International Journal of Mathematics and
Computer Science, **19**(2024), no. 1, 239–248

# Upper Bounds of the Locating Chromatic Numbers of Shadow Cycle Graphs

**Asmiati, Wenty Okzarima, Notiragayu, La Zakaria**

Department of Mathematics
Faculty of Mathematics and Natural Sciences
University Lampung
Bandar Lampung, Indonesia

email: asmiati.1976@fmipa.unila.ac.id

(Received June 30, 2022, Revised March 3, 2023 and June 21, 2023,
Accepted August 30, 2023, Published August 31, 2023)

**Abstract**

If $G$ is a connected finite simple graph with its vertices properly colored with $k$ colors appearing, then with respect to an ordering of the colors, to each vertex $v$ a $k$-tuple can be assigned whose entries give the distances in $G$ from $v$ to the different color sets. The locating chromatic number (*lcn*) of $G$ is the smallest $k$ for which a proper vertex coloring of $G$ with $k$ colors appearing exists such that the corresponding $k$-tuples are distinct. In this paper we obtain by construction upper bounds on the *lcns* of shadow cycle graphs.

## 1 Introduction

The locating chromatic number of a graph is one of the topics in graph theory studied by Chartrand et al [1]. Let $H = (V, E)$ be a connected graph and $k : V(H) \to [1, l]$ be a vertex coloring in $H$, where $k(x) \neq k(y)$ for adjacent vertices $x, y \in V(H)$. The set of vertices which receive color $i$ will be denoted $K_i$ for $i \in [1, l]$, and $\Pi = \{K_1, K_2, ..., K_l\}$ is a partition of $V(H)$. The color code of $x \in V(G)$, denoted by $k_\Pi(x)$, is the ordered $l$-tuple

**Key words and phrases:** Color classes, locating chromatic number, shadow cycle graph.
**AMS (MOS) Subject Classifications**: 05C12, 05C15.
**ISSN** 1814-0432, 2024, http://ijmcs.future-in-tech.net

$(d(x,K_1), d(x,K_2), ..., d(x,K_l)))$ where $d(v,K_i) = \min\ \{d(v,x) : x \in K_i\}$ for any $i \in [1,l]$. If the vertices of $H$ have distinct color codes, then $k$ is called an $l$-locating coloring ($l$-$lc$) of $H$. The locating chromatic number ($lcn$) of graph $H$, denoted by $\chi_L(H)$, is the smallest $l$ such that $H$ has an $l$-$lc$.

In 2002, Chartrand et al.[1] determined $lcn$ for graph classes such as paths, cycles, complete graphs, and multipartite graphs. In 2011, Asmiati et al.[2] determined $lcn$ for a homogeneous amalgamation of stars. Subsequently, Asmiati et al.[3] did so for non homogeneous amalgamation of stars, and Asmiati [4] for non-homogeneous caterpillar and firecracker graphs. Syofyan et al.[5] discussed $lcn$ for Lobster graphs. Furthermore, Behtoei and Omoomi [6] determined $lcn$ for the Kneser graph. Regarding operations of graphs, Behtoei and Omoomi [7] determined $lcn$ for the cartesian product of a paths and complete graphs, as well as for the cartesian product of two complete graphs, whilst Welyyanti et al.[8] did so for disconnected graphs. In 2020, Ghanem et al.[9] determined $lcn$ from the results of power path and cycle operations.

The study of $lcn$ and its variants continues to be of interest, as evidenced by the volume of research results on this topic. Asmiati et al. [10] determined $lcn$ for some generalized Petersen Graphs having locating chromatic number four or five, followed by a continuation of this line of research in 2018 by Asmiati et al.[11] for barbell graphs containing complete graphs or generalized Petersen graphs. In 2021, Irawan et al.[12] determined $lcn$ for an origami graph and its barbell. Later that same year, Irawan et al.[13] determined $lcn$ for subdivision of certain operation of origami barbell graphs. Furthermore, in 2023, Asmiati et al.[14] succeeded in determining $lcn$ for certain operation of origami graphs.

