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Dynamical Behavior of a Predator-Prey Model Involving Disease Spread, Predator Cannibalism, and Harvesting

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Abstract

This paper is concerned with a model of disease transmission in predator-prey, predator cannibalism, and harvesting. We demonstrate that the model's solution is bounded and positive. Then, we investigate each potential equilibrium point's existence and stability. The local stability of the model around each equilibrium point is studied by the linearizing the system using Jacobian Matrix, while the global stability is performed by defining a Lyapunov function. The model has six equilibria, which are conditionally locally asymptotically stable. Global stability analysis performed shows that all equilibria are conditionally globally asymptotically stable. To support our analytical findings, we also perform numerical simulations.

Key words and phrases: Predator-prey, disease spread, cannibalism, harvesting, equilibria, local stability, global stability.
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1 Introduction

The predator-prey models are an essential topic to study in ecology, as these interactions are crucial to understanding population dynamics and ecosystem balance.

In addition to the ecological model, the epidemiological model is developed to predict the spread of infectious diseases in populations. This model is based on assumptions about the interactions between susceptible and disease prey. In the context of ecology, the epidemiological model is used to understand how diseases can affect prey and predator populations, including [1, 2]. Research in epidemiology is expanding to consider other factors, such as harvesting and cannibalism. The predator-prey model with harvesting was introduced by a few researchers [3, 4, 5] who were showing the effect of selective harvesting on predator-prey systems.

Furthermore, cannibalism is also a phenomenon in predators. Cannibalism occurs when predators consume members of their species in order to environmental pressure. This phenomenon can help stabilize predator populations by reducing competition within the species or controlling the number of predators when prey declines. A number of researchers have examined the mathematical model of cannibalism [6, 7] and have developed models accounting for cannibalism, exploring its effects on predator-prey interactions. Therefore, cannibalism is an important aspect that needs to be considered in predator-prey models.

This study is a combination of the research of the predator-prey model with disease in the prey [1] by including harvesting in prey and predator [**?**], as well as the effect of cannibalism on predators [6]. This model's simulation results and dynamical analysis are expected to provide in-depth insights into complex interactions in predator-prey ecosystems. Previous models [8] addressed disease in prey and predator cannibalism but did not consider harvesting in both species. Our modification presents substantial impacts on the overall dynamics compared to earlier models.

2 Model Development and Basic Properties

Several assumptions led to the formation of this model, which divides prey into susceptible and infected categories. Susceptible prey is a population with an intrinsic growth rate (r) and environmental carrying capacity (k). There is natural mortality in disease prey and predators; predators are cannibalistic, and there is harvesting in both disease prey and predator.

$$\begin{aligned} \frac{dx_s}{dt} &= rx_s\left(1-\frac{x_s+x_i}{k}\right) - a_1x_sy - cx_sx_i - ex_s, \\ \frac{dx_i}{dt} &= cx_sx_i - a_2x_iy - \delta x_i, \\ \frac{dy}{dt} &= -\mu y + d_1x_sy + d_2x_iy - \frac{\beta y^2}{q+y} + \gamma y - hy. \end{aligned} \tag{2.1}$$

where x_s, x_i, y represent susceptible prey, disease prey, predator, r, k, e, a_1, c, a_2, δ, μ, d_1, d_2, β, q, γ, h respectively represent intrinsic per capita growth rate of prey population, carrying capacity, constant harvesting effort in prey, maximum consumption rate of predator population, disease transmission rate in prey population, attack rate of disease prey, natural death rate of disease prey, natural death rate of predator population, conversion rate of susceptible prey, conversion rate of disease prey, harvesting in prey population, maximum consumption rate of predator population, conversion of cannibalism into predator birth, constant harvesting effort in predator, with the following initial conditions $x_s(0) > 0$, $x_i(0) > 0$, $y(0) > 0$.

The basic properties of model 2.1 present theorems that the model's solutions are non-negative and bounded.

