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Learning the Parabola Mediated by Problem-Solving, Semiotic Representations, and Applets in GeoGebra

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Abstract

In this article we evaluate the contribution of the problem-solving approach, semiotic representations and the use of applets designed in GeoGebra when first-semester science and engineering students study the mathematical concept and application of the parabola. At the end of the classroom intervention, a significant improvement was noted in the students' performance, especially in their cognitive thinking activities; that is, in the use of algebraic language, in the appropriation of geometric concepts, and in the problem-solving performance.

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1 Introduction

The reviews, analysis, discussions, conclusions and recommendations of different types of investigations such as articles, doctoral dissertations, master's theses, among others, located in the context of didactics of mathematics, show the need and convenience of reflecting on the role that teachers and students play inside and outside the classroom; that is, a constant review and adaptation of mathematics teaching and learning processes. The majority of such works directly or indirectly suggest the permanent search and implementation of strategies, mediated by the appropriate use of tools linked to information and communication technologies that encourage, promote and optimize said processes, based on the resolution of problems. In both theory and practice, there is an emphasis on consideration and use of different ways of representing and applying mathematical concepts, as well as activities that stimulate the exchange of ideas, collaborative learning, analysis, hypothesis development, reflection, argumentation, creativity and in general the social construction of knowledge. At each level of schooling, both teachers and students repeatedly face problems of various kinds during the teaching and learning process. Such a scenario has provided, in recent decades, very useful raw material for carrying out research that, from different theoretical perspectives, aims to contribute, in one way or another, to the solution of problematic situations inherent to the said process. The development of our investigation addresses such concerns.

2 Methodology

The methodology of this work focuses on monitoring the evolution and strengthening of cognitive thinking activities and problem-solving processes is qualitative with an interpretative approach. We consider three stages:
The first stage deals with the application of a diagnostic evaluation instrument to identify, in a group of first semester students of Science and Engineering, performances and difficulties related with the mathematical concepts associated with the parabola. The second stage deals with a classroom intervention.
Finally, the third stage deals with the corresponding analysis, interpretations, conclusions and recommendations.

2.1 Instruments

2.1.1 Diagnostic test

The diagnostic test was a rubric with activities proposed to assess in a group of 36 students prior mathematical knowledge, performance in cognitive thinking activities and problem solving processes. In particular, the validation process was carried out through expert judgment. With the information obtained from the diagnostic test, it was clear that the students under observation had varying difficulties related with previous mathematical knowledge, with the use of algebraic language, difficulties to represent information about a proposed problem in a coordinate system, and therefore in the problem-solving performance.
In the instrument one application problem the proposal had to do with considering the photograph of the model of a bridge, and based on complementary information they have to represent in a coordinate system the essential aspects of the proposed problem and obtain the equation of the parabolic arch that was part of the bridge structure. Most of them presented difficulties in the use of algebraic language, in trhe conversion processes and in fact in problem-solving performance.

2.1.2 Applets in GeoGebra environment

The use of applets designed in the GeoGebra environment allows students to represent mathematical concepts and procedures in different registers; that is, through graphs that can be manipulated with dynamic geometry such as points, lines, curves and three dimensional-figures. Consequently, dynamic geometry and the interaction of the student with the applets played an important role in the appropriation of mathematical knowledge and they enabled students to link their visual and analytical thinking in the learning process thus improving mathematical communication skills and understanding of mathematical concepts that play a fundamental role in problem solving performance. In particular, three applets were designed in the GeoGebra environment with the purpose of supporting the learning of the parabola mediated by cognitive thinking activities and by the problem-solving approach.