A shadow cycle graph, denoted by $D_2(C_m)$ is a connected graph that is constructed from two cycles, namely $C_m^1$ and $C_m^2$. Let $V(C_m^1) = \{v_1, v_2, v_3, \ldots, v_m\}$ and $V(C_m^2) = \{u_1, u_2, u_3, \ldots, u_m\}$. Cycle $C_m^1$ is the outer cycle and $C_m^2$ is an image of $C_m^1$ which is located inside it, where each vertex $v_i \in V(C_m^1)$ is adjacent to vertices $u_{i-1}, u_{i+1} \in V(C_m^2)$ for $i \in [2, m-1]$, and vertex $v_1$ is adjacent to $u_2$ and $u_m$. Let $H$ and $H'$ be shadow cycle graphs, where $V(H) = \{v_i\} \cup \{u_i\}$ and $V(H') = \{v_i'\} \cup \{u_i'\}$, $i \in [1,m]$.The barbell graph of a shadow cycle graph, denoted by $B_{D_2(C_m)}$ is a graph formed from graphs $H$ and $H'$ connected by an edge $v_2v_m'$.

Thus far, There has been no study on the *lcn* for shadow cycle graphs. In light of this, the present study will discuss upper bounds of *lcns* for shadow cycle graphs and barbell shadow cycle graphs.

# 2 Main results

## 2.1 Upper bounds of the *lcns* for shadow cycle graphs

In the section, we will discuss *lcn* for shadow cycle graphs $D_2(C_m)$.

**Theorem 2.1.** *Let $D_2(C_m)$ be a shadow cycle graph for $m \geq 3$. If $m \in \{5,6,7,8\}$ then $\chi_L(D_2(C_m)) \leq 5$, $\chi_L(D_2(C_m)) \leq 6$ if $m \in \{3\}$ or odd $m \geq 9$, and $\chi_L(D_2(C_m)) \leq 8$ if $m \in \{4\}$ or even $m \geq 10$.*

**Proof.** Let $k$ be a vertex coloring with 5 colors in $D_2(C_5)$ as follows: $K_1 = \{v_1, u_3\}$; $K_2 = \{v_2, v_4\}$; $K_3 = \{v_3, v_5\}$; $K_4 = \{u_1, u_4\}$; and $K_5 = \{u_2, u_5\}$. Then, we have the following color codes: $k_\Pi(v_1) = (0,1,1,2,1)$; $k_\Pi(v_2) = (1,0,1,1,2)$; $k_\Pi(v_3) = (2,1,0,1,1)$; $k_\Pi(v_4) = (1,0,1,2,1)$; $k_\Pi(v_5) = (1,1,0,1,2)$; $k_\Pi(u_1) = (2,1,1,0,1)$; $k_\Pi(u_2) = (1,2,1,1,0)$; $k_\Pi(u_3) = (0,1,2,1,1)$; $k_\Pi(u_4) = (1,2,1,0,1)$; and $k_\Pi(u_5) = (1,1,2,1,0)$. Thus, since all vertices in $D_2(C_5)$ have distinct color codes, $\chi_L(D_2(C_5)) \leq 5$.

Let $k$ be be a vertex coloring with 5 colors in $D_2(C_6)$ as follows: $K_1 = \{v_1, v_4\}$; $K_2 = \{v_2, u_4, u_6\}$; $K_3 = \{v_3, v_6\}$; $K_4 = \{u_1, u_3, u_5\}$; and $K_5 = \{v_5, u_2, \}$. Then, we have the following color codes: $k_\Pi(v_1) = (0,1,1,2,2)$; $k_\Pi(v_2) = (1,0,1,1,2)$; $k_\Pi(v_3) = (1,1,0,2,1)$; $k_\Pi(v_4) = (0,2,1,1,1)$; $k_\Pi(v_5) = (1,2,1,1,0)$; $k_\Pi(v_6) = (1,2,0,1,1)$; $k_\Pi(u_1) = (2,1,1,0,1)$; $k_\Pi(u_2) = (1,2,1,1,0)$; $k_\Pi(u_3) = (1,1,2,0,1)$; $k_\Pi(u_4) = (2,0,1,1,1)$; $k_\Pi(u_5) = (1,1,1,0,2)$; and $k_\Pi(u_6) = (1,0,2,1,1)$. Thus, since all vertices in $D_2(C_6)$ have distinct color codes, $\chi_L(D_2(C_6)) \leq 5$.

