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On $S\Lambda_s$-connected spaces

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Abstract

In this paper, our main goal is to introduce the concept of $S\Lambda_s$-connected spaces. Moreover, we investigate some properties of $S\Lambda_s$-connected spaces.

1 Introduction

The concept of semi-open sets was first introduced by Levine [4]. Veličko [10] introduced δ-open sets, which are stronger than open sets. Park et al. [5] offered a new notion called δ-semiopen sets which are stronger than semi-open sets but weaker than δ-open sets and investigated the relationships among several types of these open sets. Caldas et al. [3] investigated some weak separation axioms by utilizing δ-semiopen sets and the δ-semiclosure operator. Moreover, Caldas et al. [2] investigated the notion of δ-Λ_s-semiclosed sets which is defined as the intersection of a δ-Λ_s-set and a δ-semiclosed set. In [1], the present authors introduced and investigated the concept of (Λ, s)-closed sets by utilizing the notions of Λ_s-sets and semi-closed sets. Srisarakham and Boonpok [9] introduced and studied the notions of $\delta(\Lambda, s)$-closed sets

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and $\delta(\Lambda,s)$-open sets. Pue-on and Boonpok [7] introduced and investigated the concepts of $\delta s(\Lambda,s)$-open sets and $\delta s(\Lambda,s)$-closed sets. In this paper, we introduce the concept of $S\Lambda_s$-connected spaces. In particular, several properties of $S\Lambda_s$-connected spaces are discussed.

2 Preliminaries

Let A be a subset of a topological space (X,τ). A subset A is called *$s(\Lambda,s)$-open* (resp. *$p(\Lambda,s)$-open*, *$\alpha(\Lambda,s)$-open*, *$\beta(\Lambda,s)$-open*)[1] if $A\subseteq[A_{(\Lambda,s)}]^{(\Lambda,s)}$ (resp. $A\subseteq[A^{(\Lambda,s)}]_{(\Lambda,s)}$, $A\subseteq[[A_{(\Lambda,s)}]^{(\Lambda,s)}]_{(\Lambda,s)}$, $A\subseteq[[A^{(\Lambda,s)}]_{(\Lambda,s)}]^{(\Lambda,s)}$). The family of all $s(\Lambda,s)$-open (resp. $p(\Lambda,s)$-open, $\alpha(\Lambda,s)$-open, $\beta(\Lambda,s)$-open) sets in a topological space (X,τ) is denoted by $s(\Lambda,s)O(X,\tau)$ (resp. $p(\Lambda,s)O(X,\tau)$, $\alpha(\Lambda,s)O(X,\tau)$, $\beta(\Lambda,s)O(X,\tau)$). The complement of a $s(\Lambda,s)$-open (resp. $p(\Lambda,s)$-open, $\alpha(\Lambda,s)$-open, $\beta(\Lambda,s)$-open) set is called *$s(\Lambda,s)$-closed* (resp. *$p(\Lambda,s)$-closed*, *$\alpha(\Lambda,s)$-closed*, *$\beta(\Lambda,s)$-closed*). The intersection of all $s(\Lambda,s)$-closed sets containing A is called the *$s(\Lambda,s)$-closure* of A and is denoted by $A^{s(\Lambda,s)}$. A subset A is called *$s(\Lambda,s)$-regular* if A is $s(\Lambda,s)$-open and $s(\Lambda,s)$-closed. The family of all $s(\Lambda,s)$-regular sets in a topological space (X,τ) is denoted by $s(\Lambda,s)r(X,\tau)$. A point x of X is called a *$\delta(\Lambda,s)$-cluster point* [9] of A if $A\cap[U^{(\Lambda,s)}]_{(\Lambda,s)}\neq\emptyset$ for every (Λ,s)-open set U of X containing x. The set of all $\delta(\Lambda,s)$-cluster points of A is called the *$\delta(\Lambda,s)$-closure* [9] of A and is denoted by $A^{\delta(\Lambda,s)}$. A subset A is called *$\delta(\Lambda,s)$-closed* [9] if $A=A^{\delta(\Lambda,s)}$. The complement of a $\delta(\Lambda,s)$-closed set is said to be *$\delta(\Lambda,s)$-open*. A subset A is called *$\delta s(\Lambda,s)$-open* [7] if $A\subseteq[A_{(\Lambda,s)}]^{\delta(\Lambda,s)}$. The complement of a $\delta s(\Lambda,s)$-open set is called *$\delta s(\Lambda,s)$-closed*. The family of all $\delta s(\Lambda,s)$-open sets in a topological space (X,τ) is denoted by $\delta s(\Lambda,s)O(X,\tau)$. A point x of X is called a *$\delta s(\Lambda,s)$-cluster point* [7] of A if $A\cap U\neq\emptyset$ for every $\delta s(\Lambda,s)$-open set U of X containing x. The set of all $\delta s(\Lambda,s)$-cluster points of A is called the *$\delta s(\Lambda,s)$-closure* [7] of A and is denoted by $A^{\delta s(\Lambda,s)}$.

