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Bounds of the Edge Covering Number of the Vertex Corona of Graphs

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Abstract

In this work, we establish an upper bound for the edge covering number of graphs resulting from the vertex corona of graphs.

1 Introduction

Covering graphs with specified subgraph structures is a fundamental research area in computer science, complexity theory, combinatorics, and operations

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research [6]. This optimization problem belongs to the class of covering problems and can be solved in polynomial time [4].

In 2004, Artes and Canoy [2] established the relationships between the edge covering number and vertex covering number of graphs. In 2023, some sharp upper bound for the edge covering number of graphs resulting from the Cartesian product and composition of two connected graphs were established [1].

An *edge cover* of a graph G is a collection U_G of edges from G such that every vertex in G is incident with an edge in U_G [3]. The smallest cardinality of an edge cover of G is the *edge covering number* of G and is given by $e_c(G) = \min\{|U_G| : U_G \text{ is an edge cover of } G\}$ [2].

2 Results

For simple graphs G and H, the *vertex corona* of G with H is the graph $G \overset{v}{\circ} H$ whose vertex-set is $V(G \overset{v}{\circ} H) = V(G) \cup \bigcup_{u \in V(G)} V(H^u)$, where H^u is a copy of H attached to $u \in V(G)$ [5]. The edge-set of $G \circ H$ is

$$E(G \overset{v}{\circ} H) = E(G) \cup \bigcup_{u \in V(G)} E(H^u) \cup \bigcup_{u \in V(G)} \{uh^u : h^u \in V(H^u)\}$$

The order of $G \overset{v}{\circ} H$ is $|V(G \overset{v}{\circ} H)| = |V(G)| + |V(G)||V(H)|$ and its size is $|E(G \overset{v}{\circ} H)| = |E(G)| + |V(G)||E(H)| + |V(G)||V(H)|$ [5].

Example 2.1. *Consider $P_3 = [a, b, c]$, $C_3 = [u, v, w, u]$, and $P_3 \overset{v}{\circ} C_3$.*

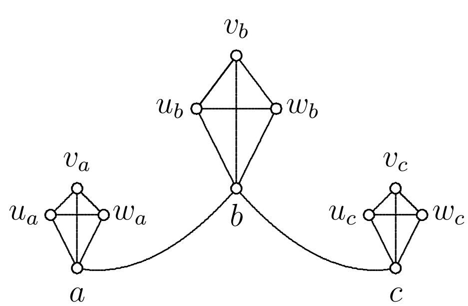

Figure 1: The vertex corona $P_3 \overset{v}{\circ} C_3$.

In the above figure, the set $\{v_a u_a, w_a a, v_b u_b, w_b b, v_c u_c, w_c c\}$ is an edge cover of $P_3 \circ C_3$. In fact, this is one of the minimal edge covers of $P_3 \circ C_3$. Hence, $e_c(P_3 \circ C_3) = 6$.

An upper bound of $G \overset{v}{\circ} H$ is established in the following result.

Theorem 2.2. *Let G be a graph of order m and H be a graph with minimum vertex degree $\delta(H) \geq 2$ having components $H_1, H_2, \ldots, H_k$. Then*

$$e_c(G \overset{v}{\circ} H) \leq m \sum_{i-1}^{k} e_c(H_i) + e_c(G).$$

Proof. For $1 \leq i \leq k$, the component H_i of H can be covered by $e_c(H_i)$ edges. Hence, $\sum_{i-1}^{k} e_c(H_i)$ edges cover each copy of H in $G \overset{v}{\circ} H$. Consequently, $m \sum_{i-1}^{k} e_c(H_i)$ edges covers all copies of H. Now, the subgraph of $G \overset{v}{\circ} H$ that is not covered is G. Thus, adding $e_c(G)$ edges to the $m \sum_{i-1}^{k} e_c(H_i)$ edges, we can cover the vertex corona $G \circ H$. Consequently, $e_c(G \overset{v}{\circ} H) \leq m \sum_{i-1}^{k} e_c(H_i) + e_c(G)$. □

The next result shows the bounds for the vertex corona of G with H, where H has p isolated vertices.

Theorem 2.3. *Let G be a graph of order m and H be a graph with p isolated vertices and having nontrivial components $H_1, H_2, \ldots, H_k$. Then*

$$e_c(G \overset{v}{\circ} H) \leq m \left(p + \sum_{i-1}^{k} e_c(H_i) \right).$$

Proof. For $1 \leq i \leq k$, the component H_i of H can be covered by $e_c(H_i)$ edges. Hence, $p + \sum_{i-1}^{k} e_c(H_i)$ edges cover the vertices of each copy of H. For each vertex $u \in V(G)$, the edge uv where $\deg_H v = 1$ covers u. Hence

$m\left(p+\sum_{i-1}^{k} e_c(H_i)\right)$ edges cover $G \overset{v}{\circ} H$. Accordingly,

$$e_c(G \overset{v}{\circ} H) \leq m\left(p+\sum_{i-1}^{k} e_c(H_i)\right).$$

The proof is complete. □

The following result on the edge covering number of the crown $C \overset{v}{\circ} K_1$ is immediate by considering pendant edges to comprise the edge cover.

Corollary 2.4. *For $n \geq 3$, $e_c(C_n \overset{v}{\circ} K_1) = n$.*

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