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Housing project management by using fully fuzzy critical path problem

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Abstract

The pentagonal ranking function is one of the distinctive methods to solve fuzzy problems. The pentagonal membership function for the fuzzy number has been derived and laws have been obtained. The novel ranking pentagonal membership is utilized to solve the fully fuzzy critical path problem. The main aim of the research was to study the Al-Suwayra residential project in Iraq including studying the actual costs of the project and determining the minimum time period to complete it. The fully fuzzy critical path problem was implemented to find the minimum cost and time period to complete the project using the proposed ranking function. The paper includes practical example to find the minimum cost of residential building in the Al-Suwayra area in Iraq, as well as determining the optimal completion time for the project. The proposed technique was applied to the given example with accompanying tables to authenticate the precision of this approach, underscoring the efficiency and dependability of the proposed method. Over and above that, the results obtained were compared with the results of previous research in the literature.

Key words and phrases: Housing Project, Critical Path Problem, Pentagonal Function, Ranking Functions.
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1 Introduction

The concept of decision making plays a significant role in various real-life scenarios such as maximizing profits or minimizing costs, and identifying critical paths. Project management and network models have been widely used techniques in operations research, with applications in various fields. The critical path technique was developed in the 1950s as a part of project management. A project network comprises a set of interrelated activities, considering factors like cost and time, tailored to each project's unique needs and characteristics taking into account the competition between those implementing the project [1]. The fuzzy network model was a topic of practical research, which was transform ambiguous numbers phenomena that have no mathematical formulas into a mathematical problem and have a solution via the ranking function. Ranking function was one of the distinct techniques to solve the fuzzy problems. In 1965, Zadeh [2] was the first to take interest fuzzy set and developed them. De [3] and Jalal [4] employed trapezoidal fuzzy numbers to solve critical path while other researchers [5, 6, 7] used scheduling with fuzzy numbers. In [8], a fuzzy critical path was solved using centroid. Other researchers [9, 10] applied fuzzy game theory. Abbass and Al-Kanani [11] applied the project network problem via trapezoidal fuzzy numbers. The aim of this research is to solve the Al-Suwayra residential project in Iraq using fuzzy critical path analysis through a pentagonal ranking function. The goal was to address the real cost and time required to complete the project with minimal cost and time. The structure of the paper is as follows:
In Section 2, we cover the essential definitions. In Section 3, we present the proposed pentagonal ranking function (PPRF). In Section 4, we apply the novel method and give a practical example. Finally, we conclude our paper in Section 5.

2 Essential Definitions

Definition 2.1. *[2] Let $\Omega \neq \phi$ be a set and let $c_0 \in \Omega$. The* **fuzzy set** *$\mathcal{B}$ in Ω is a subset of the real membership function $\mathcal{M}_{\mathcal{B}} : \Omega \rightarrow [0,1]$. The qualifier $\widetilde{\mathcal{B}}$ is defined as $\widetilde{\mathcal{B}} = \{(c_0, \mathcal{M}_{\mathcal{B}}), c_0 \in \Omega\}$.*

Definition 2.2. *[6] A* **fuzzy number** *$\widetilde{\mathcal{B}}$ is a fuzzy set on $\mathbb{R}$ with the three conditions:*
1) $\exists\ c_0 \in \mathbb{R}$ such that $\mathcal{M}_{\widetilde{\mathcal{B}}} = 1$,
2) $\widetilde{\mathcal{B}}$ is convex,

3) $\mathcal{M}_{\widetilde{\mathcal{B}}}$ *is piecewise continuous.*

3 Ranking Function

3.1 Pentagonal Function (PF) [12]

A fuzzy function for pentagonal fuzzy number $\widetilde{A}_p = (p_1, p_2, p_3, p_4, p_5 : \lambda)$ where p_1, p_2, p_3, p_4 and p_5 are real numbers and $\lambda \in [0,1]$ is defined as follows:

$$\mathcal{M}_{\widetilde{A}_p}(x) = \begin{cases} \lambda\left(\frac{x-p_1}{p_2-p_1}\right) & p_1 \le x < p_2 \\ (1-\lambda)\left(\frac{x-p_2}{p_3-p_2}\right) & p_2 \le x < p_3 \\ 1 & x = p_3 \\ (1-\lambda)\left(\frac{p_4-x}{p_4-p_3}\right) & p_3 \le x < p_4 \\ \lambda\left(\frac{p_5-x}{p_5-p_4}\right) & p_4 \le x < p_5 \\ 0 & otherwise \end{cases} \tag{3.1}$$

