International Journal of Mathematics and Computer Science
Volume **20,** Issue no. 1, (2025), 321–332
DOI: https://doi.org/10.69793/ijmcs/01.2025/sasalman

Magic Cube Constructions

Amal A. Ibrahim[1], Shatha A. Salman[2], Viean A. Zain Al-Aabdeen[3], Areej M. Abduldaim[4]

[1]Department of Mathematics
College of Education
Al-Mustansiriyah University
Baghdad, Iraq

[2,4]Mathematics and Computer Applications
Department of Applied Sciences
University of Technology
Baghdad, Iraq

[3]Control and Systems Engineering
University of Technology
Baghdad, Iraq

email: shatha.a.salman@uotechnology.edu.iq,
amal.ibrahim@uomustansiriyah.edu.iq,
viean.a.zainalaabdeen@uotechnology.edu.iq,
Areej.M.Abduldaim@uotechnology.edu.iq

(Received October 8, 2024, Accepted November 9, 2024,
Published November 13, 2024)

Abstract

Magic squares and all numbers games, in general, are of special interest because of their mysterious attraction. They seem to reveal some sort of underlying intellect that, through a premeditated strategy, creates the illusion of purposeful design a phenomenon that has a near natural equivalent. Magic squares are not immediately useful, yet they have always had a significant impact on intelligent individuals.

Key words and phrases: Construction, Magic Square, Magic Cube, Algorithm, Applications.
AMS (MOS) Subject Classifications: 93A30, 49K15.
ISSN 1814-0432, 2025, https://future-in-tech.net

They seem to hold a very valuable lesson because they are a tangible example of mathematics' symmetry, shedding light on the order that permeates the universe in both the infinitesimally small interactions between atoms and the immeasurable domain of the starry heavens. magic cube is 3-dimensional matrix with positive numbers where the sums of every row, column, and diagonals are the same number. In this article, we introduce a method for magic cube constructions.

1 Introduction

In mathematics, magic squares have attracted the attention of many researchers from around the world. To study and understand the properties of Magic squares has become a familiar concept. Appearing for the first time in Chinese legend, magic squares have attracted the attention of many people in different civilizations over decades, including ancient Babylonian civilization and India. In the middle ages, the magic square concept was transferred to Europe by the Arabs [1,2]. Originally, a 2×2 matrix, the question was on whether it can be developed into a 3×3 matrix? Kochansky [3] was the person who developed the magic square into a magic cube and it was pointed out for the first time at the end of the 19th century in a letter of Pierre de Fermat in 1640. The magic cube is one of applications of Combinatorics which have a lot of applications in other fields that [4-10].
The construction of a magic cube is more difficult compared to a magic square because the relationship between different numbers is more complex than those in a magic square. There have been many attempts to find a way to construct a magic cube. Stank [11] developed an algorithm to construct a magic cube by using ultra magic square. He also generated a magic cube by using the same method in [11] but instead of using ultra magic as a base a Duerer's matrix. Trenkler [3] introduced a new algorithm to construct a magic cube. Other researchers [12, 13, 14, 15] used the folded magic square to construct magic cubes.
In this paper, we provide steps for magic cube construction by using the formula in [16]. For many applications that use magic squares and cubes, we refer the reader to [17-19].

2 Preliminaries

In this section, we introduce some definitions related to our work.

Definition 2.1. [20] *A Latin square is a square matrix of order n in which the entries are arrays with symbols or numbers where each symbol or number occurs exactly once in every row and once in every column. Two Latin squares are called orthogonal if when one matrix is superimposed upon the other, then the entries of the resulting square will be an ordered pair of numbers such that every ordered pair occurs only once (Figures 1 and 2).*

a	b	c
b	c	a
c	a	b

Figure 1: Latin square of order 3

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

Figure 2: Orthogonal Latin squares

Definition 2.2. [21] *A magic square is said to be an ultra magic square if it satisfies the conditions that it is pandiagonal and associative Figure 3).*

$$\begin{pmatrix} 1 & 14 & 4 & 15 \\ 8 & 11 & 5 & 10 \\ 13 & 2 & 16 & 3 \\ 12 & 7 & 9 & 6 \end{pmatrix}$$

Figure 3: ultra magic square

8	1	6
3	5	7
4	9	2

Figure 4: natural magic square

4	9	2
3	5	7
8	1	6

Figure 5: associative magic square

Definition 2.3. [22], [23] *A natural magic square is $n \times n$ matrix that contains positive entries $1, 2, ..., n^2$arranged in such a way that the sum of every row, every column, and the sum of entries on the four diagonals have the same value which is called the magic constant $M_n = \frac{n}{2}(n^2 + 1)$. An associative magic square is a natural magic square where every pair of numbers symmetrically opposite the center, sum to $n^2 + 1$ (Figures 4 and 5).*

Definition 2.4. [23] *A pandiagonal magic square is a natural magic square where every pair of numbers symmetrically opposite the center, sum to n^2+1 (Figure 6).*

1	15	24	8	17
23	7	16	5	14
20	4	13	22	6
12	21	10	19	3
9	18	2	11	25

Figure 6: pandiagonal magic square.

