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The Effect of Vaccination on the Monkeypox Disease by Using Holling Type II

Shaymaa Mukhlif Shraida, May Mohammed Helal, Areej Salah Mohammed

Department of Mathematics
College of Education for Pure Science
Ibn Al-Haitham University of Baghdad
Baghdad, Iraq

email: shaimaa.m.sh@ihcoedu.uobaghdad.edu.iq, may.m@ihcoedu.uobaghdad.edu.iq, areej.s.m@ihcoedu.uobaghdad.edu.iq

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Abstract

Monkeypox virus leads to an infectious disease called Monkeypox (mpox). This disease may cause painful rashes, swollen lymph nodes, headaches and fever. Taking the vaccination for mpox can reduce these symptoms and also prevent people from becoming seriously ill. In this paper, we study the effect of vaccination on the (mpox) disease by using the functional response (Holling type II). Some analyses are made for the model and the results are confirmed by using a numerical simulation.

1 Introduction

In recent years, a lot of research on of epidemic diseases has been done, but the challenge is how to minimize the spread of these diseases through population. To study the behavior of the diseases, a mathematical model is used

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[1]-[6]. Lasisi et al. [7] used ODE to improve a mathematical model transmission of Monkey-pox and the analysis of the model was investigated. Bhunu et al. [8] represented the model as a non-linear differential equations system. Then, they discussed the stability of the conditions for the disease-free equilibrium. Sulaiman et al. [9] included the treatment and vaccination in their model as control strategies. Using standard approaches, Olumuyiwa et al. [10] used the mathematical model for Monkey-pox transmission dynamics with both the fractional order and classical differential equations, and also studied the disease stability. Furthermore, they discussed the behavior of the system's dynamic to find an adequate way to control the infection. Some researchers have proposed their disease model with interaction function (Holling types) interactions. For example, Banshidar and Swarup [11] studied prey-predator diseases with general Holling type interactions. They drive the local and global stability for the system and they got the permanence and impermanence conditions of the system. Ruiqing et al. [12] used a dynamical analysis with Holling II functional response for Hepatitis B virus to describe a fractional-order model while the response function (Holling function type II) used on predator-prey model by Jean et al. [13].

2 Mathematical Model

In this section, we describe the mathematical model for (mbox) infectious disease. Assuming that the whole human population $N(t)$ is subdivided into five sub-compartments, which are respectively, susceptible $S(t)$, exposed $E(t)$, infected $I(t)$, vaccinated V(t), and recovered $R(t)$. This gives us:
$N(t) = S(t) + E(t) + I(t) + V(t) + R(t)$, with the initial conditions:

$$S(0) > 0, E(0) \geq 0, I(0) \geq 0, V(0) > 0 \text{ and } R(0) \geq 0, \tag{2.1}$$

and the nonlinear differential equation is:

$$\begin{aligned}
\dot{S} &= \Lambda - \mu S - \rho S - \frac{\beta(1-n)SI}{1+\alpha I}, \\
\dot{E} &= \frac{\beta(1-n)SI}{1+\alpha I} - \mu E - \alpha_2 E, \\
\dot{I} &= \alpha_2 E - (\mu + \gamma + \gamma_2) I, \\
\dot{V} &= \rho S - \mu V, \\
\dot{R} &= \gamma_2 I - \mu R
\end{aligned} \tag{2.2}$$

where Λ represents the recruitment rate and μ is a parameter for death due to natural causes. The susceptible population is exposed if they get in contact with a member of the infected population through the rate β. The vaccine will be effective when $n = 1$ and ineffective when $n = 0$. The rate α_2 represents the exposed population that get infected. In the infected population, some of them will die as a result of the disease, with rate γ. The rate γ_2 represents the population that recovers while the rate ρ denotes the population that have received the vaccine. Moreover, in the right-hand side of model 2.2, the function is continuous and has continuous partial derivatives on the following space:

$$R^5_+ = \{(S, E, I, V, R) \in R^s : S(0) > 0, E(0) \geq 0, I(0) \geq 0, V(0) > 0 \text{ and } R(0) \geq 0\}. \tag{2.3}$$

Therefore, the solution of the system (2.2) exists and is unique.