3.2 The Proposed Pentagonal Ranking Function

Now, by using the γ-cut with $0 \le \gamma \le 1$ for equation (3.1), where $\inf \widetilde{A}(\gamma) = \{x \in \widetilde{A} \backslash \mathcal{M}(x) \ge \lambda\}$ is the infimum of $\widetilde{A}$, and $\sup \widetilde{A}(\gamma) = \{x \in \widetilde{A} \backslash \mathcal{M}(x) \le \lambda\}$ is the supremum of $\widetilde{A}$, we can define the γ-cut of the pentagonal function as:

$$\widetilde{A}_p(\gamma) = \begin{cases} \frac{\gamma(p_2-p_1)+\lambda p_1}{\lambda} = \inf_1\left(\widetilde{A}_p(\gamma)\right) \\ \frac{\gamma(p_3-p_2)+p_2(1-\lambda)}{(1-\lambda)} = \inf_2\left(\widetilde{A}_p(\gamma)\right) \\ \frac{\gamma(p_3-p_4)+p_4(1-\lambda)}{(1-\lambda)} = \sup_1\left(\widetilde{A}_p(\gamma)\right) \\ \frac{\gamma(p_4-p_5)+\lambda p_5}{\lambda} = \sup_2\left(\widetilde{A}_p(\gamma)\right) \end{cases} \tag{3.2}$$

where $0 \le \lambda \le 1$. Now, by applying the pentagonal membership function via equation (3.2), we obtain
$R_\gamma\left(\widetilde{A}_p\right) = \frac{1}{2}\int_0^\lambda \left[\inf_1 \widetilde{A}_p(\gamma) + \inf_2 \widetilde{A}_p(\gamma)\right] d\gamma + \frac{1}{2}\int_0^{(1-\lambda)} \left[\sup_1 \widetilde{A}_p(\gamma) + \sup_2 \widetilde{A}_p(\gamma)\right] d\gamma.$
We can simplify and deduce the following formula:

$$R_\gamma\left(\widetilde{A}_p\right) = \frac{\lambda}{4}\left[p_1 + p_2 + p_4 + p_5\right] + \frac{1-\lambda}{4}\left[p_2 + 2p_3 + p_4\right] \tag{3.3}$$

4 Practical Part

The modern world is witnessing significant advancements in civil projects, including commercial and housing developments. Developing countries have made notable progress in effectively overcoming obstacles that once hindered growth. Today, project construction heavily relies on sophisticated planning techniques. Initially, companies used the Gantt chart to determine the duration between activities and the overall start and end times of a project. This method evolved into the Program Evaluation and Review Technique (PERT), which specifies time for each activity. Eventually, PERT developed into the Critical Path Technique (CPT), which identifies the end time for each activity, ensuring a more precise project timeline. The project consists of 104 residence units on a 6080 m^2 area. Each unit has 4 floors, and the project is expected to be completed in 40 months, based on the real-time assessment using the Gantt technique. Here are the project details, displaying the real cost and time for every step in Table 1, representing the activities of the project, along with the associated costs and times for each activity.

Table 1: Display the real cost and time of the project

Activity	Costs in $	Time in weeks	Symbol of activity
Site preparation, equipment setup	1310	2	A
Site cleaning, sanitation tasks	1870	11	B
Excavation and foundation boring	8870	2	C
Asphalt groundwork, concrete base	81285	11	D
Water channel service	32100	20	E
Installing water and gas pipelines	13380	19	F
Electrical installation, wiring setup	27120	15	G
Asphalt upper structure, concrete topping	54000	18	H
Ceilings painting and waterproofing	32345	5	I
Woodworking, joinery	34840	17	J
Metalwork, forge work	24100	11	K
Building's main structure	14085	44	L
Cement wall coating, plastering	110325	16	M
Preparing the external perimeter of the units	1720	13	N
Painting, coloring	104510	24	O
Glass/ windows installation	6870	16	P
Floor tiling	108380	11	Q

We can visualize the previous activities using the network model as Figure 1 shows.