Theorem 2.1. *If the initial condition $x_s(0) \geq 0$, $x_i(0) \geq 0$, and $y(0) \geq 0$, then the solution of the model is $x_s(t) \geq 0$, $x_i(t) \geq 0$, and $y(t) \geq 0 \in \mathbb{R}^3_+$ for $t > 0$.*

Proof. If $x_s(0) = 0$, then

$$\begin{aligned} \frac{dx_s}{dt} &= rx_s\left(1-\frac{x_s+x_i}{k}\right) - a_1x_sy - cx_sx_i \\ \ln x_s &= \int\left[r\left(1-\frac{x_s+x_i}{k}\right) - a_1y - cx_i\right]dt \\ x_s &= x_s(0)e^{\int\left[r\left(1-\frac{x_s+x_i}{k}\right)-a_1y-cx_i\right]dt} > 0. \end{aligned}$$

By using the same approach, we get $x_i = x_i(0)e^{\int[cx_s-a_2y-\delta]dt} > 0$ and $y = y(0)e^{\int\left[-\mu+d_1x_s+d_2x_i-\frac{\beta y}{q+y}+\gamma\right]dt} > 0$. But $e^u > 0$. Since $x_s(0) > 0$, $x_i(0) > 0$, and $y(0) > 0$, we have $x_s(t) \geq 0$, $x_i(t) \geq 0$, and $y(t) \geq 0$. Consequently, the solution of the model is always positive. □

Theorem 2.2. *All solutions of model 2.1 in the region $\Omega = \{(x_s + x_i + y) < \frac{w}{\rho} \in \mathbb{R}^3_+\}$ are uniformly bounded.*

Proof. Choose a function defined by $v(t) = x_s(t)+x_i(t)+y(t)$, where $x_s > 0$, $x_i > 0$, $y > 0$ has the first derivative.

$$\begin{aligned}\frac{dv}{dt} + \rho v =& \frac{dx_s}{dt} + \frac{dx_i}{dt} + \frac{dy}{dt} + \rho(x_s + x_i + y)\\ &= rx_s\left(1 - \frac{x_s + x_i}{k}\right) - a_1x_sy - cx_sx_i - ex_s + cx_sx_i - a_2x_iy - \delta x_i\\ &\quad - \mu y - hy + d_1x_sy + d_2x_iy - \frac{\beta y^2}{q+y} + \gamma y + \rho(x_s + x_i + y),\end{aligned}$$

If $d_1 < a_1$ and $d_2 < a_2$, then

$$\frac{dv}{dt} + \rho v < rx_s\left(1 - \frac{x_s}{k}\right) + \rho x_s - (\rho - \delta)x_i + (\gamma - \mu - h + \rho)y.$$

By choosing $\rho < \min\{\delta, \mu + h - \gamma\}$, we have

$$\frac{dv}{dt} + \rho v < rx_s\left(1 - \frac{x_s}{k}\right) + \rho x_s = -\frac{r}{k}\left(x_s - \frac{(r+\rho)k}{2r}\right)^2 + \frac{k}{4r}(r+\rho)^2 \le \frac{k}{4r}(r+\rho)^2.$$

Thus, $\frac{dv}{dt} + \rho v(t) \le w$, where $w = \frac{k}{4r}(r+\rho)^2$. It is easy to show that the solution of the first order differential inequality satisfies $v(t) < \frac{w}{\rho} + (v(0) - \frac{w}{\rho})e^{-\rho t}$. Since $\lim_{t\to\infty} e^{-\rho t} = 0$, it is clear that $v(t)$ is uniformly bounded, which also means that all solutions of 2.1 are uniformly bounded. □

3 Equilibrium Points and Stability

We find an equilibrium point of the equation by equating the following equation. The equilibrium points and their existence conditions are illustrated in Figure 1.

Equilibrium points	**Existence conditions**
The point extiction of three population $E_0 = (0,0,0)$	
The prey extiction equilibrium point $E_1 = \left(0,0,\frac{(\mu-\gamma+h)q}{\gamma-\mu-\beta-h}\right)$	$\gamma-\beta-h<\mu<\gamma-h$ and $\gamma>\beta+h$
The disease prey and predator extiction equilibrium point $E_2 = \left(\frac{rk-ek}{r},0,0\right)$	$r>e$
The disease prey equilibrium point $E_3 = (x_s^*,0,\frac{rk-ek-rx_s^*}{a_1k})$	$x_s^* < k$
The predator extiction equilibrium point $E_4 = \left(\frac{\delta}{c},\frac{rkc-\delta-ekc}{c\,(ck+r)},0\right)$	$r<e+\frac{\delta}{kc}$
The interior equilibrium point $E_5 = (x_s^*, x_i^*, y^*)$ $x_s^* = \frac{-\omega_2\pm\sqrt{\omega_2{}^2-4\omega_1\omega_3}}{2\omega_1}$, $x_i = k_6 - k_7x_s$, dan $y = k_3x_s - k_4$	$x_s > \frac{k_6}{k_7}$ dan $x_s > \frac{k_4}{k_3}$

