International Journal of Mathematics and
Computer Science, **19**(2024), no. 1, 117–120

Some properties of $S\Lambda_s$-closed spaces

Napassanan Srisarakham, Chawalit Boonpok

Mathematics and Applied Mathematics Research Unit
Department of Mathematics
Faculty of Science
Mahasarakham University
Maha Sarakham, 44150, Thailand

email: napassanan.sri@msu.ac.th, chawalit.b@msu.ac.th

(Received May 16, 2023, Accepted July 19, 2023,
Published August 31, 2023)

Abstract

This paper is concerned with the concept of $S\Lambda_s$-closed spaces. In particular, some properties of $S\Lambda_s$-closed spaces are investigated.

1 Introduction

In 1963, Levine [4] introduced the concept of semi-open sets which is weaker than the concept of open sets in topological spaces. Veličko [8] introduced δ-open sets, which are stronger than open sets. Park et al. [5] offered a new notion called δ-semiopen sets which are stronger than semi-open sets but weaker than δ-open sets and investigated the relationships among several types of these open sets. Caldas et al. [3] investigated some weak separation axioms by utilizing δ-semiopen sets and the δ-semiclosure operator. Caldas et al. [2] investigated the notion of δ-Λ_s-semiclosed sets which is defined as the intersection of a δ-Λ_s-set and a δ-semiclosed set. In [1], the present authors introduced and investigated the concept of (Λ, s)-closed sets by utilizing the notions of Λ_s-sets and semi-closed sets. Srisarakham and Boonpok [7] introduced and studied the notions of $\delta(\Lambda, s)$-closed sets and $\delta(\Lambda, s)$-open sets.

Key words and phrases: $\delta s(\Lambda, s)$-open set, $S\Lambda_s$-closed space.
Corresponding author: Napassanan Srisarakham.
AMS (MOS) Subject Classifications: 54A05, 54D10.
ISSN 1814-0432, 2024, http://ijmcs.future-in-tech.net

Moreover, Pue-on and Boonpok [6] introduced and investigated the concepts of $\delta s(\Lambda, s)$-closed sets and $\delta s(\Lambda, s)$-open sets. In this paper, we introduce the concept of $S\Lambda_s$-closed spaces. Furthermore, some properties of $S\Lambda_s$-closed spaces are discussed.

2 Preliminaries

Let A be a subset of a topological space (X, τ). A subset A is called *$s(\Lambda, s)$-open* [1] if $A \subseteq [A_{(\Lambda,s)}]^{(\Lambda,s)}$. The family of all $s(\Lambda, s)$-open sets in a topological space (X, τ) is denoted by $s(\Lambda, s)O(X, \tau)$. The complement of a $s(\Lambda, s)$-open set is called *$s(\Lambda, s)$-closed*. The intersection of all $s(\Lambda, s)$-closed sets containing A is called the *$s(\Lambda, s)$-closure* of A and is denoted by $A^{s(\Lambda,s)}$. A subset A is called *$s(\Lambda, s)$-regular* if A is $s(\Lambda, s)$-open and $s(\Lambda, s)$-closed. The family of all $s(\Lambda, s)$-regular sets in a topological space (X, τ) is denoted by $s(\Lambda, s)r(X, \tau)$. A point x of X is called a *$\delta(\Lambda, s)$-cluster point* [7] of A if $A \cap [U^{(\Lambda,s)}]_{(\Lambda,s)} \neq \emptyset$ for every (Λ, s)-open set U of X containing x. The set of all $\delta(\Lambda, s)$-cluster points of A is called the *$\delta(\Lambda, s)$-closure* [7] of A and is denoted by $A^{\delta(\Lambda,s)}$. A subset A is called *$\delta(\Lambda, s)$-closed* [7] if $A = A^{\delta(\Lambda,s)}$. The complement of a $\delta(\Lambda, s)$-closed set is said to be *$\delta(\Lambda, s)$-open*. A subset A is called *$\delta s(\Lambda, s)$-open* [6] if $A \subseteq [A_{(\Lambda,s)}]^{\delta(\Lambda,s)}$. The complement of a $\delta s(\Lambda, s)$-open set is called *$\delta s(\Lambda, s)$-closed*. The family of all $\delta s(\Lambda, s)$-open sets in a topological space (X, τ) is denoted by $\delta s(\Lambda, s)O(X, \tau)$. A point x of X is called a *$\delta s(\Lambda, s)$-cluster point* [6] of A if $A \cap U \neq \emptyset$ for every $\delta s(\Lambda, s)$-open set U of X containing x. The set of all $\delta s(\Lambda, s)$-cluster points of A is called the *$\delta s(\Lambda, s)$-closure* [6] of A and is denoted by $A^{\delta s(\Lambda,s)}$.