Let $k$ be a vertex coloring with 5 colors in $D_2(C_7)$ as follows: $K_1 = \{v_1, v_4, u_6\}$; $K_2 = \{v_2, v_5, u_7\}$; $K_3 = \{v_3, v_5\}$; $K_4 = \{v_7, u_2, u_4\}$; and $K_5 = \{u_1, u_3, u_5\}$. Then, we have the following color codes: $k_\Pi(v_1) = (0,1,2,1,2)$; $k_\Pi(v_2) = (1,0,1,2,1)$; $k_\Pi(v_3) = (1,1,0,1,2)$; $k_\Pi(v_4) = (0,1,1,2,1)$; $k_\Pi(v_5) = (1,0,1,1,2)$; $k_\Pi(v_6) = (2,1,0,1,1)$; $k_\Pi(v_7) = (1,2,1,0,1)$; $k_\Pi(u_1) = (2,1,2,1,0)$; $k_\Pi(u_2) = (1,2,1,0,1)$; $k_\Pi(u_3) = (1,1,2,1,0)$; $k_\Pi(u_4) = (2,1,1,0,1)$; $k_\Pi(u_5) = (1,2,1,1,0)$; $k_\Pi(u_6) = (0,1,2,1,1)$; and $k_\Pi(u_7) = (1,0,1,2,1)$. Thus, since all vertices in $D_2(C_7)$ have distinct color codes, $\chi_L(D_2(C_7)) \leq 5$.

Let $k$ be a vertex coloring with 5 colors in $D_2(C_8)$ as follows: $K_1 = \{v_1, v_4, v_6\}$; $K_2 = \{v_2, v_8, u_5\}$; $K_3 = \{v_3, v_5, u_1\}$; $K_4 = \{v_7, u_2, u_7\}$; and $K_5 = \{u_3, u_6, u_8\}$. Then, we have the following color codes: $k_\Pi(v_1) = (0,1,2,1,1)$ $k_\Pi(v_2) = (1,0,1,2,1)$; $k_\Pi(v_3) = (1,1,0,1,2)$; $k_\Pi(v_4) = (0,1,1,2,1)$; $k_\Pi(v_5) = (1,2,0,1,1)$; $k_\Pi(v_6) = (0,1,1,1,2)$; $k_\Pi(v_7) = (1,1,2,0,1)$; $k_\Pi(v_8) = (1,0,1,1,2)$; $k_\Pi(u_1) = (2,1,0,1,1)$; $k_\Pi(u_2) = (1,2,1,0,1)$; $k_\Pi(u_3) = (1,1,2,1,0)$; $k_\Pi(u_4) = (2,1,1,0,1)$; $k_\Pi(u_5) = (1,0,2,1,1)$; $k_\Pi(u_6) = (2,1,1,1,0)$; $k_\Pi(u_7) = (2,1,1,0,1)$; and $k_\Pi(u_8) = (1,2,1,1,0)$. Thus, since all vertices in $D_2(C_8)$ have distinct color codes, $\chi_L(D_2(C_8)) \le 5$. Therefore, $\chi_L(D_2(C_m)) \le 5$ for $m \in \{5,6,7,8\}$. Let $k$ be a vertex coloring with 6 colors in $D_2(C_3)$ as follows: $K_1 = \{v_1\}$; $K_2 = \{v_2\}$; $K_3 = \{v_3\}$; $K_4 = \{u_1\}$; $K_5 = \{u_2\}$ and $K_6 = \{u_3\}$. Then, we have the following color codes: $D_2(C_3)$: $k_\Pi(v_1) = (0,1,1,1,2,1)$; $k_\Pi(v_2) = (1,0,1,1,1,2)$; $k_\Pi(v_3) = (1,1,0,2,1,1)$; $k_\Pi(u_1) = (1,1,2,0,1,1)$; $k_\Pi(u_2) = (2,1,1,1,0,1)$; and $k_\Pi(u_3) = (1,2,1,1,1,0)$. Thus, since all vertices in $D_2(C_3)$ have distinct color codes, $\chi_L(D_2(C_3)) \le 6$. Let $k$ be a 6-coloring in $D_2(C_m)$ for odd $m \ge 9$ as follows:

$$k(v_i) = \begin{cases} 1, & \text{odd } i \neq m \\ 2, & \text{even } i \\ 3, & m = i. \end{cases}$$

$$k(u_i) = \begin{cases} 4, & \text{odd } i \neq m \\ 5, & \text{even } i \\ 6, & m = i. \end{cases}$$

