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Slight (τ_1, τ_2)-continuity for functions

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Abstract

In this paper, we introduce the notion of slightly (τ_1, τ_2)-continuous functions. We also investigate some characterizations of slightly (τ_1, τ_2)-continuous functions.

1 Introduction

Continuity of functions has played a significant role in the theory of classical point set topology and several other branches of mathematics. This concept has been generalized by weaker and stronger forms of open sets such as semi-open sets, preopen sets, α-open sets and β-open sets. Viriyapong

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and Boonpok [15] investigated some characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets due to Boonpok and Khampakdee [2]. Duangphui et al. [9] introduced and investigated the notion of $(\mu, \mu')^{(m,n)}$-continuous functions. In particular, some characterizations of strongly $\theta(\Lambda, p)$-continuous functions, $\star$-continuous functions, $(\Lambda, p(\star))$-continuous functions, θ-$\mathscr{I}$-continuous functions, (g, m)-continuous functions, pairwise M-continuous functions were presented in [13], [1], [3], [4], [8] and [7], respectively. Jain [10] introduced the notion of slightly continuous functions. Nour [12] defined slightly semi-continuous functions as a weak form of slight continuity and investigated some characterizations of slightly semi-continuous functions. Noiri [11] introduced and studied the concept of slightly β-continuous functions. In this paper, we introduce the notion of slightly (τ_1, τ_2)-continuous functions. Moreover, we investigate several characterizations of slightly (τ_1, τ_2)-continuous functions.

2 Preliminaries

Throughout the paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [6] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [6] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [6] of A and is denoted by $\tau_1\tau_2$-Int(A). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*clopen* [6] if A is both $\tau_1\tau_2$-open and $\tau_1\tau_2$-closed. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)\beta$-*open* [5] if $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A)))$. The complement of a $(\tau_1, \tau_2)\beta$-open set is said to be $(\tau_1, \tau_2)\beta$-*closed*.

3 Slightly (τ_1, τ_2)-continuous functions

We begin this section by introducing the notion of slightly (τ_1, τ_2)-continuous functions.

Definition 3.1. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be slightly (τ_1, τ_2)-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-clopen set V of Y containing $f(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq V$.*

Theorem 3.2. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is slightly (τ_1, τ_2)-continuous;

(2) $f^{-1}(V)$ is $\tau_1\tau_2$-open in X for each $\sigma_1\sigma_2$-clopen set V of Y;

(3) $f^{-1}(V)$ is $\tau_1\tau_2$-closed in X for each $\sigma_1\sigma_2$-clopen set V of Y;

(4) $f^{-1}(V)$ is $\tau_1\tau_2$-clopen in X for each $\sigma_1\sigma_2$-clopen set V of Y.

Proof. (1) $\Rightarrow$ (2): Let V be any $\sigma_1\sigma_2$-clopen set of Y and $x \in f^{-1}(V)$. Then, $f(x) \in V$. By (1), there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq V$. Thus, $x \in U \subseteq f^{-1}(V)$ and hence $x \in \tau_1\tau_2\text{-Int}(f^{-1}(V))$. Therefore, $f^{-1}(V) \subseteq \tau_1\tau_2\text{-Int}(f^{-1}(V))$. This shows that $f^{-1}(V)$ is $\tau_1\tau_2$-open in X.

(2) $\Rightarrow$ (3): Let V be any $\sigma_1\sigma_2$-clopen set of Y. Then, $Y - V$ is $\sigma_1\sigma_2$-clopen in Y. By (2), we have $f^{-1}(Y - V) = X - f^{-1}(V)$ is $\tau_1\tau_2$-open in X. Thus, $f^{-1}(V)$ is $\tau_1\tau_2$-closed in X.

(3) $\Rightarrow$ (4): It can be shown easily.

(4) $\Rightarrow$ (1): Let $x \in X$ and V be any $\sigma_1\sigma_2$-clopen set of Y containing $f(x)$. By (4), $f^{-1}(V)$ is $\tau_1\tau_2$-open in X. Put $U = f^{-1}(V)$. Then, U is a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq V$. This shows that f is slightly (τ_1, τ_2)-continuous. □

Recall that a bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-*extremally disconnected* [14] if the $\tau_1\tau_2$-closure of every $\tau_1\tau_2$-open set U of X is $\tau_1\tau_2$-open.

Theorem 3.3. *Let (Y, σ_1, σ_2) be a (σ_1, σ_2)-extremally disconnected space. For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is slightly (τ_1, τ_2)-continuous;

(2) $\tau_1\tau_2\text{-Cl}(f^{-1}(V)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$ for every $\sigma_1\sigma_2$-open set V of Y;

(3) $f^{-1}(\sigma_1\sigma_2\text{-Int}(K)) \subseteq \tau_1\tau_2\text{-Int}(f^{-1}(K))$ for every $\sigma_1\sigma_2$-closed set K of Y.

Proof. (1) ⇒ (2): Let V be any $\sigma_1\sigma_2$-open set of Y. Then, $\sigma_1\sigma_1$-Cl(V) is a $\sigma_1\sigma_2$-clopen set of Y. By Theorem 3.2, $f^{-1}(\sigma_1\sigma_1\text{-Cl}(V))$ is $\tau_1\tau_2$-closed in X. Thus, $\tau_1\tau_2\text{-Cl}(f^{-1}(V)) \subseteq \tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))) = f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$.

(2) ⇒ (3): Let K be any $\sigma_1\sigma_2$-closed set of Y. Then, $Y - K$ is $\sigma_1\sigma_2$-open in Y. By (2), we have $X - \tau_1\tau_2\text{-Int}(f^{-1}(K)) = \tau_1\tau_2\text{-Cl}(f^{-1}(Y - K)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - K)) = X - f^{-1}(\sigma_1\sigma_2\text{-Int}(K))$ and hence $f^{-1}(\sigma_1\sigma_2\text{-Int}(K)) \subseteq \tau_1\tau_2\text{-Int}(f^{-1}(K))$.

(3) ⇒ (1): Let $x \in X$ and V be any $\sigma_1\sigma_2$-clopen set of Y containing $f(x)$. Then by (3), $x \in f^{-1}(V) = f^{-1}(\sigma_1\sigma_2\text{-Int}(V)) \subseteq \tau_1\tau_2\text{-Int}(f^{-1}(V))$. There exists a $\tau_1\tau_2$-open set U of X such that $x \in U \subseteq f^{-1}(V)$. Thus, $f(U) \subseteq V$ and hence f is slightly (τ_1, τ_2)-continuous. □

Theorem 3.4. *Let (Y, σ_1, σ_2) be a (σ_1, σ_2)-extremally disconnected space. For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is slightly (τ_1, τ_2)-continuous;

(2) $\tau_1\tau_2$-Cl$(f^{-1}(V)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$ for every $(\sigma_1, \sigma_2)\beta$-open set V of Y;

(3) $f^{-1}(\sigma_1\sigma_2\text{-Int}(K)) \subseteq \tau_1\tau_2\text{-Int}(f^{-1}(K))$ for every $(\sigma_1, \sigma_2)\beta$-closed set K of Y.

Proof. The proof is similar to that of Theorem 3.3 and it follows from Theorem 3.2 and Theorem 3.2 of [14]. □

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