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Characterizations of weakly $s(\Lambda, p)$-open functions and weakly $s(\Lambda, p)$-closed functions

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Abstract

In this article, we deal with the notions of weakly $s(\Lambda, p)$-open functions and weakly $s(\Lambda, p)$-closed functions. Moreover, we establish several characterizations of weakly $s(\Lambda, p)$-open functions and weakly $s(\Lambda, p)$-closed functions.

1 Introduction

In 1984, Rose [11] introduced and studied the notions of weakly open functions and almost open functions. Rose and Janković [10] investigated some of the fundamental properties of weakly closed functions. In 2004, Caldas and Navalagi [5] introduced two new classes of functions called weakly preopen functions and weakly preclosed functions. Weak preopenness (resp. weak preclosedness) is a generalization of weak openness (resp. weak closedness). In 2006, Caldas et al. [4] introduced and investigated the notions of weakly semi-θ-open functions and weakly semi-θ-closed functions. Noiri and Popa

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[9] studied a new class of functions called M-closed functions as functions defined between sets satisfying some conditions. In 2008, Caldas and Navalagi [3] presented the class of weak δ-openness (resp. weak δ-closedness) as a new generalization of δ-openness (resp. δ-closedness) and investigated several characterizations of weakly δ-open functions and weakly δ-closed functions. In [1], the present authors introduced and studied the notions of weakly $p(\Lambda,p)$-open functions and weakly $p(\Lambda,p)$-closed functions. Some characterizations of weakly $b(\Lambda,p)$-open functions and weakly $\delta(\Lambda,p)$-open functions were investigated in [6] and [13], respectively. Furthermore, several characterizations of (Λ,p)-closed functions and weakly $\delta(\Lambda,p)$-closed functions were established in [7] and [8], respectively. In this article, we introduce the notions of weakly $s(\Lambda,p)$-open functions and weakly $s(\Lambda,p)$-closed functions. In particular, we discuss some characterizations of weakly $s(\Lambda,p)$-open functions and weakly $s(\Lambda,p)$-closed functions.

2 Preliminaries

A subset A of a topological space (X,τ) is called (Λ,p)-*closed* [2] if $A = T\cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ,p)-closed set is called (Λ,p)-*open*. The family of all (Λ,p)-open sets in a topological space (X,τ) is denoted by $\Lambda_pO(X,\tau)$. Let A be a subset of a topological space (X,τ). A point $x\in X$ is called a (Λ,p)-*cluster point* [2] of A if $A\cap U\neq\emptyset$ for every (Λ,p)-open set U of X containing x. The set of all (Λ,p)-cluster points of A is called the (Λ,p)-*closure* [2] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ,p)-open sets of X contained in A is called the (Λ,p)-*interior* [2] of A and is denoted by $A_{(\Lambda,p)}$. A subset A of a topological space (X,τ) is said to be $s(\Lambda,p)$-*open* [2] (resp. $p(\Lambda,p)$-*open*, $\alpha(\Lambda,p)$-*open* [14], $r(\Lambda,p)$-*open* [2]) if $A\subseteq[A_{(\Lambda,p)}]^{(\Lambda,p)}$ (resp. $A\subseteq[A^{(\Lambda,p)}]_{(\Lambda,p)}$, $A\subseteq[[A_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}$, $A=[A^{(\Lambda,p)}]_{(\Lambda,p)}$). The complement of a $s(\Lambda,p)$-open (resp. $p(\Lambda,p)$-open, $\alpha(\Lambda,p)$-open, $r(\Lambda,p)$-open) set is called $s(\Lambda,p)$-*closed* (resp. $p(\Lambda,p)$-*closed*, $\alpha(\Lambda,p)$-*closed*, $r(\Lambda,p)$-*closed*). The intersection of all $s(\Lambda,p)$-closed sets of X containing A is called the $s(\Lambda,p)$-*closure* of A and is denoted by $A^{s(\Lambda,p)}$. The union of all $s(\Lambda,p)$-open sets of X contained in A is called the $s(\Lambda,p)$-*interior* of A and is denoted by $A_{s(\Lambda,p)}$. A subset A of a topological space (X,τ) is called $\theta(\Lambda,p)$-*closed* [2] if $A=A^{\theta(\Lambda,p)}$. The complement of a $\theta(\Lambda,p)$-closed set is said to be $\theta(\Lambda,p)$-*open*. A point $x\in X$ is called a $\theta(\Lambda,p)$-*interior point* [12] of A if $x\in U\subseteq U^{(\Lambda,p)}\subseteq A$ for some $U\in\Lambda_pO(X,\tau)$. The set of all $\theta(\Lambda,p)$-interior points of A is called the