Then, we have the following color codes:

$$k_\Pi(v_i) = \begin{cases} 0, & 1^{st} \text{ tuple, odd } i;\ 2^{nd} \text{ tuple, even } i;\ 3^{rd} \text{ tuple, m} = i \\ 2, & 5^{th} \text{ tuple, even } i;\ 4^{th} \text{ tuple, odd } i \neq m;\ 6^{rd} \text{ tuple, m} = i \\ i, & 3^{th} \text{ and } 6^{th} \text{ tuple, } 1 \le i \le \frac{m+1}{2} \\ m-i, & 3^{th} \text{ and } 6^{th} \text{ tuple, } i > \frac{m+1}{2} \\ 1, & \text{otherwise.} \end{cases}$$

$$k_\Pi(u_i) = \begin{cases} 0, & 4^{th} \text{ tuple, odd } i;\ 5^{th} \text{ tuple, even } i;\ 6^{th} \text{ tuple, m} = i \\ 2, & 2^{nd} \text{ tuple, even } i;\ 1^{st} \text{ tuple, odd } i \neq m;\ 3^{rd} \text{ tuple, m} = i \\ i, & 3^{rd} \text{ and } 6^{th} \text{ tuple,} 1 \le i \le \frac{m+1}{2} \\ m-i, & 3^{th} \text{ and } 6^{th} \text{ tuple, } i > \frac{m+1}{2} \\ 1, & \text{otherwise.} \end{cases}$$

Thus, since all vertices in $D_2(C_m)$ for odd $m \ge 9$ have distinct color codes, $\chi_L(D_2(C_m)) \le 6$ for odd $m \ge 9$.

Let $k$ be a vertex coloring with 8 colors in $D_2(C_4)$ as follows: $K_1 = \{v_1\}$;

$K_2 = \{v_2\}$; $K_3 = \{v_3\}$; $K_4 = \{v_4\}$; $K_5 = \{u_1\}$; $K_6 = \{u_2\}$; $K_7 = \{u_3\}$; and $K_8 = \{u_4\}$.
Then, we have the following color codes: $k_\Pi(v_1) = (0,1,2,1,2,1,2,1)$; $k_\Pi(v_2) = (1,0,1,2,1,2,1,2)$; $k_\Pi(v_3) = (2,1,0,1,2,1,2,1)$; $k_\Pi(v_4) = (1,2,1,0,1,2,1,2)$; $k_\Pi(u_1) = (2,1,2,1,0,1,2,1)$; $k_\Pi(u_2) = (1,2,1,2,1,0,1,2)$; $k_\Pi(u_3) = (2,1,2,1,2,1,0,1)$; and $k_\Pi(u_4) = (1,2,1,2,1,2,1,0)$.
Thus, since all vertices in $D_2(C_4)$ have distinct color codes, $\chi_L(D_2(C_4)) \leq 8$.
Let $k$ be an 8-coloring in $D_2(C_m)$ for even $m \geq 10$ as follows:

$$k(v_i) = \begin{cases} 1, & \text{odd } i \neq m-1 \\ 2, & \text{even } i \neq m \\ 3 & i = m-1 \\ 4, & i = m\ . \end{cases}$$

$$k(u_i) = \begin{cases} 5, & \text{odd } i \neq m-1 \\ 6, & \text{even } i \neq m \\ 7 & i = m-1 \\ 8, & i = m\ . \end{cases}$$

