begin{aligned}
(u+1)X_1(u+1) + X_1(u)\delta\Big(\frac{k}{1-k}\Big) + X_1(u)\delta + F_1(u) = 0,\\
\sum_{m=0}^{u}(-X_2(m))(u-m+1)X_1(u-m+1)\\
+\sum_{m=0}^{u}\Big(\sum_{m_1=0}^{m} X_2(m_1)X_2(m-m_1)\Big)(u-m+1)X_1(u-m+1)\\
+\sum_{m=0}^{u} X_1(m)(u-m+1)X_2(u-m+1) - kX_1(u) + F_2(u) = 0,\\
-c_1q_1(u+1)X_1(u+1) + \sum_{m=0}^{u} X_1(m)(u-m+1)X_3(u-m+1) + F_3(u) = 0,\\
(u+1)X_4(u+1) - c_2q_2 + \frac{c_2q_2}{P_a}X_2(u) + c_2X_4(u) = 0.
\end{aligned}
$$

From the above system, if $u = 0$, we obtain

$$
\begin{aligned}
X_1(1) &= -X_1(0)\delta\Big(\frac{k}{1-k}\Big) - X_1(0)\delta + \delta\Big(\frac{k}{1-k}\Big)\frac{X_1(0)}{X_2(0)},\\
X_2(1) &= \frac{1}{X_1(0)}\Big(X_2(0)X_1(1) - X_2(0)^2X_1(1) - kX_1(0) - X_2(0)X_1(0)\Big),\\
X_3(1) &= \frac{1}{X_1(0)}\Big(c_1q_1X_{1(1)} - c_1X_1(0)X_3(0)\Big),\\
X_4(1) &= c_2q_2 - \frac{c_2q_2X_2(0)}{P_a} - c_2X_4(0).
\end{aligned}
\tag{4.10}
$$

If $u = 1$, then we have

$$
\begin{aligned}
X_1(2) &= \frac{1}{2}\Big[- X_1(1)\delta\Big(\frac{k}{1-k}\Big) - X_1(1)\delta, \\
&\quad + \delta\Big(\frac{k}{1-k}\Big)\Big(\frac{X_1(1)X_2(0) - X_2(1)X_1(0)}{X_2(0)^2}\Big)\Big], \\
X_2(2) &= \frac{1}{2X_1(0)}\Big(- 2X_2(0)^2X_1(2) - 2X_2(0)X_2(1)X_1(1) \\
&\quad + 2X_1(2)X_2(0) - kX_1(1) - X_2(0)X_1(1) - X_1(0)X_2(1)\Big), \qquad (4.11) \\
X_3(2) &= \frac{1}{2X_1(0)}\Big(2c_1q_1X_1(2) - X_1(1)X_3(1) - c_1\big(X_1(1)X_3(0) \\
&\quad + X_3(1)X_1(0)\big)\Big), \\
X_4(2) &= \frac{1}{2}\Big(c_2q_2 - \frac{c_2q_2X_2(1)}{P_a} - c_2X_4(1)\Big).
\end{aligned}
$$

If $u = 2$, then we get

$$
\begin{aligned}
X_1(3) =&\frac{1}{3}\Big[- X_1(2)\delta\Big(\frac{k}{1-k}\Big) - X_1(2)\delta - \delta\Big(\frac{k}{1-k}\Big) \\
&\Big(\frac{X_1(2)}{X_2(0)} + \frac{X_2(2)X_1(0)}{X_2(0)^2} + \frac{X_2(1)^2X_1(0)}{X_2(0)^3} - \frac{X_1(1)X_2(1)}{X_2(0)^2}\Big),
\end{aligned}
$$

$$
\begin{aligned}
X_2(3) =&\frac{1}{3X_1(0)}\Big(3X_2(0)X_1(3) + X_2(1)X_1(2) - X_1(1)X_2(2) \\
&- 4X_2(0)X_2(1)X_1(2) - 3X_2(0)^2X_1(3) - 2X_2(0)X_2(2)X_1(1) \\
&- X_2(1)^2X_1(1) + kX_1(2) - X_1(1)X_2(1) - X_1(2)X_2(0) - X_2(2)X_1(0)\Big), \\
X_3(3) =&\frac{1}{3X_1(0)}\Big(c_1q_13X_1(3) - 2X_1(1)X_3(2) - X_1(2)X_3(1) \\
&- c_1\big(X_1(1)X_3(1) + X_1(2)X_3(0) + X_3(2)X_1(0)\big)\Big), \\
X_4(3) =&\frac{1}{3}\Big(c_2q_2 - \frac{c_2q_2X_2(2)}{P_a} - c_2X_4(2)\Big).
\end{aligned} \qquad (4.12)
$$

We use a similar argument for $u = 3, 4, \ldots$ to get $X_i(u+1)$ for $i = 1, 2, 3, 4$. Applying DTM to the initial conditions

$$x_{1(0)=2}, x_{2(0)} = 0.1, x_{3(0)} = 0, x_{4(0)} = 0.0015,$$

we obtain

$$X_1(0) = 2, X_2(0) = 0.1, X_3(0) = 0, X_4(0) = 0.0015.$$

Define the values of parameters as

$$\delta = 1, c_1 = 0.087, c_2 = 0.027, q_1 = 0.01, q_2 = 0.04, P_a = 10,$$

and $k = 0.5 + 0.5\tan(X_3 + X_4)$.

The differential inverse transform can be written as

$$x_i(t) = \sum_{m=0}^{\infty} X_i(m)t^m, \qquad i = 1, 2, 3, 4.$$

Then, we obtain the approximated analytical solution of AFDEs as follows:

$$\begin{aligned} x_1(t) &= 2 - 18.0481t + 48.5559t^2 + 11.1081t^3 + \dots \\ x_2(t) &= 0.1 - 0.4114t + 0.2300t^2 + 1.7103t^3 + \dots \\ x_3(t) &= -0.0079t - 0.0140t^2 - 0.0173t^3 + \dots \\ x_4(t) &= 0.0015 + 0.0010t + 5.4833 \times 10^{-4}t^2 + 3.4678 \times 10^{-4}t^3 + \dots \end{aligned} \tag{4.13}$$

5 Conclusions

Using the differential transform method (DTM) in combination with Adomian polynomials, we solved the asset flow differential equations system (AFDEs) which was represented as a mathematical model incorporating factors such as demand, supply, market price, investor preferences, and transition rates. This method provides an approximate analytical solution in the form of a convergent series that is easy to compute. By using this approach, the complicated processes underlying price changes and market behavior can be better understood and modeled. Consequently, this advancement significantly enhances financial modeling and market dynamics analysis, providing a valuable tool for interpreting and predicting market trends.

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