$\theta(\Lambda, p)$-*interior* [12] of A and is denoted by $A_{\theta(\Lambda,p)}$.

Lemma 2.1. [12] *For subsets A and B of a topological space (X, τ), the following properties hold:*

(1) $X - A^{\theta(\Lambda,p)} = [X - A]_{\theta(\Lambda,p)}$ *and* $X - A_{\theta(\Lambda,p)} = [X - A]^{\theta(\Lambda,p)}$.

(2) A *is* $\theta(\Lambda, p)$*-open if and only if* $A = A_{\theta(\Lambda,p)}$.

(3) $A \subseteq A^{(\Lambda,p)} \subseteq A^{\theta(\Lambda,p)}$ *and* $A_{\theta(\Lambda,p)} \subseteq A_{(\Lambda,p)} \subseteq A$.

(4) If $A \subseteq B$*, then* $A^{\theta(\Lambda,p)} \subseteq B^{\theta(\Lambda,p)}$ *and* $A_{\theta(\Lambda,p)} \subseteq B_{\theta(\Lambda,p)}$.

(5) If A *is* (Λ, p)*-open, then* $A^{(\Lambda,p)} = A^{\theta(\Lambda,p)}$.

3 Characterizations of weakly $s(\Lambda, p)$-open functions

In this section, we introduce the notion of weakly $s(\Lambda, p)$-open functions. Moreover, some characterizations of weakly $s(\Lambda, p)$-open functions are discussed.

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly $s(\Lambda, p)$-open if $f(U) \subseteq [f(U^{(\Lambda,p)})]_{s(\Lambda,p)}$ for each (Λ, p)-open set U of X.*

Theorem 3.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f *is weakly* $s(\Lambda, p)$*-open;*

(2) $f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{s(\Lambda,p)}$ *for every subset* A *of* X*;*

(3) $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{s(\Lambda,p)})$ *for every subset* B *of* Y*;*

(4) $f^{-1}(B^{s(\Lambda,p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda,p)}$ *for every subset* B *of* Y.

Proof. The proof follows from Theorem 3.2 of [6]. □

Theorem 3.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective function. Then the following properties are equivalent:*

(1) f *is weakly* $s(\Lambda, p)$*-open;*

(2) $[f(U)]^{s(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ *for every* (Λ, p)*-open set* U *of* X*;*

(3) $[f(K_{(\Lambda,p)})]^{s(\Lambda,p)} \subseteq f(K)$ *for every* (Λ,p)*-closed set* K *of* X.

Proof. The proof follows from Theorem 3.2 of [1]. □

Theorem 3.4. *For a function* $f : (X,\tau) \to (Y,\sigma)$*, the following properties are equivalent:*

(1) f *is weakly* $s(\Lambda,p)$*-open;*

(2) for each $x \in X$ *and each* (Λ,p)*-open set* U *of* X *containing* x*, there exists a* $s(\Lambda,p)$*-open set* V *of* Y *containing* $f(x)$ *such that* $V \subseteq f(U^{(\Lambda,p)})$.