2.1 Positivity and Boundedness

Theorem 2.1. *All solutions $S(t), E(t), I(t), V(t), R(t)$ starting from positive initial conditions in equation 2.1, and for all time $t \geq 0$ stay positive.*

Proof.
We have:

$$\begin{aligned}
&\dot{S} \mid_{S=0} = \Lambda, \\
&\dot{E} \mid_{E=s} = \frac{\beta(1-n)SI}{1+\alpha I}, \ S > 0, I \geq 0 \\
&\dot{I} \mid_{I=0} = \alpha_2 E, \ E \geq 0 \\
&\dot{V} \mid_{V=0} = \rho S, \ S > 0 \\
&\dot{R} \mid_{R=0} = \gamma_2 I, \ I \geq 0.
\end{aligned} \tag{2.4}$$

Therefore, we get the non-negative solutions. Moreover, the last equation is not dependent on the other equations and so we can leave out this equation to reduce the system to:

$$\begin{aligned}
\dot{S} &= \Lambda - \mu S - \rho S - \frac{\beta(1-n)SI}{1+\alpha I}, \\
\dot{E} &= \frac{\beta(1-n)SI}{1+\alpha I} - \mu E - \alpha_2 E, \\
\dot{I} &= \alpha_2 E - (\mu + \gamma + \gamma_2) I, \\
\dot{V} &= \rho S - \mu V,
\end{aligned} \tag{2.5}$$

Theorem 2.2. *All the solutions of our model are uniformly bounded.*

Proof.
The whole human population is:

$$\dot{N} = \dot{S} + \dot{E} + \dot{I} + \dot{V} + \dot{R} \tag{2.6}$$

After substituting the equivalent equation from our model into each item in equation 2.6 and by solving and simplifying it we get

$$supN(t) = \frac{\Lambda}{\mu}, \text{ for each } t > 0 \tag{2.7}$$

3 Existence of Equilibrium Point

The first disease-free equilibrium point is denoted by $l_{00} = (S_{00}, 0, 0, V_{00})$. Substituting this point in model 2.5 gives the following:

$$S_{00} = \frac{\Lambda}{\mu + \rho}, V_{00} = \frac{\rho}{\mu} \frac{\Lambda}{\mu + \rho}. \tag{3.8}$$

The endemic equilibrium point in model 2.5 is denoted by $l_{11} = (S_{11}, E_{11}, I_{11}, V_{11})$, so from model 2.5 we can get

$$S_{11} = \frac{\Lambda\alpha_2(\mu+\rho)\alpha(\mu+\rho) + \Lambda\alpha_2\beta(1-n)[(\mu+\rho)-1] + [(\mu+\alpha_2)(\mu+\gamma+\gamma_2)(\mu+\rho)]}{\alpha_2(\mu+\rho)[\alpha(\mu+\rho)+\beta(1-n)]} \tag{3.9}$$

$$E_{11} = \frac{\mu+\gamma+\gamma_2}{\alpha_2} \frac{\alpha_2\Lambda\beta(1-n) - (\mu+\alpha_2)(\mu+\gamma+\gamma_2) + (\mu+\rho)}{(\mu+\alpha_2)(\mu+\gamma+\gamma_2)[\alpha(\mu+\rho)+\beta(1-n)]} \tag{3.10}$$

$$I_{11} = \frac{\alpha_2\Lambda\beta(1-n) - (\mu+\alpha_2)(\mu+\gamma+\gamma_2) + (\mu+\rho)}{(\mu+\alpha_2)(\mu+\gamma+\gamma_2)[\alpha(\mu+\rho)+\beta(1-n)]} \tag{3.11}$$