Then, we have the following color codes:

$$k_\Pi(v_i) = \begin{cases} 0, & 1^{st} \text{ tuple, odd } i \neq m-1;\ 2^{nd} \text{ tuple, even } i \neq m;\ 3^{rd} \text{ tuple, odd } i = m-1; \\ & 4^{th} \text{ tuple, even } i = m \\ 2, & 6^{th} \text{ tuple, even } i \neq m;\ 5^{th} \text{ tuple, odd } i;\ 8^{th} \text{ tuple, } i = m \\ i, & 4^{th} \text{ and } 8^{th} \text{ tuple, } 1 \leq i \leq \frac{m}{2} \\ i+1, & 3^{rd} \text{ and } 7^{th} \text{ tuple, } 1 \leq i \leq \frac{m}{2} \\ m-i, & 4^{th} \text{ and } 8^{th} \text{ tuple, } i > \frac{m}{2} \\ (m-i)-1, & 3^{th} \text{ and } 7^{th} \text{ tuple, } i > \frac{m}{2} \\ 1, & \text{otherwise.} \end{cases}$$

$$k_\Pi(u_i) = \begin{cases} 0, & 5^{th} \text{ tuple, odd } i \neq m-1; 6^{th} \text{ tuple, even } i \neq m;\ 7^{th} \text{ tuple, odd } i = m-1; \\ & 8^{th} \text{ tuple, even } i = m \\ 2, & 2^{nd} \text{ tuple, even } i \neq m;\ 1^{st} \text{ tuple, odd } i;\ 4^{th} \text{ tuple, } i = m \\ i, & 4^{th} \text{ and } 8^{th} \text{ tuple, } 1 \leq i \leq \frac{m}{2} \\ i+1, & 3^{rd} \text{ and } 7^{th} \text{ tuple, } 1 \leq i \leq \frac{m}{2} \\ m-i, & 4^{th} \text{ and } 8^{th} \text{ tuple, } i > \frac{m}{2} \\ (m-i)-1, & 3^{th} \text{ and } 7^{th} \text{ tuple, } i > \frac{m}{2} \\ 1, & \text{otherwise.} \end{cases}$$

Thus, since all vertices of $D_2(C_m)$ for even $m \geq 10$ have distinct color codes, $\chi_L(D_2(C_m)) \leq 8$ for even $m \geq 10$.

## 2.2 Upper bounds of the *lcns* for barbell shadow cycle graphs

**Theorem 2.2.** *$\chi_L(B_{D_2(C_m)}) \leq 6$ if $m \geq 3$ is odd, and $\chi_L(B_{D_2(C_m)}) \leq 8$ if $m \geq 4$ is even.*

**Proof. Case 1.** (odd $m$)

Let $k$ be a vertex coloring with 6 colors in $B_{D_2(C_m)}$ for odd $m \geq 3$ as follows:

$$k(v_i) = \begin{cases} 1, & \text{odd } i \neq m \\ 2, & \text{even } i \\ 3, & i = m\ . \end{cases}$$

$$k(u_i) = \begin{cases} 4, & \text{odd } i \neq m \\ 5, & \text{even } i \\ 6, & i = m. \end{cases}$$

$$k(v_i^{'}) = \begin{cases} 2, & \text{odd } i \neq m \\ 4, & \text{even } i \\ 1, & i = m. \end{cases}$$

$$k(u_i^{'}) = \begin{cases} 3, & \text{odd } i \neq m \\ 6, & \text{even } i \\ 5, & i = m. \end{cases}$$

Then, we have the following color codes:

$$k_\Pi(v_i) = \begin{cases} 0, & 1^{st} \text{ tuple, odd } i \neq m;\ 2^{nd} \text{ tuple, even } i;\ 3^{rd} \text{ tuple, } i = m-1; \\ 1, & 1^{st} \text{ tuple, even } i;\ 2^{nd} \text{ tuple, odd } i \neq m \\ 2, & 5^{th} \text{ tuple, even } i;\ 4^{th} \text{ tuple, odd } i \neq m;\ 6^{th} \text{ tuple, m} = i \\ i, & 3^{rd} \text{ and } 6^{th} \text{ tuple, } 1 \leq i < \frac{m+1}{2} \\ m-i, & 3^{rd} \text{ and } 6^{th} \text{ tuple, } i \geq \frac{m+1}{2}. \end{cases}$$

