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Slightly $(\tau_1, \tau_2)s$-continuous functions

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Abstract

In this paper, we introduce the notion of slightly $(\tau_1, \tau_2)s$-continuous functions. Moreover, we investigate several characterizations of slightly $(\tau_1, \tau_2)s$-continuous functions.

1 Introduction

The notion of slightly continuous functions was introduced by Jain [14]. Nour [15] defined slightly semi-continuous functions as a weak form of slight continuity and investigated some characterizations of slightly semi-continuous functions. Duangphui et al. [13] introduced and investigated the notion of $(\mu, \mu')^{(m,n)}$-continuous functions. Viriyapong and Boonpok [19] investigated some characterizations of (Λ, sp)-continuous functions by utilizing the notions

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of (Λ, sp)-open sets and (Λ, sp)-closed sets due to Boonpok and Khampakdee [6]. Moreover, some characterizations of strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\star$-continuous functions, $\theta(\star)$-precontinuous functions, $(\Lambda, p(\star))$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise almost M-continuous functions, $\delta p(\Lambda, s)$-continuous functions, (τ_1, τ_2)-continuous functions and almost (τ_1, τ_2)-continuous functions were presented in [18], [3], [4], [5], [7], [8], [12], [11], [17], [1] and [2], respectively. Sangviset et al. [16] introduced and studied the concept of slightly (m, μ)-continuous functions. In this paper, we introduce the notion of slightly $(\tau_1, \tau_2)s$-continuous functions. We also investigate several characterizations of slightly $(\tau_1, \tau_2)s$-continuous functions.

2 Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [10] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*clopen* [10] if A is both $\tau_1\tau_2$-open and $\tau_1\tau_2$-closed. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [10] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [10] of A and is denoted by $\tau_1\tau_2$-Int(A). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)s$-*open* [9] (resp. $(\tau_1, \tau_2)p$-*open* [9]) if $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A))$ (resp. $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A)))$. The complement of a $(\tau_1, \tau_2)s$-open (resp. $(\tau_1, \tau_2)p$-open) set is said to be $(\tau_1, \tau_2)s$-*closed* (resp. $(\tau_1, \tau_2)p$-*closed*). Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $(\tau_1, \tau_2)s$-closed sets of X containing A is called the $(\tau_1, \tau_2)s$-*closure* [9] of A and is denoted by (τ_1, τ_2)-sCl(A). The union of all $(\tau_1, \tau_2)s$-open sets of X contained in A is called the $(\tau_1, \tau_2)s$-*interior* [9] of A and is denoted by (τ_1, τ_2)-sInt(A).

Lemma 2.1. *For subsets A and B of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) If $A \subseteq B$, then (τ_1, τ_2)-sCl$(A) \subseteq (\tau_1, \tau_2)$-sCl$(B)$.

(2) A is $(\tau_1, \tau_2)s$-closed if and only if $A = (\tau_1, \tau_2)$-$sCl(A)$.

(3) (τ_1, τ_2)-$sCl(X - A) = X - (\tau_1, \tau_2)$-$sInt(A)$.

3 Slightly $(\tau_1, \tau_2)s$-continuous functions

We begin this section by introducing the notion of slightly $(\tau_1, \tau_2)s$-continuous functions.

Definition 3.1. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be slightly $(\tau_1, \tau_2)s$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-clopen set V of Y containing $f(x)$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $f(U) \subseteq V$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be slightly $(\tau_1, \tau_2)s$-continuous if f has this property at every point of X.*

Theorem 3.2. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is slightly $(\tau_1, \tau_2)s$-continuous;

(2) $f^{-1}(V)$ is $(\tau_1, \tau_2)s$-open in X for each $\sigma_1\sigma_2$-clopen set V of Y;

(3) $f^{-1}(V)$ is $(\tau_1, \tau_2)s$-closed in X for each $\sigma_1\sigma_2$-clopen set V of Y;

(4) for each $x \in X$ and for each $\sigma_1\sigma_2$-clopen set V of Y containing $f(x)$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $f(U) \subseteq V$.

Proof. (1) $\Rightarrow$ (2): Let V be any $\sigma_1\sigma_2$-clopen set V of Y and $x \in f^{-1}(V)$. Then $f(x) \in V$. Since f is slightly $(\tau_1, \tau_2)s$-continuous, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $f(U) \subseteq V$. Thus $x \in U \subseteq f^{-1}(V)$ and hence $x \in (\tau_1, \tau_2)$-sInt$(f^{-1}(V))$. This implies that $f^{-1}(V) \subseteq (\tau_1, \tau_2)$-sInt$(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is $(\tau_1, \tau_2)s$-open in X.

(2) $\Leftrightarrow$ (3): Obvious.

(2) $\Rightarrow$ (1): Let $x \in X$ and V be any $\sigma_1\sigma_2$-clopen set V of Y containing $f(x)$. Then $x \in f^{-1}(V) = (\tau_1, \tau_2)$-sInt$(f^{-1}(V))$. There exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $U \subseteq f^{-1}(V)$. Thus $f(U) \subseteq V$ and hence f is slightly $(\tau_1, \tau_2)s$-continuous at x. This shows that f is slightly $(\tau_1, \tau_2)s$-continuous.

(1) $\Leftrightarrow$ (4): Obvious. □

Recall that a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*connected* [10] if X cannot be written as the union of two disjoint nonempty $\tau_1\tau_2$-open sets.

Definition 3.3. *A bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)s$-connected if X cannot be written as the union of two disjoint nonempty $(\tau_1, \tau_2)s$-open sets.*

Theorem 3.4. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a slightly $(\tau_1, \tau_2)s$-continuous surjection and (X, τ_1, τ_2) is a $(\tau_1, \tau_2)s$-connected space, then (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-connected.*

Proof. Assume that (Y, σ_1, σ_2) is not $\sigma_1\sigma_2$-connected. Then, there exist nonempty $\sigma_1\sigma_2$-open sets U and V of Y such that $U \cap V = \emptyset$ and $U \cup V = Y$. Therefore, U and V are $\sigma_1\sigma_2$-clopen sets of Y. Since f is slightly $(\tau_1, \tau_2)s$-continuous, $f^{-1}(U)$ and $f^{-1}(U)$ are $(\tau_1, \tau_2)s$-open sets of X. Moreover, we have $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ and $f^{-1}(U) \cup f^{-1}(V) = Y$. Since f is surjective, $f^{-1}(U)$ and $f^{-1}(V)$ are nonempty. Thus (X, τ_1, τ_2) is not $(\tau_1, \tau_2)s$-connected. This is a contradiction and hence (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-connected. □

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