$$V_{11} = \frac{\rho}{\mu} \frac{\Lambda\alpha_2(\mu+\rho)\alpha(\mu+\rho) + \Lambda\alpha_2\beta(1-n)[(\mu+\rho)-1] + [(\mu+\alpha_2)(\mu+\gamma+\gamma_2)(\mu+\rho)]}{\alpha_2(\mu+\rho)[\alpha(\mu+\rho)+\beta(1-n)]} \tag{3.12}$$

with the conditions that leave S_{11}, E_{11}, I_{11} and V_{11} positive:

$$\begin{aligned} &(1): \Lambda\alpha_2\left[\alpha(\mu+\rho)+\beta(1-n)\right] + (\mu+\rho) > \alpha_2\beta(1-n) \\ &(2): \alpha_2\Lambda\beta(1-n) > (\mu+\alpha_2)(\mu+\gamma+\gamma_2)(\mu+\rho) \end{aligned} \tag{3.13}$$

4 Local and Global Stability

In this section we investigate the local and global stability for the equilibrium points of model 2.5. Now, by using the Jacobian matrix for the point$(S_{00}, 0, 0, V_{00})$, we find that the local stability for the first point is satisfied if $(\mu + \alpha_2)(\mu + \gamma + \gamma_2) - \alpha_2\beta(1-n)S > 0$ is satisfied, while the local stability for the point $(S_{11}, E_{11}, I_{11}, V_{11})$ is satisfied with the conditions:

$$\max\left(\frac{-\alpha_2\beta(1-n)S}{(\mu+\gamma+\gamma_3)(1+\alpha I)^2}, \frac{\beta(1-n)I}{(1+\alpha I)}, \frac{-\alpha_2\beta(1-n)S}{(\mu+\alpha_2)(1+\alpha I)^2)}\right) < -(\mu+\rho) - \frac{\beta(1-n)I}{(1+\alpha I)}) \tag{4.14}$$

By using Lyapunov function (see [14]), we prove the equilibrium points are globally stable.

Theorem 4.1. *The conditions below lead to the global asymptotic stability of* L_{00}

$$(\frac{\rho}{2})^2 < \frac{\mu+\rho}{2}\frac{\mu}{2} \text{ and } S - S_{00} > 1. \tag{4.15}$$

Proof.
By using Lyapunov function, we get

$$L_{00} = \frac{(S-S_{00})^2}{2} + \frac{(E-E_{00})^2}{2} + \frac{(I-I_{00})^2}{2} + \frac{(V-V_{00})^2}{2} \tag{4.16}$$

The derivative of L_{00} corresponding to the model solutions is

$$\frac{L_{00}}{dt} = (S-S_{00})\frac{dS}{dt} + (E-E_{00})\frac{dE}{dt} + (I-I_{00})\frac{dI}{dt} + (V-V_{00})\frac{dV}{dt} \tag{4.17}$$

Now, we simplify the equation 4.17 to get

$$(V-V_{00})\frac{dV}{dt} = -\left[\sqrt{\mu+\rho}(S-S_0) - \sqrt{\mu}(V-V_{00}\right] - (S-S_{00}-1)\frac{\beta(1-n)SI}{1+\alpha I} \tag{4.18}$$

Therefore, equation 4.18 is Lyapunov function. This proves that the conditions 4.15 are satisfied.

Theorem 4.2. L_{11} *has global asymptotic stability under the following conditions:*

$$\left(\frac{\beta(1-n)S_{11}}{(1+\alpha I)(1+\alpha I_{11})}\right)^2 < \frac{2}{3}\left((\mu+\rho)+\frac{\beta(1-n)I}{1+\alpha I}\right)(\mu+\gamma+\gamma_2) \quad (4.19)$$

$$\left(\frac{\beta(1-n)I}{1+\alpha I}\right)^2 < \frac{2}{3}\left[(\mu+\rho)+\frac{\beta(1-n)I}{1+\alpha I}\right](\mu+\alpha_2) \quad (4.20)$$

$$\rho^2 < \left[(\mu+\rho)+\frac{\beta(1-n)I}{1+\alpha I}\right]\frac{\mu}{12}, \quad (4.21)$$

and

$$\left[\frac{\beta(1-n)S_{11}}{(1+\alpha I)(1+\alpha I_{11})}+\alpha_2\right]^2 < (\mu+\alpha_2)(\mu+\gamma+\gamma_2) \quad (4.22)$$

