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Reduction of Congestion in Data Transfer Using Modified Bulk Service Rule

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Abstract

In a finite buffer queuing system, the congestion mostly occurs due to the higher blocking probability. In this article, the author has presented a Modified Bulk Service (MBS) rule for the finite buffer queueing system under assumptions that the server can accept a customer during ongoing service if serving batch size is lower; however, the time spent in serving the lower batch size is elapsed. Various performance metrics are discussed.

1 Introduction and Model Description

In the study of queueing models 'bulk service queue' playing a vital role. Various bulk service rule are available in the literature [1, 2, 3], etc. Baily [1] was the first to introduce the bulk service queue with fixed batch size, there after GBS rule ([2]) comes in literature under two threshold limits of batch size under assumption that when a service is initiated no customer can join during ongoing service, leads to the higher blocking probability if buffer space is finite. To overcome this congestion, one proposed the Modified Bulk Service (MBS) rule. In MBS rule, the service capacity is allowed to change in between the serving batch (if serving a batch of size$< b$) when an arrival occurs, however, the (incomplete) time spent in serving the lower batch size is

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elapsed. Further, if there is no customer in queue, and the server is busy with a batch of size $'r(< b)`$, and in between an arrival occurs then the server is capable of deciding whether not to include (with probability p) or to include (with probability $q = 1-p$) the new arrival in the ongoing batch service, and serve the new batch size of '$r+1'$ customers. When system is full and server is busy in serving lower batch size can start serving the higher batch size if more arrivals. The recent literature with MBS rule can be found in infinite buffer setup only [5, 6]. For the finite buffer, denote $M/G^{(a\to b)}/1/N$ as [5].

2 Steady State Analysis

Customers arrives according to Poisson process with rate λ and service time is generally distributed. The modified bulk service $(a \to b)$ rule directs the server to serve a batch. System size is $N + b$. Let $S(.)$ be the i.i.d. service time of a batch, with probability density function (pdf) $s(.)$, Laplace-Stieltjes transform (LST) $s^*(.)$ and mean service time $\tilde{s}$. Consider at time t, $N_q(t) \equiv$ queue length, and $N_s(t) \equiv$ server content during busy period.

2.1 Joint probability distributions at arbitrary epoch and departure epoch

Consider $U(t)$, the remaining service time, and define the joint probabilities, at time t: $P_{n,0}(t) \equiv prob.\{N_q(t) = n,\ N_s(t) = 0\}$; $\quad 0 \le n \le a-1$, and $P_{n,r}(x,t)dx \equiv prob.\{N_q(t) = n,\ N_s(t) = r,\ x \le U(t) \le x + dx\}$; $\quad 0 \le n \le N$, $a \le r \le b$, and in steady state: $\lim\limits_{t\to\infty} P_{n,0}(t) = P_{n,0}$; $0 \le n \le a-1$, and $\lim\limits_{t\to\infty} P_{n,r}(x,t) = P_{n,r}(x)$; $0 \le n \le N$, $a \le r \le b$. In similar fashion, define the steady state joint probabilities at departure epoch $p^+_{n,r}$ and $p^+_n \equiv \sum_{r=a}^{b} p^+_{n,r}\ (0 \le n \le N)$. Analyzing the Kolmogrov equations one can obtain the following closed form expression of joint probabilities. We deduce that

$$p^+_{n,r} \;=\; \sigma P_{n,r}(0),\ 0 \le n \le N,\ a \le r \le b. \tag{2.1}$$

where $\sigma^{-1} = \sum_{n=0}^{N}\sum_{r=a}^{b} P_{n,r}(0)$ is the value of the proportionality constant and is found in following theorem.

Theorem 2.1. *The value of σ as appeared in (2.1), is given by*

$$\sigma^{-1} = \sum_{n=0}^{N}\sum_{r=a}^{b} P_{n,r}(0) = \frac{1 - \sum_{n=0}^{a-1} P_{n,0} - \sum_{r=a}^{b-1}\left[\lambda P_{N,r}\tilde{s} + \lambda P^{*(1)}_{N,r}(0) + q\lambda P_{0,r}\tilde{s} + q\lambda P^{*(1)}_{0,r}(0)\right]}{\sum_{n=0}^{N} p^+_n \tilde{s}}, \tag{2.2}$$

where, $\sum_{r=a}^{b} p^+_{n,r} = p^+_n$, $P^*_{n,r}(\theta)$ and $s^*(0) = \tilde{s}$ represent the LST of $P_{n,r}(x)$ and $S(x)$, respectively and $P^{*(1)}_{n,r}(\theta)$ represents its first derivative .

Proof. The steps are similar to that of [4].