3 Some properties of $S\Lambda_s$-closed spaces

In this section, we introduce the concept of $S\Lambda_s$-closed spaces. Moreover, we discuss some properties of $S\Lambda_s$-closed spaces.

Lemma 3.1. *For a subset A of a topological space (X, τ), the following properties hold:*

(1) If A is a $s(\Lambda, s)$-regular set, then it is $\delta s(\Lambda, s)$-open.

(2) If A is a $\delta s(\Lambda, s)$-open set, then it is $s(\Lambda, s)$-open.

(3) If A is a $s(\Lambda, s)$-open set, then $A^{s(\Lambda,s)}$ is $s(\Lambda, s)$-regular.

Let A be a subset of a topological space (X, τ). A point x of X is called a $\theta s(\Lambda, s)$-*cluster point* of A if $A \cap U^{s(\Lambda,s)} \neq \emptyset$ for every $s(\Lambda, s)$-open set U of X containing x. The set of all $\theta s(\Lambda, s)$-cluster points of A is called the $\theta s(\Lambda, s)$-*closure* of A and is denoted by $A^{\theta s(\Lambda,s)}$. A subset A is called $\theta s(\Lambda, s)$-*closed* if $A = A^{\theta s(\Lambda,s)}$. The complement of a $\theta s(\Lambda, s)$-closed set is called $\theta s(\Lambda, s)$-*open*.

Lemma 3.2. *Let (X, τ) be a topological space. Then, $V^{\theta s(\Lambda,s)} = V^{\delta s(\Lambda,s)} = V^{s(\Lambda,s)}$ for each $V \in s(\Lambda, s)O(X, \tau)$.*

Definition 3.3. *A topological space (X, τ) is said to be $S(\Lambda, s)$-closed if, for every cover $\{V_\gamma \mid \gamma \in \nabla\}$ of X by $s(\Lambda, s)$-open sets of X, there exists a finite subset ∇_0 of ∇ such that $X = \bigcup_{\gamma \in \nabla_0} V_\gamma^{s(\Lambda,s)}$.*

Theorem 3.4. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is $S\Lambda_s$-closed.

(2) For every $\delta s(\Lambda, s)$-open cover $\{V_\gamma \mid \gamma \in \nabla\}$ of X, there exists a finite subset ∇_0 of ∇ such that $X = \bigcup_{\gamma \in \nabla_0} V_\gamma^{s(\Lambda,s)}$.

(3) For every $\delta s(\Lambda, s)$-open cover $\{V_\gamma \mid \gamma \in \nabla\}$ of X, there exists a finite subset ∇_0 of ∇ such that $X = \bigcup_{\gamma \in \nabla_0} V_\gamma^{\delta s(\Lambda,s)}$.

Proof. (1) $\Rightarrow$ (2): Suppose that (X, τ) is $S\Lambda_s$-closed. Let $\{V_\gamma \mid \gamma \in \nabla\}$ be a $\delta s(\Lambda, s)$-open cover of X. By Lemma 3.1, $\delta s(\Lambda, s)O(X, \tau) \subseteq s(\Lambda, s)O(X, \tau)$ and there exists a finite subset ∇_0 of ∇ such that $X = \bigcup_{\gamma \in \nabla_0} V_\gamma^{s(\Lambda,s)}$.

(2) $\Rightarrow$ (3): Let $\{V_\gamma \mid \gamma \in \nabla\}$ be a $\delta s(\Lambda, s)$-open cover of X. By Lemma 3.1, $\delta s(\Lambda, s)O(X, \tau) \subseteq s(\Lambda, s)O(X, \tau)$ and it follows from Lemma 3.2 that $V_\gamma^{\delta s(\Lambda,s)} = V_\gamma^{s(\Lambda,s)}$ for each $\gamma \in \nabla$.