Figure 1: Equilibrium points and existence conditions

The eigenvalues of the Jacobian matrix are used to determine the linear approximation around each equilibrium point in order to investigate the local stability behavior of the model 2.1.

$$J = \begin{bmatrix} r-\frac{2rx_s}{k}-\frac{rx_i}{k}-a_1y-cx_i-e & \frac{2rx_s}{k}-cx_s & -a_1x_s \\ cx_i & cx_s-a_2y-\delta & -a_2x_i \\ d_1y & d_2y & -\mu+d_1x_s+d_2x_i+\gamma-h-\frac{2\beta y(q+y)-\beta y^2}{(q+y)^2} \end{bmatrix}. \tag{3.2}$$

Additionally, the following theorem yields the result.

Theorem 3.1. *The following Theorem describes the model's equilibrium points' local stability.*

i. *The equilibrium* $E_0 = (0,0,0)$ *is locally asymptotically stable if* $r < e$ *and* $\gamma > \mu + h$.

ii. $E_1 = \left(0,0,\frac{(\mu-\gamma-h)q}{\gamma-\mu-\beta-h}\right)$ *is locally asymptotically stable if* $r < a_1\frac{(\mu-\gamma+h)q}{\gamma-\mu-\beta-h} + e$.

iii. $E_2 = \left(\frac{(r-e)k}{r},0,0\right)$ *is locally asymptotically stable if* $k < \min\left\{\frac{\delta r}{c(r-e)}, \frac{r(\mu+h-\gamma)}{d_1(r-e)}\right\}$ *and* $r > 2e$.

iv. $E_3 = \left(x_s^*,0,\frac{rk-ek-rx_s^*}{a_1k}\right)$ *is locally asymptotically stable.*

v. $E_4 = \left(\frac{\delta}{c}, \frac{rkc-\delta-ekc}{c(ck+r)}, 0\right)$ *is locally asymptotically stable*

vi. $E_5 = (x_s^*, x_i^*, y^*)$ *is locally asymptotically stable if* $\rho_1 > 0$, $\rho_3 > 0$, *and* $\rho_1\rho_2 - \rho_3 > 0$.

Proof. i. The eigenvalues of the Jacobian matrix E_0 are $\lambda_1 = r - e$, $\lambda_2 = -\delta < 0$, and $\lambda_3 = \gamma - \mu - h$. Then E_0 is locally asymptotically stable if $r > e$, and $\gamma > \mu + h$.

ii. The eigenvalues of the Jacobian matrix E_1 are $\lambda_1 = r - a_1\frac{(\mu-\gamma+h)q}{\gamma-\mu-\beta-h}$, $\lambda_2 = -a_2\frac{(\mu-\gamma+h)q}{\gamma-\mu-\beta-h} - \delta < 0$, and $\lambda_3 = -\frac{\beta q y^*}{(q+y^*)^2} < 0$. So, E_1 is locally asymptotically stable if $r < a_1\frac{(\mu-\gamma-h)q}{\gamma-\mu-\beta-h}$.

iii. The eigenvalues of the Jacobian matrix E_2 are $\lambda_1 = 2e - r$, $\lambda_2 = ck - \delta - \frac{cek}{r} < 0$, and $\lambda_3 = \gamma + d_1k - \mu - h - \frac{d_1ek}{r}$. So E_2 is locally asymptotically stable if $r > e$, and $k < \min\left\{\frac{\delta r}{c(r-e)}, \frac{r(\mu+h-\gamma)}{d_1(r-e)}\right\}$.