3 Properties of $S\Lambda_s$-connected spaces

In this section, we introduce the concept of $S\Lambda_s$-connected spaces. Moreover, we investigate some properties of $S\Lambda_s$-connected spaces.

Definition 3.1. *A topological space (X,τ) is called $S\Lambda_s$-connected if X cannot be expressed as a disjoint union of two nonempty $s(\Lambda,s)$-open sets.*

Lemma 3.2. [8] *For a subset A of a topological space (X, τ), the following properties hold:*

(1) If A is a $s(\Lambda, s)$-regular set, then it is $\delta s(\Lambda, s)$-open.

(2) If A is a $\delta s(\Lambda, s)$-open set, then it is $s(\Lambda, s)$-open.

(3) If A is a $s(\Lambda, s)$-open set, then $A^{s(\Lambda,s)}$ is $s(\Lambda, s)$-regular.

Lemma 3.3. [8] *Let (X, τ) be a topological space. Then, $V^{\theta s(\Lambda,s)} = V^{\delta s(\Lambda,s)} = V^{s(\Lambda,s)}$ for each $V \in s(\Lambda, s)O(X, \tau)$.*

Theorem 3.4. *For a topological space (X, τ), the following properties are equivalent:*

(1) $V^{(\Lambda,s)} = X$ for every nonempty (Λ, s)-open set V of X;

(2) (X, τ) is $S\Lambda_s$-connected;

(3) X cannot be expressed by the disjoint union of two nonempty $\delta s(\Lambda, s)$-open sets;

(4) $V^{\delta s(\Lambda,s)} = X$ for every nonempty $\delta s(\Lambda, s)$-open set V of X.

Proof. (1) $\Leftrightarrow$ (2): The proof follows from Theorem 4.3 of [6].

(2) $\Rightarrow$ (3): Suppose that there exist two nonempty $\delta s(\Lambda, s)$-open sets V_1, V_2 such that $V_1 \cap V_2 = \emptyset$ and $V_1 \cup V_2 = X$. Since $\delta s(\Lambda, s)O(X, \tau) \subseteq s(\Lambda, s)O(X, \tau)$, this shows that (X, τ) is not $S\Lambda_s$-connected.

(3) $\Rightarrow$ (4): Suppose that $V^{\delta s(\Lambda,s)} \neq X$ for some nonempty $\delta s(\Lambda, s)$-open set V of X. Then $X - V^{\delta s(\Lambda,s)} \neq \emptyset$ and $X = (X - V^{\delta s(\Lambda,s)}) \cup V^{\delta s(\Lambda,s)}$. Since $\delta s(\Lambda, s)O(X, \tau) \subseteq s(\Lambda, s)O(X, \tau)$ by Lemma 3.2 and Lemma 3.3 $V^{\delta s(\Lambda,s)} = V^{s(\Lambda,s)} \in s(\Lambda, s)r(X, \tau)$. Moreover, since $s(\Lambda, s)r(X, \tau) \subseteq \delta s(\Lambda, s)O(X, \tau)$, $(X - V^{\delta s(\Lambda,s)})$ and $V^{\delta s(\Lambda,s)}$ are $\delta s(\Lambda, s)$-open.