(3) $\Rightarrow$ (1): Let $\{V_\gamma \mid \gamma \in \nabla\}$ be a $s(\Lambda, s)$-open cover of X. Then, we have $X = \bigcup_{\gamma \in \nabla_0} V_\gamma^{s(\Lambda,s)}$. By Lemma 3.1, $V_\gamma^{s(\Lambda,s)} \in s(\Lambda, s)r(X, \tau) \subseteq \delta s(\Lambda, s)O(X, \tau)$ and there exists a finite subset ∇_0 of ∇ such that $X = \bigcup_{\gamma \in \nabla_0} [V_\gamma^{s(\Lambda,s)}]^{\delta s(\Lambda,s)}$. By Lemma 3.2, $[V_\gamma^{s(\Lambda,s)}]^{\delta s(\Lambda,s)} = [V_\gamma^{s(\Lambda,s)}]^{s(\Lambda,s)} = V_\gamma^{s(\Lambda,s)}$ and hence $X = \bigcup_{\gamma \in \nabla_0} V_\gamma^{s(\Lambda,s)}$. Thus, (X, τ) is $S\Lambda_s$-closed. □

Theorem 3.5. *A topological space (X, τ) is $S\Lambda_s$-closed if and only if for every $\theta s(\Lambda, s)$-open cover $\{V_\gamma \mid \gamma \in \nabla\}$ of X, there exists a finite subset ∇_0 of ∇ such that $X = \bigcup_{\gamma \in \nabla_0} V_\gamma$.*

Proof. Let $\{V_\gamma \mid \gamma \in \nabla\}$ be a $\theta s(\Lambda,s)$-open cover of X. For each $x \in X$, there exists $\gamma(x) \in \nabla$ such that $x \in V_{\gamma(x)}$. Since $V_{\gamma(x)}$ is $\theta s(\Lambda,s)$-open, there exists $G_{\gamma(x)} \in s(\Lambda,s)O(X,\tau)$ such that $x \in G_{\gamma(x)} \subseteq G_{\gamma(x)}^{s(\Lambda,s)} \subseteq V_{\gamma(x)}$. Since $\{G_{\gamma(x)} \mid x \in X\}$ is a $s(\Lambda,s)$-open cover of X, there exist finite points, say, $x_1, x_2, ..., x_n$ such that $X = \bigcup_{i=1}^{n} G_{\gamma(x_i)}^{s(\Lambda,s)}$. Thus, $X = \bigcup_{i=1}^{n} V_{\gamma(x_i)}$.

Conversely, let $\{V_\gamma \mid \gamma \in \nabla\}$ be a $s(\Lambda,s)$-open cover of X. By Lemma 3.1, $\{V_\gamma^{s(\Lambda,s)} \mid \gamma \in \nabla\}$ is a $s(\Lambda,s)$-regular cover of X and hence a $\theta s(\Lambda,s)$-open cover of X. Thus, there exists a finite subset ∇_0 of ∇ such that $X = \bigcup_{\gamma \in \nabla_0} V_\gamma^{s(\Lambda,s)}$. This shows that (X,τ) is $S\Lambda_s$-closed. □

Acknowledgment. This research project was financially supported by Mahasarakham University.

References

[1] C. Boonpok, C. Viriyapong, On some forms of closed sets and related topics, Eur. J. Pure Appl. Math., **16,** no. 1, (2023), 336–362.

[2] M. Caldas, M. Ganster, D. N. Georgiou, S. Jafari, T. Noiri, δ-semiopen sets in topological spaces, Topology Proc., **29,** no. 2, (2005), 369–383.

[3] M. Caldas, D. N. Georgiou, S. Jafari, T. Noiri, More on δ-semiopen sets, Note di Matematica, **22,** no. 2, (2003), 1–14.

[4] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, **70,** (1963), 36–41.

[5] J. H. Park, B. Y. Lee, M. J. Son, On δ-semiopen sets in topological spaces, J. Indian Acad. Math., **19,** (1997), 59–67.

[6] P. Pue-on, C. Boonpok, On $\delta s(\Lambda,s)$-open sets in topological spaces, Int. J. Math. Comput. Sci., **18,** no. 4, (2023), 749–753.

[7] N. Srisarakham, C. Boonpok, On characterizations of $\delta p(\Lambda,s)$-$\mathscr{D}_1$ spaces, Int. J. Math. Comput. Sci., **18,** no. 4, (2023), 743–747.

[8] N. V. Veličko, H-closed topological spaces, Amer. Math. Soc. Transl., **78,** no. 2, (1968), 102–118.