Proof.
To prove this case, we use the Lyapunov function as follows:

$$L_{11} = \frac{(S-S_{11})^2}{2} + \frac{(E-E_{11})^2}{2} + \frac{(I-I_{11})^2}{2} + \frac{(V-V_{11})^2}{2} \quad (4.23)$$

The derivative of L_{11} corresponding to the model solutions is

$$\frac{L_{11}}{dt} = (S-S_{11})\frac{dS}{dt} + (E-E_{11})\frac{dE}{dt} + (I-I_{11})\frac{dI}{dt} + (V-V_{11})\frac{dV}{dt} \quad (4.24)$$

By simplifying equation 4.24, we obtain:

$$\frac{L_{11}}{dt} = A^* + B^* + C^* + D^* \quad (4.25)$$

$A^* = -\frac{(S-S_{11})^2}{3}\left((\mu+\rho)-\frac{\beta(1-n)I}{1+\alpha I}\right) - \frac{\beta(1-n)(S-S_{11})(I-I_{11})S_1}{(1+\alpha I)(1+\alpha I_{11})} - (\mu+\gamma+\gamma_2)(I-I_{11})^2$
$B^* = -\frac{(S-S_{11})^2}{3}\left((\mu+\rho)+\frac{\beta(1-n)I}{1+\alpha I}\right) + \frac{\beta(1-n)(E-E_{11})(S-S_{11})I_{11}}{(1+\alpha I)} - \frac{(E-E_{11})^2}{2}(\mu+\alpha_2)$
$C^* = -\left[\frac{(S-S_{11})^2}{3}\left((\mu+\rho)+\frac{\beta(1-n)I}{1+\alpha I}\right) + (V-V_{11})(S-S_{11})\rho - (V-V_{11})^2\mu\right] - \frac{(E-E_{11})^2}{2}(\mu+\alpha_2)$
$D^* = -(E-E_{11})(I-I_{11})\left[\frac{\beta(1-n)S_{11}}{(1+\alpha I)(1+\alpha I_{11})} + \alpha_2 + (\mu+\gamma+\gamma_2)\frac{(I-I_{11})^2}{2}\right]$

This yields a Lyapunov function and the conditions 4.19-4.22 are satisfied.

5 Numerical Simulation

In this section, a numerical simulation is used to confirm our results and to get a better understanding of the effects of Holling and vaccine parameters. Figure 1 discussed the different values of $\alpha = 0.1, 0.01, 0.001$ and $0,0001$ (Holling parameter). In Figure 1 (A), as $\alpha = 0.1$ (blue line) is big, the number of susceptible population is big and it gets smaller and smaller as α gets smaller. Therefore, by using Holling parameter we can keep the number of susceptible population large, thus less people will get exposed or infected. Figure 1 (B), represents the exposed population for different values of *alpha*. As α gets smaller (blue line to black line), less people get infected as Figure 1 (C) shows. Figure 1(D) explains that as α tends to grow (blue line), less communication will happen between the susceptible and infected people therefore more people will have a chance to receive the vaccine. Different values of ρ have been discussed in Figure 2, where Figure 2 (A) explains that with the vaccine parameter $\rho = 10$ (black line), many people will have the vaccine so they will move to the vaccination compartment. Meanwhile, in Figure 2 (B) and 2 (C) the vaccine parameter has no effect because the vaccination only works on susceptible individuals and not those who have been exposed or already infected. For Figure 2 (D), we can see the effect of the vaccination on people as $\rho = 0.1, 2, 6, 10$ moving from blue line to black line respectively, indicating that more people will get the vaccine.

