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On a Certain Sequence of Sequence t-Neo Balancing Numbers

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Abstract

In this paper, we investigate the sequence t-Neo balancing numbers by using the properties of Pell's equation and the Brahmagupta's identity for generalized certain sequences.

1 Introduction

In this paper, we delve into the sequence $\{a_m = 2m-1\}$ related to t-Neo balancing numbers, employing the properties of Pell's equation [10] and Brahmagupta's identity [4, 5, 6]. Panda and Behera [1] defined the balancing numbers which served as the catalyst for many generalized researches of balancing numbers [2, 3, 7, 8, 9]. Panda [8] defined a certain sequence of real numbers $\{a_m\}$ to be a sequence of balancing numbers. Dash and Ota [2, 3] defined t-balancing numbers and hence a sequence t-balancing numbers $\{a_m\}$. Chailangka and Pakapongpun [7] defined neo balancing numbers $n \in \mathbb{N}$ by the Diophantine equation

$$1+2+3+\cdots+(n-1) = (n-1)+(n+0)+(n+1)+\cdots+(n+r). \quad (1.1)$$

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2 The Certain Sequences on Sequence t-Neo Balancing Numbers

In this section, we show the origin of sequence t-Neo balancing numbers in another way with sequence t-balancing numbers. Let $\{a_n\}_{n=1}^{\infty}$ be a real sequence. The sequence $\{a_m\}$ is called a sequence t-Neo balancing numbers if a_m satisfies the Diophantine equation

$$a_1 + a_2 + a_3 + \cdots + a_{m-1} = a_{m+t-1} + a_{m+t} + a_{m+t+1} + \cdots + a_{m+t+r}, \quad (2.2)$$

for some integer r.

2.1 The sequence $\{a_m = 2m - 1\}$ on sequence t-Neo balancing numbers

We investigate the sequence $\{a_m = 2m - 1\}$, which is called the sequence t-Neo balancing numbers if

$$1+3+\cdots+(2m-3) = (2m+2t-3)+(2m+2t-1)+\cdots+(2m+2t+2r-1).$$

Now, we have $2n = 2r + 6 + \sqrt{8r^2 + 24r + 8rt + 16t + 16}$.

2.2 The recurrence relations for the sequence t-Neo balancing number

Theorem 2.1. *If $a_m = 2m - 1$ and $n \geq 3$, then the recurrence relations for the sequence t-Neo balancing number's index is*

$$m_{2n-1} = 6m_{2n-3} - m_{2n-5} - 6. \quad (2.3)$$

Moreover, the recurrence relations for the sequence t-Neo balancing number is

$$a_{m_{2n-1}} = 6a_{m_{2n-3}} - a_{m_{2n-5}} + 4t - 8. \quad (2.4)$$

Proof. Since $\{a_m\}$ is a sequence t-Neo balancing number and m is a t-Neo balancing number, $8r^2+24r+8rt+16t+16$ is a perfect square. Then we can let $y = \sqrt{8r^2 + 24r + 8rt + 16t + 16}$. Let $x = 4r + 6 + 2t$. Then we obtain

$$x^2 - 2y^2 = (2(t-1))^2 - 8. \quad (2.5)$$

Therefore, we have the triplet

$$(a, b, k) = (2(t-1), 2, (2(t-1))^2 - 8). \tag{2.6}$$

for the equation (2.5). Afterwards, we consider the Pell's equation

$$x^2 - 2y^2 = 1. \tag{2.7}$$

Then we get the expanded solution [10] of equation (2.7)

$$\bar{x}_n = \frac{1}{2}[(3+2\sqrt{2})^n + (3-2\sqrt{2})^n] \tag{2.8}$$

$$\bar{y}_n = \frac{1}{2\sqrt{2}}[(3+2\sqrt{2})^n - (3-2\sqrt{2})^n]. \tag{2.9}$$

Therefore, we have the triplet $(x_n, y_n, 1)$ for equation (2.7). Thus, we compose the triplets by Brahmagupta's identity ([4, 5, 6]) and obtain the important sequences with general terms

$$\begin{array}{ccc} X_n = a\bar{x}_n + 2b\bar{y}_n & \text{and} & X_n^* = a\bar{x}_n - 2b\bar{y}_n \\ Y_n = b\bar{x}_n + a\bar{y}_n & & Y_n^* = b\bar{x}_n - a\bar{y}_n \end{array}$$

which are the solution for equation (2.5). Therefore, we have

$$\begin{aligned} 2X_n &= (3+2\sqrt{2})^n(a+b\sqrt{2}) + (3-2\sqrt{2})^n(a-b\sqrt{2}) \\ 2\sqrt{2}Y_n &= (3+2\sqrt{2})^n(a+b\sqrt{2}) - (3-2\sqrt{2})^n(a-b\sqrt{2}) \end{aligned}$$

and

$$\begin{aligned} 2X_n^* &= (3+2\sqrt{2})^n(a-b\sqrt{2}) + (3-2\sqrt{2})^n(a+b\sqrt{2}) \\ 2\sqrt{2}Y_n^* &= -(3+2\sqrt{2})^n(a-b\sqrt{2}) + (3-2\sqrt{2})^n(a+b\sqrt{2}). \end{aligned}$$

Then we obtain two sequences $\{X_n\}$ and $\{Y_n\}$ satisfying the recurrence relations

$$\begin{aligned} X_n &= 6X_{n-1} - X_{n-2} \\ Y_n &= 6Y_{n-1} - Y_{n-2}. \end{aligned}$$

Since we have already found $2n = 2r + 6 + \sqrt{8r^2 + 24r + 8rt + 16t + 16}$, $x = 4r + 6 + 2t$ and $y = \sqrt{8r^2 + 24r + 8rt + 16t + 16}$, we get the index's relation

$$m_n = 6m_{n-1} - m_{n-2} + 2t - 6.$$

Since we have defined the sequence $\{a_m = 2m - 1\}$, we obtain the important recurrence relation

$$a_{m_n} = 6a_{m_{n-1}} - a_{m_{n-2}} + 4(t - 2).$$

$\square$

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