Theorem 2.2. *The steady state probabilities $P_{0,r}$, $P_{n,r}$, and $P_{N,r}$; $0 \leq n \leq N$, $a \leq r \leq b$, are given in terms of p_n^+ and $p_{n,r}^+$ as follows:*

$$P_{0,a} = (\lambda\sigma)^{-1}\left(\sum_{i=0}^{a} p_i^+ - p_{0,a}^+\right),\ 0 \leq n \leq a-1, \tag{2.3}$$

$$P_{0,r} = (\lambda\sigma)^{-1}\left[\sum_{i=0}^{r-a-1} q^i\left(\sum_{i=0}^{a} p_{r-i}^+ - p_{0,r-i}^+\right) + q^{r-a}\left(\sum_{i=0}^{a} p_i^+ - p_{0,a}^+\right)\right],\ a+1 \leq r \leq b, \tag{2.4}$$

$$P_{n,a} = (\lambda\sigma)^{-1}\left[p\left(\sum_{i=0}^{a} p_i^+ - p_{0,a}^+\right) - \sum_{i=1}^{n} p_{i,a}^+\right],\ 1 \leq n \leq N-1, \tag{2.5}$$

$$\begin{aligned} P_{n,r} &= (\lambda\sigma)^{-1} p\left\{\sum_{i=0}^{r-a-1} q^i\left(p_{r-i}^+ - p_{0,r-i}^+\right) + q^{r-a}\left(\sum_{i=0}^{a} p_i^+ - p_{0,a}^+\right)\right\} \\ &\quad -(\lambda\sigma)^{-1}\sum_{i=1}^{n} p_{i,r}^+, \qquad a+1 \leq r \leq b-1,\ 1 \leq n \leq N-1, \end{aligned} \tag{2.6}$$

$$P_{n,b} = P_{0,b} + (\lambda\sigma)^{-1}\left(\sum_{i=1}^{n}\sum_{r=a}^{b} p_{b+i,r}^+ - \sum_{i=1}^{n} p_{i,b}^+\right),\ 1 \leq n \leq N-b, \tag{2.7}$$

$$P_{n,b} = P_{0,b} + (\lambda\sigma)^{-1}\left(\sum_{i=1}^{N-b}\sum_{r=a}^{b} p_{b+i,r}^+ - \sum_{i=1}^{n} p_{i,b}^+\right),\ N-b+1 \leq n \leq N-1, \tag{2.8}$$

$$P_{N,a} = (\lambda\sigma)^{-1}\left[p\left(\sum_{i=0}^{a} p_i^+ - p_{0,a}^+\right) - \sum_{i=1}^{N} p_{i,a}^+\right], \tag{2.9}$$

$$\begin{aligned} P_{N,r} &= (\lambda\sigma)^{-1} p\sum_{k=a+1}^{r}\left\{\sum_{i=0}^{k-a-1} q^i\left(p_{k-i}^+ - p_{0,k-i}^+\right) + q^{k-a}\left(\sum_{i=0}^{a} p_i^+ - p_{0,a}^+\right)\right\} \\ &\quad +(\lambda\sigma)^{-1}\left\{p\left(\sum_{i=0}^{a} p_i^+ - p_{0,a}^+\right) - \sum_{i=1}^{N} p_i^+\right\},\ a+1 \leq r \leq b-1. \end{aligned} \tag{2.10}$$

$$P_{N,b} = \lambda P_{N,b-1}\tilde{s} - \lambda P_{N-1,b}^{*(1)}(0) \tag{2.11}$$

Proof. The steps are similar to that of [4].
$\{N_q(t_i), N_s(t_i)\}$ will constitute a 2-D Markov chain with state space $\{(n,r) :\ 0 \leq n \leq N,\ a \leq r \leq b\}$. Define $\pi = (\pi_0^+, \pi_1^+, ..., \pi_N^+)$ and $\pi_n^+ = (p_{n,a}^+, p_{n,a+1}^+, ..., p_{n,b}^+)$. Once $p_{n,r}^+$ are obtained, $P_{n,r}$ are known. Here, $p_{n,r}^+$ can be obtained by solving $\pi P = \pi$, where P is the transition probability matrix, of dimension $N(b-a+1) \times N(b-a+1)$.

3 Performance measure

Average server content: $L_s = \sum_{r=a}^{b} \sum_{n=0}^{N} r P_{n,r} / \sum_{n=0}^{N} \sum_{r=a}^{b} P_{n,r}$; Average queue length: $L_q = \sum_{n=0}^{a-1} n \left(P_{n,0} + \sum_{r=a}^{b} P_{n,r} \right) + \sum_{n=a}^{N} \sum_{r=a}^{b} n P_{n,r}$; Average system length:
$L = \sum_{n=0}^{a-1} n P_{n,0} + \sum_{n=a}^{N+a} \sum_{r=a}^{min(b,n)} n P_{n-r,r} + \sum_{n=N+a+1}^{N+b} \sum_{r=n-N}^{b} n P_{n-r,r}$; Using the Little's law, average waiting in queue $(W_q) = L_q/\bar{\lambda}$ and in the system $(W) = L/\bar{\lambda}$, where $\bar{\lambda}$ is the effective arrival rate, given by $\bar{\lambda} = \lambda (1 - P_B)$; and the blocking probability, is given by, $P_B = P_{N,b}$.

4 Conclusion

In this article, author has successfully obtained the joint probability distribution of queue length and server content at arbitrary epoch in terms of departure epoch for a finite buffer bulk service queue where server renders the service using MBS rule. Comparing the expression of blocking probability of the existing bulk service rules, it is established the fact that using MBS rule the blocking probability can be reduced, which can reduce congestion. Correlated arrivals under MBS rule would be further extension of this work.

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