$$k_\Pi(u_i) = \begin{cases} 0, & 4^{th} \text{ tuple, odd } i \neq m;\ 5^{th} \text{ tuple, even } i;\ 6^{th} \text{ tuple, } i = m-1 \\ 1, & 1^{st} \text{ tuple, odd } i;2^{nd} \text{ tuple, even } i \\ 2, & 2^{nd} \text{ tuple, even } i\text{: } 1^{st} \text{ tuple, odd } i \neq m;\ 3^{rd} \text{ tuple, } i = m \\ i, & 3^{rd} \text{ and } 6^{th} \text{ tuple, } 1 \leq i < \frac{m+1}{2} \\ m-i, & 3^{rd} \text{ and } 6^{th} \text{ tuple, } i \geq \frac{m+1}{2}. \end{cases}$$

$$k_\Pi(v_i') = \begin{cases} 0, & 2^{nd} \text{ tuple, odd } i \neq m;\ 4^{th} \text{ tuple, even } i;\ 1^{st} \text{ tuple, } i = m-1 \\ 1, & 3^{rd} \text{ tuple, even } i;\ 4^{th} \text{ tuple, odd } i \neq m \\ 2, & 6^{th} \text{ tuple, even } i;\ 3^{rd} \text{ tuple, odd } i \neq m;\ 5^{th} \text{ tuple, m} = i \\ i, & 1^{st} \text{ and } 6^{th} \text{ tuple, } 1 \leq i < \frac{m+1}{2} \\ m-i, & 1^{st} \text{ and } 6^{th} \text{ tuple, } i \geq \frac{m+1}{2}. \end{cases}$$

$$k_{\Pi}(u'_i) = \begin{cases} 0, & 3^{rd} \text{ tuple, odd } i \neq m;\ 6^{th} \text{ tuple, even } i;\ 5^{th} \text{ tuple, } i = m-1 \\ 1, & 6^{th} \text{ tuple, odd } i;\ 2^{nd} \text{ tuple, even } i \\ 2, & 4^{th} \text{ tuple, even } i;\ 2^{nd} \text{ tuple, odd } i \neq m;\ 1^{st} \text{ tuple, } m = i \\ i, & 1^{st} \text{ and } 4^{th} \text{ tuple, } 1 \leq i < \frac{m+1}{2} \\ m-i, & 1^{st} \text{ and } 4^{th} \text{ tuple, } i \geq \frac{m+1}{2}. \end{cases}$$

Thus, since all vertices in $B_{D_2(C_m)}$ for odd $m \geq 3$ have distinct color codes, $\chi_L(B_{D_2(C_m)}) \leq 6$ for odd $m \geq 3$.

**Case 2.** (even $m$). Let $k$ be a vertex coloring with 8 colors in $B_{D_2(C_m)}$ for even $m \geq 4$ as follows:

$$k(v_i) = \begin{cases} 1, & \text{odd } i \neq m-1 \\ 2, & \text{even } i \neq m \\ 3, & i = m-1 \\ 4, & i = m\ . \end{cases}$$

$$k(u_i) = \begin{cases} 5, & \text{odd } i \neq m-1 \\ 6, & \text{even } i \neq m \\ 7, & i = m-1 \\ 8, & i = m\ . \end{cases}$$

$$k(v'_i) = \begin{cases} 1, & \text{odd } i \neq m-1 \\ 7, & \text{even } i \neq m \\ 3, & i = m-1 \\ 5, & i = m\ . \end{cases}$$

$$k(u'_i) = \begin{cases} 4, & \text{odd } i \neq m-1 \\ 6, & \text{even } i \neq m \\ 2, & i = m-1 \\ 8, & i = m\ . \end{cases}$$

Then, we have the following color codes:

$$k_{\Pi}(v_i) = \begin{cases} 0, & 1^{st} \text{ tuple, odd } i \neq m;\ 2^{nd} \text{ tuple, even } i \neq m;\ 3^{rd} \text{ tuple, } i = m-1; \\ & 4^{th} \text{ tuple, } i = m \\ 2, & 6^{th} \text{ tuple, even } i \neq m;\ 4^{th} \text{ tuple, even } i \neq m;\ 5^{th} \text{ tuple, odd } m = i; \\ & 8^{th} \text{ tuple, } i = m \\ i, & 4^{th} \text{ and } 8^{th} \text{ tuple, } 1 \leq i < \frac{m}{2} \\ i+1, & 3^{rd} \text{ and } 7^{th} \text{ tuple, } 1 \leq i < \frac{m}{2} \\ m-1, & 4^{th} \text{ and } 8^{th} \text{ tuple, } i > \frac{m}{2} \\ (m-i)-1, & 3^{rd} \text{ and } 7^{th} \text{ tuple, } i > \frac{m}{2} \\ 1, & \text{otherwise.} \end{cases}$$