iv. The eigenvalues of $E_3 = (x_s^*, 0, \frac{rk-ek-rx_s^*}{a_1k})$ are $\lambda_1 = cx_s^* - \delta - a_2\frac{(rk-ek-rx_s^*)}{a_1k} < 0$ and $\lambda_{2,3}$ fulfilled $J_1(E_3) = \begin{vmatrix} -\frac{rx_s^*}{k} & -a_1x_s^* \\ d_1y^* & -\frac{\beta q y^*}{(q+y^*)^2} \end{vmatrix}$. Then, we get $\det(J_1(E_3)) > 0$ and $\mathrm{Trace}(J_1(E_3)) < 0$, so E_3 is locally asymptotically stable.

v. The eigenvalues from E_4 are $\lambda_1 = \gamma - \mu - h + d_1\frac{\delta}{c} + d_2\frac{rkc-r\delta-ekc}{ck+r} < 0$, if $\gamma + d_1\frac{\delta}{c} + d_2\frac{rkc-r\delta-ekc}{ck+r} < \mu + h$. $\lambda_{2,3}$ fulfilled characteristic equation $J_1(E_4) = \begin{vmatrix} \frac{-r\delta}{ck} & \frac{-\delta(r+ck)}{ck} \\ \frac{rkc-ekc-r\delta}{ck+r} & 0 \end{vmatrix}$, then we get $\det(J_1(E_4)) = \frac{(ekc+r\delta-rkc)(\delta(2r-ck))}{(ck+r)ck} > 0$ if $\frac{ck}{2} > r$ and $\mathrm{trace}(J_1(E_4)) < 0$. So, E_4 locally asymptotically stable.

vi. By substituting $E_5 = (x_s^*, x_i^*, y^*)$ to the the Jacobian matrix 3.2, we get

$$J(E_5) = \begin{bmatrix} B_{11} - \lambda & B_{12} & B_{13} \\ B_{21} & B_{22} - \lambda & B_{23} \\ B_{31} & B_{32} & B_{33} - \lambda \end{bmatrix}, \tag{3.3}$$

with

$B_{11}(E_5) = -\frac{rx_s^*}{k}$, $B_{12}(E_5) = \frac{rx_s^*}{k} - cx_s^*$, $B_{13}(E_5) = a_1x_s^*$, $B_{21}(E_5) = cx_i^*$, $B_{22}(E_5) = cx_s^* - a_2y^* - \delta$, $A_{23}(E_5) = -a_2x_i^*$, $B_{31}(E_5) = d_1y^*$, $B_{32}(E_5) = d_2y^*$, $B_{33}(E_5) = -\frac{\beta q y^*}{(q+y^*)^2}$.

The characteristic equation corresponds to the Jacobian matrix.

$\lambda^3 + \rho_1\lambda^2 + \rho_2\lambda + \rho_3 = 0$, with $\rho_1 = -\tilde{\rho}_1$, $\rho_2 = -\tilde{\rho}_2$, and $\rho_3 = -\tilde{\rho}_3$

$$\rho_1 = B_{11} + B_{22} + B_{33},$$
$$\rho_2 = B_{13}B_{31} + B_{12}B_{21} + B_{23}B_{32} - B_{11}B_{22} - B_{22}B_{33} - B_{11}B_{33},$$
$$\rho_3 = B_{11}B_{22}B_{33} + B_{12}B_{23}B_{31} + B_{13}B_{21}B_{32} - B_{11}B_{23}B_{32} - B_{22}B_{13}B_{31} - B_{33}B_{12}B_{21}.$$

E_5 is locally asymptotically stable according to the Hurwitz criterion $\rho_1 > 0$, $\rho_1\rho_2 > \rho_3$, and $\rho_3 > 0$, and unstable if neither of these conditions is met.

We shall prove the global stability of the model by constructing a suitable Lyapunov function. □

Theorem 3.2. *The coexistence equilibrium* $E_5 = (x_s^*, x_i^*, y^*)$ *of the model 2.1 is globally asymptotically stable if* $\frac{l_1 r}{K} > \frac{(l_1a_1+l_3d_1)^2}{2}$ *and* $\frac{l\beta q}{(q+y^*)(q+y)} > \frac{(l_1a_1+l_3d_1)^2}{2}$.