Definition 2.5. [24] *A 2-dimension matrix is called a magic square of square if the entries of the matrix are square values (Figure 7).*

68^2	29^2	41^2	37^2
17^2	31^2	79^2	32^2
59^2	28^2	23^2	61^2
11^2	77^2	8^2	49^2

Figure 7: magic square of squares

3 Methods to Construct a Magic Cube

3.1 First Constructions Method

The construction of a magic cube depends on the formula
$q(i,j,k) = [s(i,r(j,k)-1].n^2 + m(i,s(j,k))\forall i,j,k \in N$
This method can be used to generate a magic cube of odd and even numbers.
The steps for creating every layer of the magic cube will be as follows:
Step 1: Find all the coordinates of the first layer. This means that the value of i in the first layer will be 1 and the only values that will change are j and k as follows:

$q(1,j,k) = [s(i,r(j,k)-1].n^2 + m(i,s(j,k)) \qquad \forall i,j,k \in N$

Step 2. For the second layer we will find the coordinates only for the border of the layer. This means finding the coordinates of the first and the last row and column such as:

1. The entries of the first row can be found by taking the values of i and j as 2 and 1 respectively and the change will be only in the k value
$q(2,1,k) = [s(i,r(j,k)-1].n^2 + m(i,s(j,k)) \qquad \forall, k \in N$
2. The entries of the last row can be found such as in the previous point and the change will be by taking the value of j as n and the rest is as mentioned precisely.
$q(2,n,k) = [s(i,r(j,k)-1].n^2 + m(i,s(j,k)) \qquad \forall k \in N$
3. The entries of the first column can be found by taking the values of i and k as 2 and 1 respectively and the change will be only in the j value.
$q(2,j,1) = [s(i,r(j,k)-1].n^2 + m(i,s(j,k)) \qquad \forall j \in N$
4. The entries of the last column can be found such as in the previous point and the change will be by taking the value of k as n and the rest is as

mentioned precisely.
$q(2,j,n) = [s(i, r(j,k) - 1].n^2 + m(i, s(j,k)) \qquad \forall j \in N$
Step 3. The third layer can be created such as in the previous step this method uses all the layers between the first and the last layer.
Step 4. The last layer is constructed by finding all the coordinates as follows:
$q(n,j,k) = [s(i, r(j,k) - 1].n^2 + m(i, s(j,k)) \qquad \forall j,k \in N$
This means the value of i will be n and the values of j and k will change. In this way, we will obtain magic cube where the sum of the rows and columns equal to the magic cube constant.
Example (1): Now, the construction of a magic cube of order 5 is given using the following two orthogonal Latin square and ultra-magic square:

$$\text{R}=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix} \; S = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \end{pmatrix}$$
$$\text{m}=\begin{pmatrix} 1 & 15 & 22 & 18 & 9 \\ 23 & 19 & 6 & 5 & 12 \\ 10 & 2 & 13 & 24 & 16 \\ 14 & 21 & 20 & 7 & 3 \\ 17 & 8 & 4 & 11 & 25 \end{pmatrix}$$

Solution:
$q(i,j,k) = [s(i, r(j,k) - 1].n^2 + m(i, s(j,k))$
$q(1,1,1) = [s(1, r(1,1)) - 1].5^2 + m(1, s(1,1))$
$= [s(1,1) - 1].25 + m(1,1)$
q(1,1,1) $= 1$

Step 2. The entries of the first row are
$q(2,1,k) \qquad \forall k \in N$
$q(2,1,1) = 48 \qquad \forall k = 1, \cdots, 5$
The entries in the last row are
$q(2,n,k) \qquad \forall k \in N$
$q(2,5,1) = 62 \qquad \forall k = 1, \cdots, 5 and j = 5$ and so on
The entries in the first column are
$q(2,j,1) \qquad \forall j \in N$
$q(2,1,1) = 48 \qquad \forall j = 1, \cdots, 5$ and so on.
The entries in the last column are $q(2,j,n) \qquad \forall j \in N$

$q(2,1,5) = 12 \qquad \forall j = 1, \cdots, 5 \qquad and \qquad k = 5$
and so on. Then the second layer will be as follows:

48	69	81	105	12
19				123
106				94
80				56
62	98	119	6	30

The third and fourth layers can be calculated in the same way as in step two. Then the third layer will be:

60	77	113	24	41
27				10
13				102
124				88
91	110	2	38	74

The fourth layer is:

89	121	20	32	53
71				39
45				21
7				120
103	14	46	70	82

In the same way as in step one, we can calculate all the entries of the last layer:

117	8	29	61	100
83	104	11	50	67
54	86	125	17	33
36	75	92	108	4
25	42	58	79	111

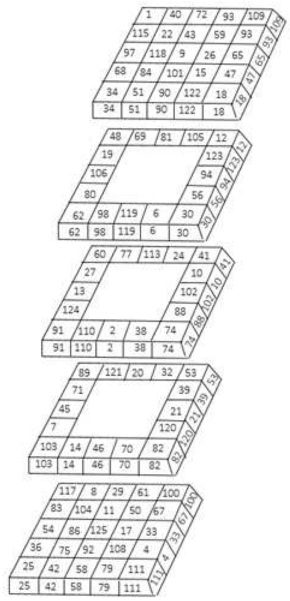


Figure 8: Magic cube of order 5

3.2 Second construction method

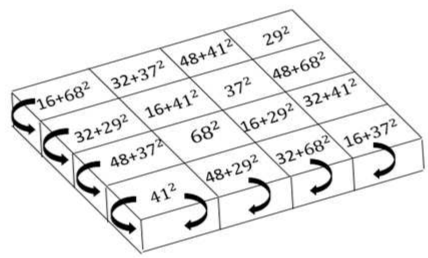
In this method, we will use the same formula as in the previous method but instead of using ultra magic square, we will use a magic square of square as a base for the construction. The steps for the construction are similar to the steps mentioned in the first method.

Example (2): Construction magic cube of order 4×4 by using the following two orthogonal Latin squares and magic square of square

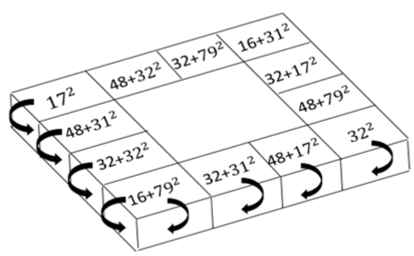
$$\mathrm{R}=\begin{pmatrix} 4 & 3 & 2 & 1 \\ 3 & 4 & 1 & 2 \\ 2 & 1 & 4 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix} \quad S=\begin{pmatrix} 1 & 4 & 3 & 2 \\ 2 & 3 & 4 & 1 \\ 4 & 1 & 2 & 3 \\ 3 & 2 & 1 & 4 \end{pmatrix}$$

$$\mathrm{m}=\begin{pmatrix} 68^2 & 29^2 & 41^2 & 37^2 \\ 17^2 & 31^2 & 79^2 & 32^2 \\ 59^2 & 28^2 & 23^2 & 61^2 \\ 11^2 & 77^2 & 8^2 & 49^2 \end{pmatrix}$$

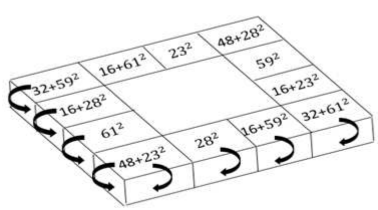
Solution:
Step one: Calculate the first layer:



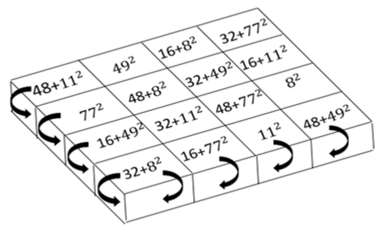
Step two: Calculate the coordinates of the second layer:



Step three: Compute the coordinates of the third layer:



Step four: Find the coordinates of all cells in the fourth layer and the last:



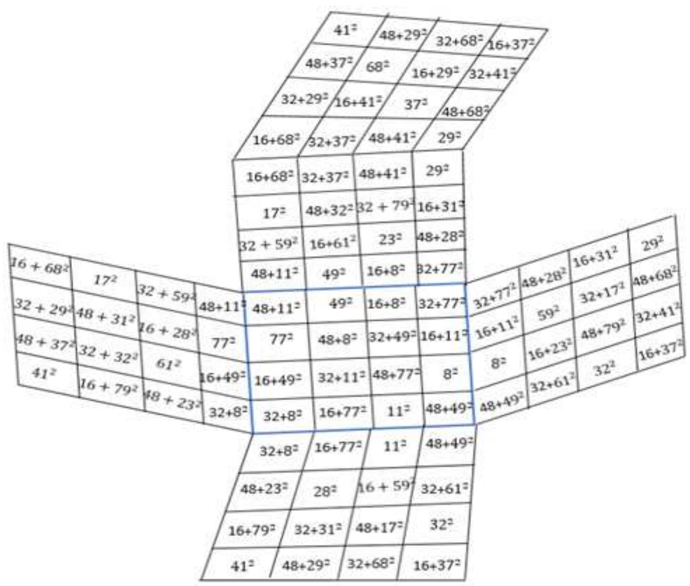


Figure 9: Magic cube of order 4

References

[1] C. Schuyler, Islamic and Indian magic squares part I and II, History of religions, **8,** (1969), 181–209, 271–299.

