International Journal of Mathematics and
Computer Science, **20**(2025), no. 1, 89–93
DOI: https://doi.org/10.69793/ijmcs/01.2025/khampakdee

Slight $(\tau_1, \tau_2)s$-continuity for multifunctions

Jeeranunt Khampakdee[1], **Supannee Sompong**[2], **Chawalit Boonpok**[1]

[1]Mathematics and Applied Mathematics Research Unit
Department of Mathematics
Faculty of Science
Mahasarakham University
Maha Sarakham, 44150, Thailand

[2]Department of Mathematics and Statistics
Faculty of Science and Technology
Sakon Nakhon Rajbhat University
Sakon Nakhon, 47000, Thailand

email: jeeranunt.k@msu.ac.th, s_sompong@snru.ac.th,
chawalit.b@msu.ac.th

(Received June 2, 2024, Revised July 3, 2024,
Accepted July 11, 2024, Published July 17, 2024)

Abstract

In this paper, we introduce the notions of upper and lower slightly $(\tau_1, \tau_2)s$-continuous multifunctions. Moreover, we investigate several characterizations of upper and lower slightly $(\tau_1, \tau_2)s$-continuous multifunctions.

1 Introduction

Jain [11] introduced the notion of slightly continuous functions. Nour [14] defined slightly semi-continuous functions as a weak form of slight continuity and investigated some characterizations of slightly semi-continuous func-

Key words and phrases: Upper slightly $(\tau_1, \tau_2)s$-continuous multifunction, lower slightly $(\tau_1, \tau_2)s$-continuous multifunction.
AMS (MOS) Subject Classifications: 54C08, 54C60, 54E55.
The corresponding author is Jeeranunt Khampakdee.
ISSN 1814-0432, 2025, http://ijmcs.future-in-tech.net

tions. Duangphui et al. [10] introduced and studied the notion of $(\mu, \mu')^{(m,n)}$-continuous functions. Moreover, some characterizations of strongly $\theta(\Lambda, p)$-continuous functions, (Λ, sp)-continuous functions, $\star$-continuous functions and θ-$\mathscr{I}$-continuous functions were presented in [16], [17], [2] and [6], respectively. Sangviset et al. [15] introduced and investigated the concept of slightly (m, μ)-continuous functions. Noiri and Popa [13] introduced the notion of slightly m-continuous multifunctions and studied the relationships among m-continuity, almost m-continuity, weak m-continuity and slight m-continuity for multifunctions. Laprom et al. [12] introduced and investigated the notion of $\beta(\tau_1, \tau_2)$-continuous multifunctions. Furthermore, several characterizations of $\star$-continuous multifunctions, $\beta(\star)$-continuous multifunctions, weakly $(\tau_1, \tau_2)\alpha$-continuous multifunctions, almost weakly (τ_1, τ_2)-continuous multifunctions and slightly (Λ, sp)-continuous multifunctions were established in [8], [5], [18], [4] and [3], respectively. In this paper, we introduce the notions of upper and lower slightly $(\tau_1, \tau_2)s$-continuous multifunctions. We also investigate several characterizations of upper and lower slightly $(\tau_1, \tau_2)s$-continuous multifunctions.

2 Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [9] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*clopen* [9] if A is both $\tau_1\tau_2$-open and $\tau_1\tau_2$-closed. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [9] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [9] of A and is denoted by $\tau_1\tau_2$-Int(A). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)s$-*open* if $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A))$. The complement of a $(\tau_1, \tau_2)s$-open set is said to be $(\tau_1, \tau_2)s$-*closed*. The intersection of all $(\tau_1, \tau_2)s$-closed sets of X containing A is called the $(\tau_1, \tau_2)s$-*closure* [7] of A and is denoted by (τ_1, τ_2)-sCl(A). The union of all $(\tau_1, \tau_2)s$-open sets of X contained in A is called the $(\tau_1, \tau_2)s$-*interior* [7] of A and is denoted by (τ_1, τ_2)-sInt(A).

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, following [1] we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$.

3 Upper and lower slightly $(\tau_1, \tau_2)s$-continuous multifunctions

We begin this section by introducing the notion of upper slightly $(\tau_1, \tau_2)s$-continuous multifunctions.

Definition 3.1. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper slightly $(\tau_1, \tau_2)s$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-clopen set V of Y containing $F(x)$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $F(U) \subseteq V$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper slightly $(\tau_1, \tau_2)s$-continuous if F has this property at every point of X.*

Theorem 3.2. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper slightly $(\tau_1, \tau_2)s$-continuous;

(2) $F^+(V)$ is $(\tau_1, \tau_2)s$-open in X for every $\sigma_1\sigma_2$-clopen set V of Y;

(3) $F^-(V)$ is $(\tau_1, \tau_2)s$-closed in X for every $\sigma_1\sigma_2$-clopen set V of Y;

(4) for each $x \in X$ and each $\sigma_1\sigma_2$-clopen set V of Y such that $x \in F^+(V)$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $U \subseteq F^+(V)$.