$$k_{\Pi}(u_i) = \begin{cases} 0, & 5^{th} \text{ tuple, odd } i \neq m;\ 6^{th} \text{ tuple, even } i \neq m;\ 7^{th} \text{ tuple, } i = m-1; \\ & 8^{th} \text{ tuple, } i = m \\ 2, & 2^{nd} \text{ tuple, even } i \neq m;\ 1^{st} \text{ tuple, odd } i;\ 4^{th} \text{ tuple, m= } i \\ i, & 4^{th} \text{ and } 8^{th} \text{ tuple, } 1 \leq i < \frac{m}{2} \\ i+1, & 3^{rd} \text{ and } 7^{th} \text{ tuple, } 1 \leq i < \frac{m}{2} \\ m-1, & 4^{th} \text{ and } 8^{th} \text{ tuple, } i \geq \frac{m}{2} \\ (m-i)-1, & 3^{rd} \text{ and } 7^{th} \text{ tuple, } i \geq \frac{m}{2} \\ 1, & \text{otherwise.} \end{cases}$$

$$k_{\Pi}(v'_i) = \begin{cases} 0, & 1^{st} \text{ tuple, odd } i \neq m; 7^{th} \text{ tuple, even } i \neq m;\ 3^{rd} \text{ tuple, } i = m-1; \\ & 5^{th} \text{ tuple, } i = m \\ 2, & 6^{th} \text{ tuple, even } i;\ 4^{th} \text{ tuple, odd } i;\ 8^{th} \text{ tuple, m= } i \\ i, & 5^{th} \text{ and } 8^{th} \text{ tuple, } 1 \leq i < \frac{m}{2} \\ i+1, & 2^{nd} \text{ and } 3^{rd} \text{ tuple, } 1 \leq i < \frac{m}{2} \\ m-1, & 5^{th} \text{ and } 8^{th} \text{ tuple, } i \geq \frac{m}{2} \\ (m-i)-1, & \text{for } 3^{nd} \text{ and } 3^{rd} \text{ tuple, } i \geq \frac{m}{2} \\ 1, & \text{otherwise.} \end{cases}$$

$$k_{\Pi}(u'_i) = \begin{cases} 0, & 4^{th} \text{ tuple, odd } i \neq m;\ 6^{th} \text{ tuple, even } i \neq m; \\ & 8^{rd} \text{ tuple, } i = m-1;\ 2^{nd} \text{ tuple, } i = m \\ 2, & 7^{th} \text{ tuple, even } i \neq m{:}1^{st} \text{ tuple, odd } i;\ 5^{th} \text{ tuple, m= } i \\ i, & 5^{th} \text{ and } 8^{th} \text{ tuple, } 1 \leq i < \frac{m}{2} \\ i+1, & 2^{nd} \text{ and } 3^{rd} \text{ tuple, } 1 \leq i < \frac{m}{2} \\ m-1, & 5^{th} \text{ and } 8^{th} \text{ tuple, } i \geq \frac{m}{2} \\ (m-i)-1, & 3^{nd} \text{ and } 3^{rd} \text{ tuple, } i \geq \frac{m}{2} \\ 1, & \text{otherwise.} \end{cases}$$

Thus, since all vertices in $B_{D_2(C_m)}$ for even $m \geq 4$ have distinct color codes, $\chi_L(B_{D_2(C_m)}) \leq 8$ for even $m \geq 4$.

Figure 1 is an example lc for barbel shadow cycle graph $B_{D_2(C_5)}$.