[2] S.V. Cammann, Old Chinese magic squares, Verlag für Recht und Gesellschaft, (1962).

[3] M. Trenkler, Connections-magic squares, cubes and matchings, Applications of Modern Mathematical Methods, Ljubljana, (2001), 191199.

[4] Iman S. Hadeed, Shatha A. Salman, Adaya J. Hayder, The General Formula for Ehrhart Series of a Special Polytope in R^d, Technologies

and Materials for Renewable Energy, Environment and Sustainability, The 16th Int'l Conf., Athens–Greece, May 19–21, 2021, Populated on August 17, 2022.

[5] S.A. Salman, Abd-Almeer, Clotting detection in the vascular network, Indonesian Journal of Electrical Engineering and Computer Science, **20,** no. 2, (2020), 1103–1108.

[6] Shatha A. Salman, Roweda M. Mohammed, On the Coefficients of the Ehrhart Tensor Polynomials, In 1st. Information Technology to Enhance e-learning and Other Application, IEEE, (2020), 7–11.

[7] Shatha A. Salman, On the number of lattice points in n-dimensional space with an application," Computer Science, **16,** no. 2, (2021), 705–712.

[8] Shatha A. Salman, Abbas G. Rajij, Solid angle polynomials and its applications in Geometric Combinatorics, In International Conference on Current Research in Computer Science and Information Technology, IEEE, (2017), 63–68.

[9] Shatha A. Salman, Iman S. Hadeed, Fourier Transform of a Polytope and its Applications in Geometric Combinatorics. In Second Al-Sadiq International Conference on Multidisciplinary in IT and Communication Science and Applications, IEEE, (2017), 255–259.

[10] S.A. Salman, Abd-Almeer, The Minimum Cost Connected Subgraph for the Vascular Network, Energy Procida, **157**,(2019), 128–134.

[11] I.W. Stanek, D.I.M. Sinaga, Magic Mathematics Based on New Matrix Transformations (2D and 3D) for Interdisciplinary Physics, Mathematics, Engineering and Energy Management. In Products and Services from R and D to Final Solutions, IntechOpen, (2010).

[12] O.A. Dawood, A.M.S. Rahma, A.M.J.A. Hossen, Generalized method for constructing magic cube by folded magic squares, International Journal of Intelligent Systems and Applications, **8,** no. 1, (2016).

[13] Shatha A. Salman, Amal A. Ibrahim, "Transportation polytopes and its relation to graph theory", Journal of Al-Qadisiyah for computer science and mathematics, **11,** no. 4, (2019), 102–110.

[14] Shatha A. Al-Najjar, A. R. Nuha, Fuad AAl-Heety, "Computation of Odd Magic Square Using a New Approach with Some Properties," Engineering and Technology Journal, **30,** no. 7, (2012).

[15] Amal A. Ibrahim, Shatha A. Salman, "Some Properties of Magic Squares of Distinct Squares and Cubes", Al-Mustansiriyah Journal of Science, **30,** no. 3, (2019).

[16] S.H. Jamel, T. Herawan, M.M. Deris, "Steps for constructing magic cube using two orthogonal Latin squares and a magic square". In Malaysian Tech. Univ. Conf. Eng. Tech., (2009).

[17] Shatha A. Salman, Fatema A. Sadiq, Viean A. Zain Al-Aabdeen, "Construction of Cubic Magic with Cryptography Application", Journal of Discrete Mathematical Sciences and Cryptography, **27,** no. 5, (2024),1641–1651.

[18] Shatha A. Salman, Amal A. Ibrahim, "A new algorithm for water retention on magic square", Indonesian Journal of Electrical Engineering and computer Science, **2,** no. 2,(2020), 1062–1069.

[19] Shatha A. Salman, "Magic Circles Cryptosystem", Italian Journal of Pure and Applied Mathematics, **47,**(2022), 905–913.

[20] A.D. Keedwell, J. Dénes, " Latin Squares and Their Applications", Elsevier, 2015.

[21] R.P. Nordgren, On properties of special magic square matrices, Linear algebra and its applications, **437,** no. 8, (2012), 2009–2025.

[22] P. Fahimi, R. Javadi, "An introduction to magic squares and their physical applications", 2016.

[23] W.S. Andrews, Magic squares and cubes, Cosimo Inc., 2004.

[24] J.P. Robertson, Magic squares of squares, Mathematics Magazine, **69,** no. 4, (1993), 289–293.