(4) $\Rightarrow$ (1): Let V be any nonempty (Λ, s)-open set of X. Then, $V^{(\Lambda,s)}$ is $r(\Lambda, s)$-closed and hence $s(\Lambda, s)$-regular. Thus, $V^{(\Lambda,s)}$ is $\delta s(\Lambda, s)$-open and $X = [V^{(\Lambda,s)}]^{\delta s(\Lambda,s)} = [V^{(\Lambda,s)}]^{s(\Lambda,s)} = V^{(\Lambda,s)}$. □

Theorem 3.5. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is $S\Lambda_s$-connected;

(2) $V^{\delta s(\Lambda,s)} = X$ for every nonempty $V \in \beta(\Lambda, s)O(X, \tau)$;

(3) $V^{\delta s(\Lambda,s)} = X$ *for every nonempty* $V \in s(\Lambda, s)O(X, \tau)$*;*

(4) $V^{\delta s(\Lambda,s)} = X$ *for every nonempty* $V \in p(\Lambda, s)O(X, \tau)$*;*

(5) $V^{\delta s(\Lambda,s)} = X$ *for every nonempty* $V \in \alpha(\Lambda, s)O(X, \tau)$*;*

(6) $V^{\delta s(\Lambda,s)} = X$ *for every nonempty* $V \in (\Lambda, s)O(X, \tau)$*.*

Proof. (1) $\Rightarrow$ (2): Let V be any nonempty $\beta(\Lambda, s)$-open set and U be any nonempty $\delta s(\Lambda, s)$-open set. Then, $[V^{(\Lambda,s)}]_{(\Lambda,s)} \neq \emptyset$ and $U_{(\Lambda,s)} \neq \emptyset$. Thus, by Theorem 3.4,

$$\begin{aligned}\emptyset \neq U_{(\Lambda,s)} \cap [V^{(\Lambda,s)}]_{(\Lambda,s)} &\subseteq U \cap [V^{(\Lambda,s)}]_{(\Lambda,s)} \\ &\subseteq U \cap (V \cup [V^{(\Lambda,s)}]_{(\Lambda,s)}) = U \cap V^{s(\Lambda,s)} \subseteq U \cap V^{\delta s(\Lambda,s)}.\end{aligned}$$

Since $U \in \delta s(\Lambda, s)O(X, \tau)$, $U \cap V \neq \emptyset$. This shows that $V^{\delta s(\Lambda,s)} = X$.

(6) $\Rightarrow$ (1): Let U, V be any nonempty $\delta s(\Lambda, s)$-open sets. Since

$$\delta s(\Lambda, s)O(X, \tau) \subseteq s(\Lambda, s)O(X, \tau)$$

and $V_{(\Lambda,s)} \neq \emptyset$, we have $\emptyset \neq U \cap V_{(\Lambda,s)} \subseteq U \cap V$. This shows that $V^{\delta s(\Lambda,s)} = X$ for every nonempty $V \in \delta s(\Lambda, s)O(X, \tau)$. Thus, by Theorem 3.4, (X, τ) is $S\Lambda_s$-connected.

The other implications are obvious since $(\Lambda, s)O(X, \tau) \subseteq \alpha(\Lambda, s)O(X, \tau) \subseteq s(\Lambda, s)O(X, \tau) \cap p(\Lambda, s)O(X, \tau)$ and $s(\Lambda, s)O(X, \tau) \cup p(\Lambda, s)O(X, \tau) \subseteq \beta(\Lambda, s)O(X, \tau)$. $\square$

Corollary 3.6. *For a topological space* (X, τ)*, the following properties are equivalent:*

(1) (X, τ) *is* $S\Lambda_s$*-connected;*

(2) $U \cap V \neq \emptyset$ *for any nonempty sets* $U \in \beta(\Lambda, s)O(X, \tau)$ *and* $V \in \delta s(\Lambda, s)O(X, \tau)$*;*

(3) $U \cap V \neq \emptyset$ *for any nonempty sets* $U \in p(\Lambda, s)O(X, \tau)$ *and* $V \in \delta s(\Lambda, s)O(X, \tau)$*;*

(4) $U \cap V \neq \emptyset$ *for any nonempty sets* $U \in s(\Lambda, s)O(X, \tau)$ *and* $V \in \delta s(\Lambda, s)O(X, \tau)$*;*

(5) $U \cap V \neq \emptyset$ *for any nonempty sets* $U \in \alpha(\Lambda, s)O(X, \tau)$ *and* $V \in \delta s(\Lambda, s)O(X, \tau)$*;*

(6) $U \cap V \neq \emptyset$ for any nonempty sets $U \in (\Lambda, s)O(X, \tau)$ and $V \in \delta s(\Lambda, s)O(X, \tau)$;

(7) $U \cap V \neq \emptyset$ for any nonempty sets $U \in \delta s(\Lambda, s)O(X, \tau)$ and $V \in \delta s(\Lambda, s)O(X, \tau)$.

Proof. This is an immediate consequence of Theorems 3.4 and 3.5. □

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