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Doubly Isolate Domination in the Corona of Graphs

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Abstract

The doubly dominating set of a graph G is a set of dominating set $S \subseteq V(G)$ such that both $\langle S\rangle$ and $\langle V(G)\setminus S\rangle$ have an isolated vertex. The cardinality of a doubly isolate dominating set with minimum cardinality is called doubly isolate domination number and is denoted by $\gamma_{00}(G)$. In this paper, we characterize the doubly isolate dominating set for the corona and lexicographic product of two graphs.

1 Introduction

For simplicity, a simple graph $G = (V, E)$ with $V(G)$ as vertex set and $E(G)$ as edge set is called a graph G. Two vertices $u, v \in V(G)$ are said to be neighbors (or adjacent) if $uv \in E(G)$. The set of all neighbors of u is denoted by $N(u)$ and it is called the open neighborhood of u; i.e., $N(u) = \{x \in V(G) : xv \in E(G)\}$. The closed neighborhood of u, denoted by $N[u]$, is given by $N[u] = N(u) \cup \{u\}$ [6].

A set $S \subseteq V(G)$ is said to be a *dominating set* of G if $|N_G[S]| = |V(G)|$. The minimum cardinality of the dominating set is called the *domination*

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number, denoted by $\gamma(G)$ [7]. A dominating set $S \subseteq V(G)$ is called an isolate dominating set if the subgraph $\langle S\rangle$ induced by S has an isolated vertex. The minimum cardinality of an isolate dominating set is called the *isolate domination number* and is denoted by $\gamma_0(G)$. This new variant of domination was introduced in 2013 by Hamid and Balamurugan in [8, 9, 10] and further studied in [2]. Moreover, characterization of the isolate domination of some special graphs such as cocktail party, graph, crown graph, gear graph, and jump graph were derived in [3].

An isolate dominating set $S \subseteq V(G)$ is called a *doubly isolate dominating set* of G if $\langle V(G) \setminus S\rangle$ has an isolated vertex. The minimum cardinality of a doubly isolate dominating set is called *doubly isolate domination number*, denoted by $\gamma_{00}(G)$. The concept of doubly isolate domination was introduced by Arriola [1], who characterized both the lower and upper bound of the doubly isolate domination and established some other properties.

2 Doubly Isolate Domination in the Corona of Graphs

The *corona* of two graphs G and H, denoted by $G \circ H$, is the graph obtained by taking one copy of G of order n and n copies of H, and then joining the i-th vertex of G to every vertex in the i-th copy of H. For every $v \in V(G)$, we denote by H^v the copy of H whose vertices are joined or attached to the vertex v. For each $v \in V(G)$, we use the notations $T^v \subseteq V(H^v)$ and $D^v \subseteq V(\langle\{v\}\rangle + H^v)$.

There are few studies about the domination in the corona of graphs. For instance, Arriola and Canoy [4] characterized the doubly connected domination of the corona as well as the $(1, 2)$-domination in the corona of garphs [5].

Lemma 2.1. [1] *Let G be any graph of order $n \geq 3$. If $\Delta(G) = n - 1$ and $\delta(G) \geq 2$, then G has no doubly isolate dominating set.*

For the corona of two graphs G and H, we have:

Proposition 2.2. *If G is trivial and H has $\delta(H) \geq 1$, then $G \circ H$ has no doubly isolate dominating set.*

Proof. Let H be any graph with $\delta(H) \leq 1$ of order n. Then, $\delta(G \circ H) \leq 1$. Since G is trivial, $\Delta(G \circ H) = n - 1$. By Lemma 2.1, $G \circ H$ has no doubly isolate dominating set. $\square$

The next theorem is a characterization of the doubly isolate dominating set for the corona of two graphs.

Theorem 2.3. *Let G be any nontrivial graph and H be any graph. Then a nonempty subset $C \subseteq V(G \circ H)$ is a doubly isolate dominating set of $G \circ H$ if and only if*

$$C = \bigcup_{v \in V(G)} D^v$$

and the following hold:

(i.) $D^u = T^u$ *is a dominating set of* H^u *for each* $u \notin D^u$*;*

(ii.) there exists $x \in V(G)$ *such that* D^x *has an isolated vertex and* $N_G(x) \cap C = \varnothing$ *whenever* $D^x = \{x\}$*;*

(iii.) there exists $y \in V(G)$ *such that* $V(\langle\{y\}\rangle + H^y) \setminus D^y$ *has an isolated vertex and* $N_G(y) \subseteq C$ *whenever* $D^y = H^y$.

Proof. Suppose $C \subseteq V(G \circ H)$ is a doubly isolate dominating set of $G \circ H$. Clearly,

$$C = \bigcup_{v \in V(G)} D^v.$$

Let $u \notin D^u$. Since C is a dominating set of $G \circ H$, $D^u = T^u$ is a dominating set of H^u. Also, since C has an isolated vertex and for any $u, v \in V(G)$, $D^u \cap D^v = \varnothing$, it follows that D^x has an isolated vertex for some $x \in V(G)$. If $D^x = \{x\}$ and since x is an isolated vertex in C, then every element of $N(x)$ does not belong to C; i.e., $N_G(x) \cap C = \varnothing$. Similarly, since $V(G \circ H) \setminus C$ has an isolated vertex, $V(\langle\{y\}\rangle + H^y) \setminus D^y$ has an isolated vertex for some $y \in V(G)$. If $D^y = H^y$ and since y is an isolated vertex in $V(G \circ H) \setminus C$, then every element of $N_G(y)$ must be in C; i.e., $N_G(y) \subseteq C$.

The converse is clear. $\square$

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