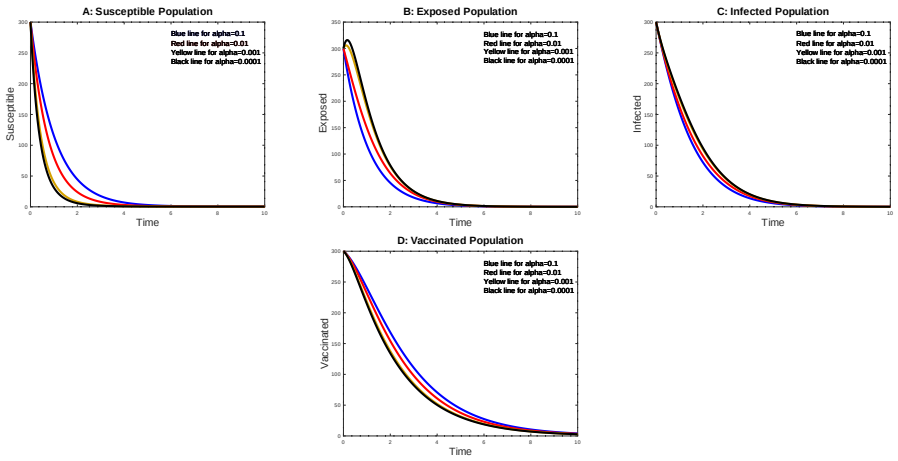

Figure 1: Result of simulation, where $\Lambda = 0.3$, $\rho = 0.4$, $n = 0.4$, $\alpha_2 = 0.5$. $\mu = 0.5$, $\gamma = 0.1$, $\gamma_2 = 0.5$, $\beta = 0.01$ with different values of α.

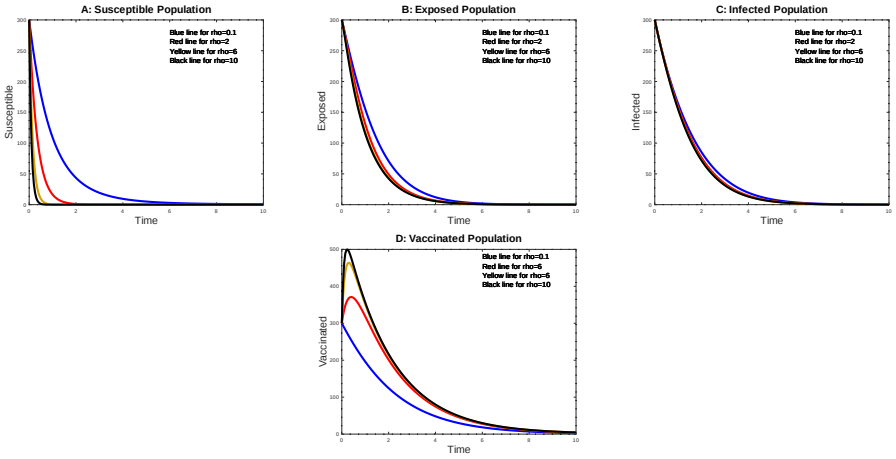

Figure 2: Result of simulation, where $\Lambda = 0.3$, $n = 0.4$, $\alpha = 0.01$ $\alpha_2 = 0.5$. $\mu = 0.5$, $\gamma = 0.1$, $\gamma_2 = 0.5$, $\beta = 0.01$ with different values of ρ.

6 Concluding Remarks

Monkeypox virus (mpox) has been investigated in this paper . The effect of the functional response (Holling type II) and the vaccination is discussed. As the Holling function parameter gets large, disease transmission can be somewhat controlled. Furthermore, increasing the value of the vaccination parameter prevents people from contracting this virus. A numerical simulation was used and confirmed our results.

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