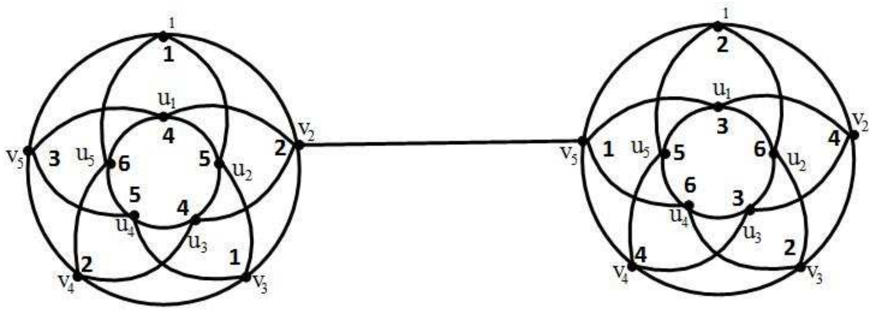


Figure 1: A minimum locating coloring for $B_{D_2(C_5)}$

# 3 Conclusions

In this paper, we obtained upper bounds of the locating chromatic numbers for shadow cycle graphs and its barbell. However, we have not precisely determined the lower bound of the locating chromatic number for these graphs, so this will be investigated in subsequent research. In addition, research on locating chromatic numbers for other operations of shadow cycle graphs is also of interest.

**Acknowledgment.** This work was partially supported by a research grant from the Directorate of Research and Community Services at Ministry of Education and Culture of Republic of Indonesia

# References

[1] G. Chartrand, D. Erwin, M.A. Henning, P.J. Slater, P. Zhang, The locating-chromatic number of a graph, Bulletin of the Institute of Combinatorics and Its Applications, **36**, (2002) 89–101.

[2] Asmiati, H. Assiyatun, E.T. Baskoro, Locating-Chromatic Number of Amalgamation of Stars, ITB J. Sci.,**43A**, (2011), 1–8.

[3] Asmiati, The locating-chromatic number of non-homogeneous amalgamation of stars, Far East Journal of Mathematical Sciences, **93(1)**, (2014), 89–96.

[4] Asmiati, On the locating-chromatic number of non-homogeneous caterpillars and firecracker graphs, Far East Journal of Mathematical Science, **100,** no. 8,(2016), 1305–1316.

[5] D.K. Syofyan, E.T. Baskoro, H. Assiyatun, On the locating-chromatic number of homogeneous lobsters, AKCE International Journal of Graphs and Combinatorics, **10,** no. 3, (2013), 245–252.

[6] A. Behtoei, B. Omoomi, On the locating chromatic number of Kneser graphs, Discrete Applied Mathematics, **159,** (2011), 2214–2221.

[7] A. Behtoei, B. Omoomi, On the locating chromatic number of cartesian product of graphs, Ars Combinatoria, **126,** (2016),221–235.

[8] D. Welyyanti, R. Simanjuntak, S. Uttunggadewa, E.T. Baskoro, The locating-chromatic number of disconnected graphs, Far East J. Math. Sci., **94,** (2014), 169–182.

[9] M. Ghanem, Al-ezeh, A. Dabour, Locating chromatic number of powers of path and cycles, Symmetry, **11,** (389), (2019),2–6.

[10] Asmiati, Wamiliana, Devriyadi, L. Yulianti, On Some Petersen Graphs Having Locating Chromatic Number Four or Five, Far East Journal of Mathematical Sciences, **102,** no. 4, (2017), 769–778.

[11] Asmiati, I.K.S.G. Yana, L. Yulianti, On the Locating Chromatic Number of Certain Barbell Graph, International Journal of Mathematical Sciences, (2018), 1–5.

[12] A. Irawan, Asmiati, L. Zakaria, K. Muludi, The Locating Chromatic Number of Origami Graphs, Algorithms,**14,** (167) (2021), 1–15.

[13] A. Irawan, Asmiati, L. Zakaria, K. Muludi, B. Utami, Subdivision of Certain Barbell Operation of Origami Graphs has Locating-Chromatic Number Five, International Journal of Computer Science and Network Security, **21,** no. 9,(2021), 79–85.

[14] Asmiati, A. Irawan, A. Nuryaman, K. Muludi, The Locating Chromatic Number for Certain Operation of Origami Graphs, Mathematics and Statistics, **11,** no. 1, (2023), 101–106.