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Cycles in the Corona of Graphs: A Polynomial Representation

Cerina A. Villarta[1], Precious Jane F. Lapura[2], Phoebe Jean J. Domen[3], Sherlyn U. Sappayani[3], Bayah J. Amiruddin-Rajik[3], Rosalio G. Artes Jr.[3]

[1]Central Mindanao University
8710 Musuan, Bukidnon, Philippines

[2]University of Science and Technology of Southern Philippines
9000 Cagayan de Oro City, Philippines

[3]Mindanao State University
Tawi-Tawi College of Technology and Oceanography
7500 Bongao, Tawi-Tawi, Philippines

email: f.cerina.villarta@cmu.edu.ph, preciousjane.lapura@ustp.edu.ph, phoebejeandomen@msutawi-tawi.edu.ph, sherlynsappayani@msutawi-tawi.edu.ph, bayahamiruddin@msutawi-tawi.edu.ph, rosalioartes@msutawi-tawi.edu.ph

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Abstract

In this work, we characterize the induced cycles in the vertex corona of graphs. Moreover, we establish the induced cycle polynomial of graphs resulting from this binary graph operation.

1 Introduction

The study of graph representations in terms of polynomials has captured the interests of discrete mathematicians because of their contributions in the

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areas of Biology, Physics and Chemistry [1]. Maldo and Artes [3] explored the geodetic independence polynomial of graphs. Villarta, Eballe, and Artes [4] pioneered the work on induced path polynomials and established results for graphs under some binary operations. Madalim et al. [2] introduced the concept of induced cycle polynomial of graphs and established some of its properties.

In this paper, we extend the work in [2] to graphs resulting from the vertex corona of two cyclic connected graphs. The *induced cycle polynomial* of a cyclic graph G is given by

$$\Gamma_{ic}(G;x) = \sum_{i=3}^{c(G)} c_i(G)x^i,$$

where $c_i(G)$ is the number of induced cycles in G of order i and $c(G)$ is the cardinality of the maximum cycle in G [2].

2 Results on Vertex Corona of Graphs

Following is the definition of the vertex corona of graphs.

Definition 2.1. The *vertex corona* of G with H, denoted by $G \overset{v}{\circ} H$, is the graph with vertex-set

$$V(G \overset{v}{\circ} H) = V(G) \cup \bigcup_{v \in V(G)} V(H^v),$$

where H^v is a copy of H attached to $v \in V(G)$. We denote by (v,u) a vertex in H^v, where $u \in V(H)$. The edge-set is given by

$$E(G \overset{v}{\circ} H) = E(G) \cup \bigcup_{v \in V(G)} E(H^v) \cup \bigcup_{v \in V(G)} \{vw : w \in V(H^v)\}.$$

Observe that if $u, v \in V(G)$ and w is in a copy of H, then $\{u,v,w\}$ cannot be in a cycle in $G \overset{v}{\circ} H$.

The following result is straightforward.

Lemma 2.2. *If S induces a cycle in $G \overset{v}{\circ} H$ and $|S \cap V(G)| \geq 2$, then $S \subseteq V(G)$.*

The above lemma asserts that if S induces a cycle in $G \overset{v}{\circ} H$ and S contains at least two vertices from G, then S must be in G.

Lemma 2.3. *If S induces a cycle in $G \overset{v}{\circ} H$, then S cannot contain elements from different copies of H.*

Proof. Let S be a cycle-inducing subset of $V(G \overset{v}{\circ} H)$. Suppose, on the contrary, that there exist $u, v \in V(G)$ such that S intersects both H^u and S^v. Now, u is a cut-vertex of $G \overset{v}{\circ} H$. This means that there is no path joining a vertex in H^u and a vertex from H^v. Hence, S cannot induce a cycle in $G \overset{v}{\circ} H$. □

The above lemma asserts that if $S \cap V(H^v) \neq \varnothing$, then $S \cap V(H^u) = \varnothing$ for every $u \in V(G) \setminus \{v\}$.

Next, we characterize the induced cycles in the vertex corona of graphs.

Lemma 2.4. *Let G and H be cyclic connected graphs. A subset S of $V(G \overset{v}{\circ} H)$ induces a cycle in $G \overset{v}{\circ} H$ if and only if it satisfies one of the following:*

(*i*) *S induces a cycle in G*

(*ii*) *S induces a cycle in a copy H^v of H attached to $v \in V(G)$.*

(*iii*) *$S = \{v\} \cup S_{H^v}$, where $v \in V(G)$ and S_{H^v} induces a K_2 in H^v, a copy of H attached to $v \in V(G)$.*

Proof. Let $S \subseteq V(G \overset{v}{\circ} H)$. Suppose S induces a cycle in $G \overset{v}{\circ} H$. If S contains two vertices from G, then, by Lemma 2.2, $S \subseteq V(G)$. Hence, S induces a cycle in G and (i) is satisfied.

Suppose that $|S \cap V(G)| = 1$. Let $S \cap V(G) = \{v\}$. Then $S \setminus \{v\} \subseteq V(H^v)$, by Lemma 2.3. If $|S \cap V(H^v)| \geq 3$, then $\deg_G(v) \geq 3$. Hence, $\langle S \rangle$ is not a cycle in $G \overset{v}{\circ} H$. Consequently, $|S \cap V(H^v)| = 2$ with $\langle S \cap V(H^v) \rangle = K_2$ in H^v. Hence, (iii) is satisfied.

Now, if $S \cap V(G) = \varnothing$, then S induces a cycle in a copy of H. Thus, (ii) is satisfied.

The converse follows easily. □

Now, we establish the main result.

Theorem 2.5. *Let G and H be cyclic connected graphs. Then*

$$\Gamma_{ic}(G \overset{v}{\circ} H; x) = \Gamma_{ic}(G; x) + |V(G)|\Gamma_{ic}(H; x) + |V(G)||E(H)|x^3.$$

Proof. The first term follows from Lemma 2.4 (i). Note that $G \overset{v}{\circ} H$ has $|V(G)|$ copies of H. By Lemma 2.4 (ii), we have the second term. Now for each $e = wz \in V(H^v)$, the set $\{w, z, v\}$ induces a triangle in $G \overset{v}{\circ} H$. Summing up gives the third term. The result follows by combining the terms. □

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