Proof. (1) ⇒ (2): Let $x \in X$ and U be any (Λ,p)-open set of X containing x. Since f is weakly $s(\Lambda,p)$-open, $f(x) \in f(U) \subseteq [f(U^{(\Lambda,p)})]_{s(\Lambda,p)}$. Put $V = [f(U^{(\Lambda,p)})]_{s(\Lambda,p)}$, then V is a $s(\Lambda,p)$-open set of Y containing $f(x)$ such that $V \subseteq f(U^{(\Lambda,p)})$.

(2) ⇒ (1): Let U be any (Λ,p)-open set of X and $y \in f(U)$. It follows from (2) that $V \subseteq f(U^{(\Lambda,p)})$ for some $s(\Lambda,p)$-open set V of Y containing y. Thus $y \in V \subseteq [f(U^{(\Lambda,p)})]_{s(\Lambda,p)}$ and hence $f(U) \subseteq [f(U^{(\Lambda,p)})]_{s(\Lambda,p)}$. This shows that f is weakly $s(\Lambda,p)$-open. □

The proof of the following theorem is straightforward and thus is omitted.

Theorem 3.5. *For a function* $f : (X,\tau) \to (Y,\sigma)$*, the following properties are equivalent:*

(1) f *is weakly* $s(\Lambda,p)$*-open;*

(2) $f(K_{(\Lambda,p)}) \subseteq [f(K)]_{s(\Lambda,p)}$ *for every* (Λ,p)*-closed set* K *of* X*;*

(3) $f([U^{(\Lambda,p)}]_{(\Lambda,p)}) \subseteq [f(U^{(\Lambda,p)})]_{s(\Lambda,p)}$ *for every* (Λ,p)*-open set* U *of* X*;*

(4) $f(U) \subseteq [f(U^{(\Lambda,p)})]_{s(\Lambda,p)}$ *for every* $p(\Lambda,p)$*-open set* U *of* X*;*

(5) $f(U) \subseteq [f(U^{(\Lambda,p)})]_{s(\Lambda,p)}$ *for every* $\alpha(\Lambda,p)$*-open set* U *of* X.

4 Characterizations of weakly $s(\Lambda,p)$-closed functions

We begin this section by introducing the notion of weakly $s(\Lambda,p)$-closed functions.

Definition 4.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly $s(\Lambda, p)$-closed if $[f(K_{(\Lambda,p)})]^{s(\Lambda,p)} \subseteq f(K)$ for each (Λ, p)-closed set K of X.*

Theorem 4.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is weakly $s(\Lambda, p)$-closed;

(2) $[f(U)]^{s(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for every (Λ, p)-open set U of X.

Proof. The proof follows from Theorem 4.1 of [1]. □

Theorem 4.3. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is weakly $s(\Lambda, p)$-closed;

(2) $[f(U)]^{s(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ every (Λ, p)-open set U of X;

(3) $[f(K_{(\Lambda,p)})]^{s(\Lambda,p)} \subseteq f([K_{(\Lambda,p)}]^{(\Lambda,p)})$ every (Λ, p)-closed set K of X;

(4) $[f(K_{(\Lambda,p)})]^{s(\Lambda,p)} \subseteq f([K_{(\Lambda,p)}]^{(\Lambda,p)})$ every $r(\Lambda, p)$-closed set K of X;

(5) $[f(K_{(\Lambda,p)})]^{s(\Lambda,p)} \subseteq f(K)$ every $p(\Lambda, p)$-closed set K of X;

(6) $[f(K_{(\Lambda,p)})]^{s(\Lambda,p)} \subseteq f(K)$ every $\alpha(\Lambda, p)$-closed set K of X.

Proof. (1) ⇒ (2): The proof follows from Theorem 4.2.

(2) ⇒ (3): Let K be any (Λ, p)-closed set of X. Then $K_{(\Lambda,p)}$ is (Λ, p)-open in X and by (2), $[f(K_{(\Lambda,p)})]^{s(\Lambda,p)} \subseteq f([K_{(\Lambda,p)}]^{(\Lambda,p)})$.