2.2 Procedures

The second stage began by exhibiting the presence of the parabola in different contexts, for example, in different types of constructions (bridges, building, among others monuments), in telecommunications (signal receiving antennas), in the military field and in aerospace research with the launching of projectiles. After that, theoretical aspects were discussed with some application problems whose solutions were supported with the applets designed in the GeoGebra environment. Activities were proposed to be developed individually and in groups, for example, questionnaires and workshops mediated by semiotic representations and problem solving whose solution demand analytical performance and the interaction student-student, student-teacher and student-applets which allowed students to overcome difficulties in learning the theoretical foundation and application of the parabola. The intervention in the classroom was focused on improving the learning of the parabola through activities that strengthened mathematical reasoning, use of algebraic language, mathematical communication and problem-solving performance processes. The classroom intervention was developed during 8 sessions of two hours each. In the first three sessions, the mathematical and geometric foundations necessary to understand the parabola were exposed and discussed by using the first applets and the concept of parabola was represented graphically, step by step, through dynamic geometry and its definition was enunciated as the geometric locus of a point that moves in a plane in such a way that its distance from a fixed line, located in the plane, is always equal to its distance from a fixed point of the plane, and that fixed point does not belong to the line. The fixed point is called the focus and the fixed line is called the directrix of the parabola. Subsequently, using the same applet the geometric and algebraic properties of the parabola were discussed and, in particular, when its vertex is at any point in the plane and its symmetry axis is parallel to the X coordinate axis or parallel to the Y coordinate axis. In this way, the concept of parabola was exhibited in various representation registers; e.g., verbal register, graphic register, tabular register and algebraic register. Some activities were focused on representing the parabola from the canonical form to the corresponding general form and vice versa. In addition, from a graphic representation to obtain their respective algebraic representations. The following figure with its associated elements; that is, directrix, focus, axis of symetry, vertex, and focal chord was generated with dynamic geometry using the first applet.

The first applet can be reviewed clicking the next code.

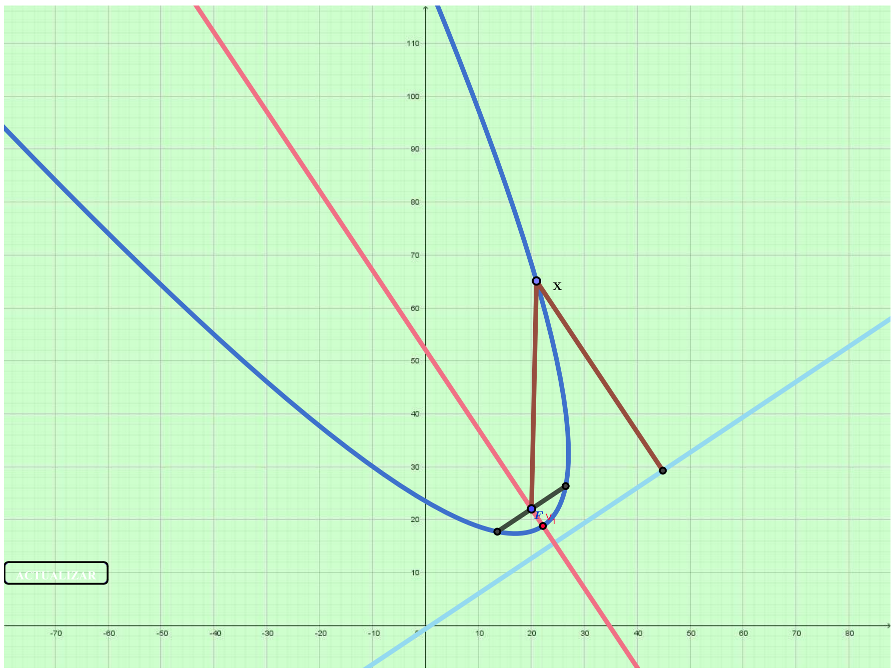

Figure 1: Parabola generated with dynamic geometry

Subsequently, with the second applet training and evaluation activities were proposed to monitor the concepts and skills acquired by the students under observation. For example, graphs of parabolas are proposed and students must identify their equation in standard form. Then, through algebraic manipulations, they rewrite them in their corresponding general form or vice versa. By carrying out these activities, mathematical knowledge is strengthened and at the same time the difficulties linked to cognitive treatment activities and cognitive conversion activities are overcome. The second applet can be reviewed by clicking in the following code:

Finally, a third and last applet was implemented and can be reviewed clicking in the following code:

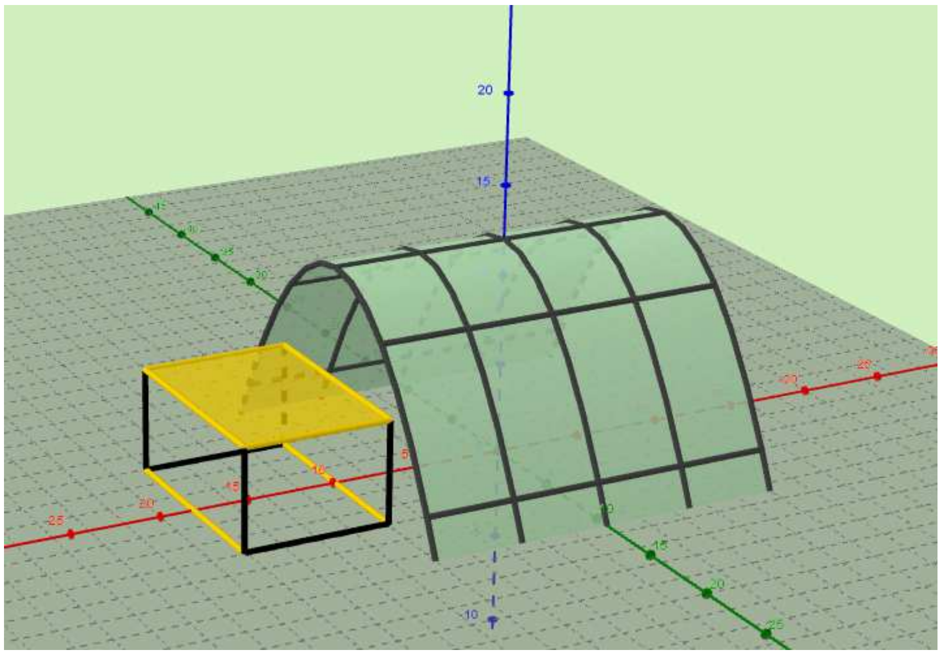

Figure 2: parabolic tunnel

The following figure was generated with dynamic geometry by using the third applet:

Problems were proposed with applications and training activities of the parabola. For example, this applet dynamically recreates a tunnel in the shape of a parabolic arch and the student must obtain the height of the focus, the equation of the parabolic arc in standard and general forms. Moreover, given a vehicle with a certain width, calculate the maximum height it must be in order for it to enter the tunnel, see figure 2.

3 Results

In each of the previous stages, a scenario was sought where learning is a process of social construction that requires interaction between student-student, student-teacher and student-applets. This interaction contributed to strengthening the students' capacities and overcoming the difficulties of learning the parabola; e.g., improvement of the use of algebraic language skills to graphically represent problematic situations, appropriation of geometric concepts and improvements in problem solving-performance.

4 Conclusions

1. Student-student, student-applets and student-teacher interactions positively impact the learning of mathematical concepts and applications associated with the parabola because the use of dynamic geometry contributes to strengthening the skills linked to cognitive activities of thinking and problem solving processes.

2. Finally, the didactic strategy of teaching the parabola based on semiotic representations, problem solving and with the support of applets designed in the GeoGebra environment provides favorable evidence for the use of strategies or tools that can give teachers the opportunity to use them in their improvement processes.

5 Recommendations

1. The learning of the parabola mediated by problem solving, semiotic representations, the use of applets in the GeoGebra environment, seeking a scenario where learning is a process of social construction that demands, on the part of the learners, interaction between student-student, student-teacher and student-applets contribute to the strengthening of algebraic management, the cognitive processes of conversion, the strengthening of problem-solving processes and the overcoming of learning difficulties in the parabola.

2. It is recommended to consider the process of evolution of the cognitive activities of thinking, problem solving and the use of software in the teaching and learning processes of mathematics. For instance, the GeoGebra software has great potential, especially in learning geometry since it allows mathematical concepts to be illustrated simultaneously through the manipulation of graphs, tables, equations and symbolic calculation, thus allowing students to master and understand concepts and procedures related to concepts and applications, particularly of the parabola.

3. The software considered in this work can be download and reviewed via: https://www.geogebra.org/download

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