Proof. (1) $\Rightarrow$ (2): Let V be any $\sigma_1\sigma_2$-clopen set V of Y and $x \in F^+(V)$. Then $F(x) \subseteq V$. By (1), there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $F(U) \subseteq V$. Thus $x \in U \subseteq F^+(V)$ and hence $x \in (\tau_1, \tau_2)$-sInt$(F^+(V))$. Therefore, $F^+(V) \subseteq (\tau_1, \tau_2)$-sInt$(F^+(V))$ and so $F^+(V)$ is $(\tau_1, \tau_2)s$-open in X.

(2) $\Leftrightarrow$ (3): Obvious.

(2) $\Rightarrow$ (1): Let $x \in X$ and V be any $\sigma_1\sigma_2$-clopen set V of Y containing $F(x)$. Then $x \in F^+(V) = (\tau_1, \tau_2)$-sInt$(F^+(V))$. There exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $U \subseteq F^+(V)$. Thus $F(U) \subseteq V$ and hence

F is upper slightly $(\tau_1, \tau_2)s$-continuous at x. This shows that F is upper slightly $(\tau_1, \tau_2)s$-continuous.

(1) $\Leftrightarrow$ (4): Obvious. $\square$

Definition 3.3. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower slightly $(\tau_1, \tau_2)s$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-clopen set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $F(z) \cap V \neq \emptyset$ for each $z \in U$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower slightly $(\tau_1, \tau_2)s$-continuous if F has this property at every point of X.*

Theorem 3.4. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower slightly $(\tau_1, \tau_2)s$-continuous;

(2) $F^-(V)$ is $(\tau_1, \tau_2)s$-open in X for every $\sigma_1\sigma_2$-clopen set V of Y;

(3) $F^+(V)$ is $(\tau_1, \tau_2)s$-closed in X for every $\sigma_1\sigma_2$-clopen set V of Y;

(4) for each $x \in X$ and each $\sigma_1\sigma_2$-clopen set V of Y such that $x \in F^-(V)$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $U \subseteq F^-(V)$.

Proof. The proof is similar to that of Theorem 3.2. $\square$

Acknowledgment. This research project was financially supported by Mahasarakham University.

References

[1] C. Berge, Espaces topologiques fonctions multivoques, Dunod, Paris, 1959.

[2] C. Boonpok, On some spaces via topological ideals, Open Math., **21,** (2023), 20230118.

[3] C. Boonpok, J. Khampakdee, Slight (Λ, sp)-continuity and Λ_{sp}-extremally disconnectedness, Eur. J. Pure Appl. Math., **15,** no. 3, (2022), 1180–1188.

[4] C. Boonpok, C. Viriyapong, Upper and lower almost weak (τ_1, τ_2)-continuity, Eur. J. Pure Appl. Math., **14,** no. 4, (2021), 1212–1225.

[5] C. Boonpok, Upper and lower $\beta(\star)$-continuity, Heliyon, **7,** (2021), e05986.

[6] C. Boonpok, On characterizations of $\star$-hyperconnected ideal topological spaces, J. Math., (2020), 9387601.

[7] C. Boonpok, $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions, Heliyon, **6,** (2020), e05367.

[8] C. Boonpok, On continuous multifunctions in ideal topological spaces, Lobachevskii J. Math., **40,** no. 1, (2019), 24–35.

[9] C. Boonpok, C. Viriyapong, M. Thongmoon, On upper and lower (τ_1, τ_2)-precontinuous multifunctions, J. Math. Computer Sci., **18,** (2018), 282–293.

[10] T. Duangphui, C. Boonpok, C. Viriyapong, Continuous functions on bigeneralized topological spaces, Int. J. Math. Anal., **5,** no. 24, (2011), 1165–1174.

[11] R. C. Jain, The role of regularly open sets in general topology, Ph.D. Thesis, Meerut University, Meerut, 1980.

[12] K. Laprom, C. Boonpok, C. Viriyapong, $\beta(\tau_1, \tau_2)$-continuous multifunctions on bitopological spaces, J. Math., (2020), 4020971.

[13] T. Noiri, V. Popa, Slightly m-continuous multifunctions, Bull. Inst. Math. Acad. Sinica (New Series), **1,** no. 4, (2006), 485–501.

[14] T. M. Nour, Slightly semi-continuous functions, Bull. Calcutta Math. Soc., **87,** no. 2, (1995), 187–190.

[15] P. Sangviset, C. Boonpok, C. Viriyapong, Slightly (m, μ)-continuous functions, Far East J. Math. Sci., **85,** no. 2, (2014), 165–176.

[16] M. Thongmoon, C. Boonpok, Strongly $\theta(\Lambda, p)$-continuous functions, Int. J. Math. Comput. Sci., **19,** no. 2, (2024), 475–479.

[17] C. Viriyapong, C. Boonpok, (Λ, sp)-continuous functions, WSEAS Trans. Math., **21,** (2022), 380–385.

[18] C. Viriyapong, C. Boonpok, $(\tau_1, \tau_2)\alpha$-continuity for multifunctions, J. Math., (2020), 6285763.