(3) ⇒ (4): This is obvious since every $r(\Lambda, p)$-closed set is (Λ, p)-closed.

(4) ⇒ (5): Let K be any $p(\Lambda, p)$-closed set of X. Then we have $[K_{(\Lambda,p)}]^{(\Lambda,p)} \subseteq K$. Since $[K_{(\Lambda,p)}]^{(\Lambda,p)}$ is $r(\Lambda, p)$-closed, by (4), $[f(K_{(\Lambda,p)})]^{s(\Lambda,p)} \subseteq [f([[K_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)})]^{s(\Lambda,p)} \subseteq f([[[K_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)}) = f([K_{(\Lambda,p)}]^{(\Lambda,p)})$.

(5) ⇒ (6): This is obvious since every $\alpha(\Lambda, p)$-closed set is $p(\Lambda, p)$-closed.

(6) ⇒ (1): Let K be any (Λ, p)-closed set of X. Then K is $\alpha(\Lambda, p)$-closed in X. Using (6), we have $[f(K_{(\Lambda,p)})]^{s(\Lambda,p)} \subseteq f(K)$. This shows that f is weakly $s(\Lambda, p)$-closed. □

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References

[1] C. Boonpok, M. Thongmoon, Weakly $p(\Lambda, p)$-open functions and weakly $p(\Lambda, p)$-closed functions, Int. J. Anal. Appl., **22,** (2024), 10.

[2] C. Boonpok, C. Viriyapong, On (Λ, p)-closed sets and the related notions in topological spaces, Eur. J. Pure Appl. Math., **15,** no. 2, (2022), 415–436.

[3] M. Caldas, G. Navalagi, On generalizing δ-open functions, Pro Math., **22,** nos, 43-44, (2008), 101–119.

[4] M. Caldas, S. Jafari, G. Navalagi, Weak forms of open and closed functions via semi-θ-open sets, Carpathian J. Math., **22,** nos. 1-2, (2006), 21–31.

[5] M. Caldas, G. Navalagi, On weak forms of preopen and preclosed functions, Archivum Math., **40,** (2004), 119–128.

[6] N. Chutiman, C. Boonpok, Some properties of weakly $b(\Lambda, p)$-open functions, Int. J. Math. Comput. Sci., **19,** no. 2, (2024), 497–501.

[7] J. Khampakdee, C. Boonpok, Properties of (Λ, p)-closed functions, Int. J. Math. Comput. Sci., **19,** no. 2, (2024), 481–484.

[8] C. Klanarong, C. Boonpok, Characterizations of weakly $\delta(\Lambda, p)$-closed functions, Int. J. Math. Comput. Sci., **19,** no. 2, (2024), 503–507.

[9] T. Noiri, V. Popa, A unified theory of closed functions, Bull. Math. Soc. Math. Roumanie Tome, **49(97),** no. 4, (2006), 371–382.

[10] D. A. Rose, D. S. Janković, Weakly closed functions and Hausdorff spaces, Math. Nachr., **130,** (1987), 105–110.

[11] D. A. Rose, On weak openness and almost openness, Int. J. Math. Math. Sci., **7,** (1984), 35–40.

[12] M. Thongmoon, C. Boonpok, Strongly $\theta(\Lambda, p)$-continuous functions, Int. J. Math. Comput. Sci., **19,** no. 2, (2024), 475–479.

[13] N. Srisarakham, C. Boonpok, On weakly $\delta(\Lambda, p)$-open functions, Int. J. Math. Comput. Sci., **19,** no. 2, (2024), 485–489.

[14] N. Viriyapong, C. Boonpok, On (Λ, p)-extremally disconnected spaces, Int. J. Math. Comput. Sci., **18,** no. 2, (2023), 289–293.