Proof. The theorem can be proven by defining a Lyapunov function $V_3 = V_{31} + V_{32} + V_{33}$, where

$$V_3(x_s, x_i, y) = l_1\left(x_s - x_s^* - x_s^* \ln\frac{x_s}{x_s^*}\right) + l_2\left(x_i - x_i^* - x_i^* \ln\frac{x_i}{x_i^*}\right) + l_3\left(y - y^* - y^* \ln\frac{y}{y^*}\right).$$

So, we get the derivative of $V_3(x_s, x_i, y)$.

$$\begin{aligned}\frac{dV_3}{dt} =& -\frac{l_1 r}{k}(x_s - x_s^*)^2 - \frac{l_3\beta q(y-y^*)^2}{(q+y^*)(q+y)} - \left(\frac{l_1 r}{k} + l_1 c - l_2 c\right)(x_s - x_s^*)(x_i - x_i^*)\\ &+ (l_1a_1 - l_3d_1)(x_s - x_s^*)(y - y^*) - (l_2a_2 - l_3d_2)(x_i - x_i^*)(y - y^*).\end{aligned}$$

If $l_1 = 1$, $l_2 = \frac{r}{kc} + 1$, $l_3 = \left(\frac{r}{kc} + 1\right)\frac{a_2}{d_2}$, then

$$\begin{aligned}\frac{dV_3}{dt} \leq& -\frac{l_1 r}{k}(x_s - x_s^*)^2 - \frac{l_3\beta q(y-y^*)^2}{(q+y^*)(q+y)} + \frac{(l_1a_1 + l_3d_1)^2}{2}(x_s - x_s^*)^2 + \frac{(l_1a_1 + l_3d_1)^2}{2}(y - y^*)^2.\\ =& -\left[\frac{l_1 r}{k} - \frac{(l_1a_1 + l_3d_1)^2}{2}\right](x_s - x_s^*)^2 - \left[\frac{l_3\beta q}{(q+y^*)(q+y)} - \frac{(l_1a_1 + l_3d_1)^2}{2}\right](y - y^*)^2.\end{aligned}$$

So, $\frac{dV_3}{dt} \leq 0$ if $\frac{l_1 r}{k} > \frac{(l_1a_1+l_3d_1)^2}{2}$ and $\frac{l_3\beta q}{(q+y^*)(q+y)} > \frac{(l_1a_1+l_3d_1)^2}{2}$. □

4 Numerical Simulation

Model 2.1 is numerically simulated in this section using Runge-Kutta 4th order. Confirmation of the dynamics analysis conclusions and the existence