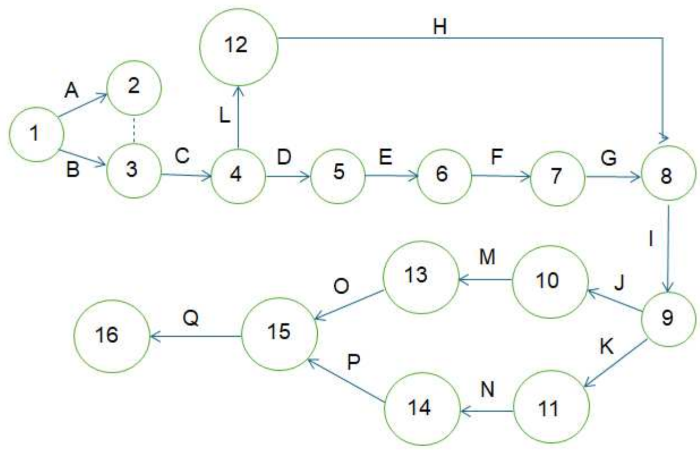

Figure 1: The time and cost network description for activities of the project

The critical path of this project in the crisp method for the minimum real cost is \$327375 and the min real time is 133 weeks as Figure 2 shows.

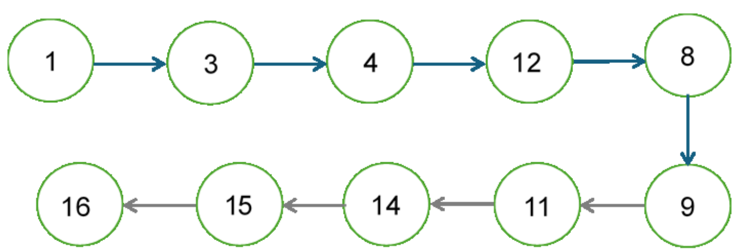

Figure 2: The time and cost network description for activities of the project

4.1 The Pentagonal Functions to Determine the Fuzzy Critical Path

The pentagonal fuzzy number was applied to calculate the minimum cost for our project. Let $\Delta_1 = 100$, $\Delta_2 = 150$. Consider the pentagonal fuzzy numbers $(p_1 - \Delta_1, p_2 + \Delta_1, p_3, p_4 - \Delta_2, p_5 + \Delta_2)$. Using equation (3.3) and the fuzzy numbers $\widetilde{A}_p = (p_1, p_2, p_3, p_4, p_5 : \lambda)$ with weight function $\lambda \in [0, 1]$, choose $\lambda = 0.1$ and $\lambda = 0.9$ to compute the PPRF in equation (3.3). The results for each activity were demonstrated in Table 2.

Table 2: Computation of the cost using PPRF for each activities in two cases when $\lambda = 0.1$ and $\lambda = 0.9$

Real Costs $	Pentagonal Fuzzy number for Costs $	PPRF $\lambda = 0.1$	PPRF $\lambda = 0.9$	Using ranking function in [13]
1310	(1210,1410,1310,1160,1460)	1298.75	1308.75	1303.75
1870	(1770,1970,1870,1720,2020)	1858.75	1868.75	1863.75
8870	(8770,8970,8870,8720,9020)	8858.75	8868.75	8863.75
81285	(81185,81385,81285, 81135, 81435)	81273.75	81283.75	81278.75
32100	(32000, 32200, 32100, 31950,32250)	32088.75	32098.75	32093.75
13380	(13280, 13480, 13380, 13230,13530)	13368.75	13378.75	13373.75
27120	(27020,27220, 27120, 26970, 27270)	27108.75	27118.75	27113.75
54000	(53900, 54100, 54000,53850,54150)	53988.75	53998.75	53993.75
32345	(32245, 32445,32345, 32195,32495)	32333.75	32343.75	32338.75
34840	(34740, 34940,34840,34690,34990)	34828.75	34838.75	34833.75
24100	(24000,24200,24100, 23950, 24250)	24088.75	24098.75	24093.75
14085	(13985, 14185,14085,13935,14235)	14073.75	14083.75	14078.75
110325	(110225,110425,110325,110175,110475)	110313.75	110323.75	110318.75
1720	(1620, 1820, 1720, 1570, 1870)	1708.75	1718.75	1713.75
104510	(104410, 104610,104510,104360,104660)	104498.75	104508.75	104503.75
6870	(6770, 6970, 6870, 6720, 7020)	6858.75	6868.75	6863.75
108380	(108280,108480,108380,108230,108530)	108368.75	108378.75	108373.75

The fully fuzzy critical path of this project when using PPRF to compute the cost when $\lambda = 0.1$ was \$327273.8 and when $\lambda = 0.9$ it was \$327363.8. According to [13], the cost was \$327318.8.

Now, to measure the fuzzy shortest time to complete the project using PPRF, let $\Delta_1 = 0.5$, $\Delta_2 = 1$. Consider the pentagonal fuzzy numbers $(p_1 - \Delta_1, p_2 + \Delta_1, p_3, p_4 - \Delta_2, p_5 + \Delta_2)$. Using (3.3) where the fuzzy numbers $\widetilde{A}_p = (p_1, p_2, p_3, p_4, p_5 : \lambda)$ with weight function $\lambda \in [0, 1]$. Table 3 demonstrates all results.

Table 3: Computation of the period using PPRF for each activity in two cases $\lambda = 0.1$ and $\lambda = 0.9$

Real Time (week)	Fuzzy time PPRF	$\lambda = 0.1$	$\lambda = 0.9$	Using ranking function [13]
2	(1.5,2.5,2,1,3)	1.8875	1.9875	1.9375
11	(10.5,11.5,11,10,12)	10.8875	10.9875	10.9375
2	(1.5,2.5,2,1,3)	1.8875	1.9875	1.9375
11	(10.5,11.5,11,10,12)	10.8875	10.9875	10.9375
20	(19.5,20.5,20,19,21)	19.8875	19.9875	19.9375
19	(18.5,19.5,19,18,20)	18.8875	18.9875	18.9375
15	(14.5,15.5,15,14,16)	14.8875	14.9875	14.9375
18	(17.5,18.5,18,17,19)	17.8875	17.9875	17.9375
5	(4.5,5.5,5,4,6)	4.8875	4.9875	4.9375
17	(16.5,17.5,17,16,18)	16.8875	16.9875	16.9375
11	(10.5,11.5,11,10,12)	10.8875	10.9875	10.9375
44	(43.5,44.5,44,43,45)	43.8875	43.9875	43.9375
16	(15.5,16.5,16,15,17)	15.8875	15.9875	15.9375
13	(12.5,13.5,13,12,14)	12.8875	12.9875	12.9375
24	(23.5,24.5,24,23,25)	23.8875	23.9875	23.9375
16	(15.5,16.5,16,15,17)	15.8875	15.9875	15.9375
11	(10.5,11.5,11,10,12)	10.8875	10.9875	10.9375

The shortest fully fuzzy critical path of this project by using PPRF to compute the time when $\lambda = 0.1$ was 131.9875 weeks and when $\lambda = 0.9$ was 132.8875 weeks and when we use the reference [13] the solution is 132.4375 weeks. The best and minimum time when $\lambda = 0.1$ and was demonstrated in Table 3. Moreover, the results are better than the real time which is 133 weeks.

5 Conclusion

The Al-Suwayra housing project in Iraq underwent analysis using a fully fuzzy critical path approach incorporating a proposed pentagonal function formulated in this study. A comparative analysis was conducted between the actual results derived using conventional methods for project costs and scheduling and the outcomes of the proposed technique, affirming its viability and precision in attaining the requisite solution. Furthermore, the determination of the shortest fully fuzzy critical path was found in two cases, particularly when $\lambda = 0.1$ and $\lambda = 0.9$, demonstrated the latter as the optimal solution, as evidenced in Tables 2 and 3. Comparative evaluation against reference [13] substantiated the superior performance of the proposed methodology.

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