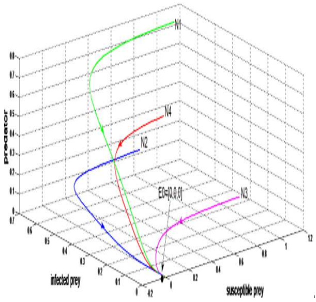

Figure 2: Numerical Simulation of E_0

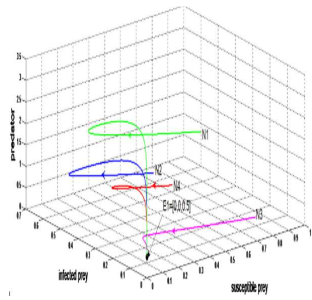

Figure 3: Numerical Simulation of E_1

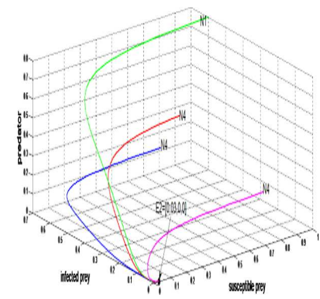

Figure 4: Numerical Simulation of E_2

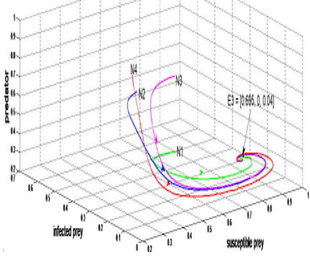

Figure 5: Numerical Simulation of E_3

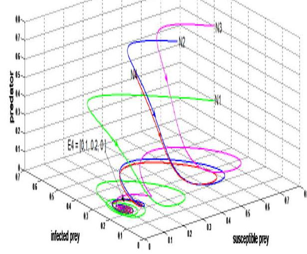

Figure 6: Numerical Simulation of E_4

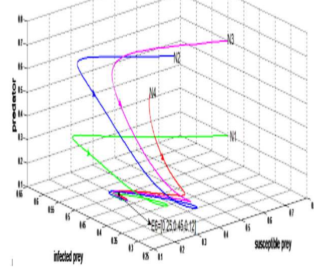

Figure 7: Numerical Simulation of E_5

of equilibrium points are the goals of the numerical simulations. Table 1 presents a possible selection of parameter values because there is currently no available data pertaining to our suggested model. Table 1 shows model 2.1 numerical simulations with different parameters.

Table 1: Parameter Value

Parameter	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5	Simulation 6
r	0.5	0.2	0.5	1	0.4	1.5
K	0.05	1	0.05	2	2	1.5
α_1	0.3	0.5	0.3	1	0.3	0.5
e	0.8	0.2	0.2	0.2	0.1	0.5
c	0.5	0.5	0.5	0.5	1	0.5
α_2	1	0.2	1	1	1	0.2
δ	0.2	0.1	0.2	0.3	0.1	0.1
d_1	0.5	0.2	0.5	0.5	0.5	0.3
d_2	0.6	1	1	0.6	0.6	1
β	0.1	0.2	0.2	0.1	0.1	0.3
q	0.5	0.5	0.5	0.5	0.5	1
γ	0.5	0.5	0.5	0.5	0.5	1
μ	0.8	0.3	0.8	0.2	0.2	1
h	0.6	0.1	0.6	0.6	0.6	0.5

Simulation E_0 was carried out by choosing the parameters in Table 1 simulation 1, with stability conditions $r = 0.5 < e = 0.8$ and $\gamma = 0.5 <$

$\mu + h = 1.4$. Furthermore, by using the parameters in Table 1 simulation 2, $E_1 = [0, 0, 0.5]$ exists, because existence condition are met; i.e., $\gamma - \beta - h = 0.2 < \mu = 0.3 < \gamma - h = 0.4$. By choosing an initial value $N_1 = [1, 0.6, 0.8]$, $N_2 = [0.7, 0.6, 0.2]$, $N_3 = [0.8, 0.1, 0.2]$, $N_4 = [0.6, 0.4, 0.5]$, we get E_1 is locally asymptotically stable, with stability condition $r = 0.2 < a_1 \left(\frac{(\mu-\gamma-h)q}{\gamma-\mu-\beta-h}\right) + e = 1.7$. This result is consistent with the eigenvalues of the Jacobian matrix showing that E_1 is locally asymptotically stable.

Numerical simulation E_2 was carried out by using the parameters in Table 1, simulation 3. $E_2 = (0.03, 0, 0)$ exists. The local stability condition fulfilled in $r = 0.5 > 2e = 0.4$ and $k = 0.05 < \min\left\{\frac{\delta r}{c(r-e)} = 2.667, \frac{r(\mu+h-\gamma)}{d_1(r-e)} = 3\right\}$. This figure illustrates that disease prey and predator are on the verge of extinction.

Numerical simulation in Figure 6 was carried out by using the parameters in Table 1 simulation 5. $E_4 = \left(\frac{\delta}{c}, \frac{rkc-\delta-ekc}{c(ck+r)}, 0\right)$ exists and the stability condition is fulfilled. Numerical simulation E_5 was carried out by using the parameters in Table 1 simulation 6. $E_5 = (0.3, 0.2, 0.1)$ exists. The local stability condition is fulfilled.

5 Conclusion

We have developed a model that explains how three species interact: susceptible prey and harvesting, diseased prey and predator cannibalism and harvesting. We demonstrate that every model solution is limited and non-negative. The dynamical behavior of the model, particularly the behavior of solutions around the equilibrium point, has been demonstrated. In this model, there are six equilibria. All equilibria are conditionally stable and locally asymptotically stable. Runge Kutta 4th order numerical simulations provide the basis for all local stability. Additionally, by establishing Lyapunov functions, we have demonstrated the global stability of the model of specific equilibrium locations.

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