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Upper and lower faintly (Λ, sp)-continuous multifunctions

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Abstract

In this paper, we deal with the concepts of upper and lower faintly (Λ, sp)-continuous multifunctions. Moreover, we characterize upper and lower faintly (Λ, sp)-continuous multifunctions

1 Introduction

In 1982, Long and Herrington [4] introduced the concept of faintly continuous functions. Several characterizations of faintly continuous functions were presented in [5, 9]. In 1990, Noiri and Popa [8] introduced and investigated three weaker forms of faint continuity: faint semi-continuity, faint precontinuity, and faint β-continuity. In 2020, Noiri and Popa [6] introduced a new class of multifunctions called faintly M-continuous multifunctions and investigated the notion of faintly m-I-continuous multifunctions which is a generalization of faintly M-continuous multifunctions. In this paper, we introduce the notions of upper and lower faintly (Λ, sp)-continuous multifunctions. Moreover, we characterize upper and lower faintly (Λ, sp)-continuous multifunctions.

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2 Preliminaries

Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be β-open [1] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$. The complement of a β-open set is called β-closed. The family of all β-open sets of a topological space (X, τ) is denoted by $\beta(X, \tau)$. A subset $\Lambda_{sp}(A)$ [7] is defined as follows: $\Lambda_{sp}(A) = \cap\{U \mid A \subseteq U, U \in \beta(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_{sp}-*set* [7] if $A = \Lambda_{sp}(A)$. A subset A of a topological space (X, τ) is called (Λ, sp)-*closed* [3] if $A = T \cap C$, where T is a Λ_{sp}-set and C is a β-closed set. The complement of a (Λ, sp)-closed set is called (Λ, sp)-*open*. The family of all (Λ, sp)-open sets in a topological space (X, τ) is denoted by $\Lambda_{sp}O(X, \tau)$. Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, sp)-*cluster point* [3] of A if $A \cap U \neq \emptyset$ for every (Λ, sp)-open set U of X containing x. The set of all (Λ, sp)-cluster points of A is called the (Λ, sp)-*closure* [3] of A and is denoted by $A^{(\Lambda,sp)}$. The union of all (Λ, sp)-open sets contained in A is called the (Λ, sp)-*interior* [3] of A and is denoted by $A_{(\Lambda,sp)}$.

Definition 2.1. [3] *Let A be a subset of a topological space (X, τ). The $\theta(\Lambda, sp)$-closure of A, $A^{\theta(\Lambda,sp)}$, is defined as follows:*

$$A^{\theta(\Lambda,sp)} = \{x \in X \mid A \cap\ U^{(\Lambda,sp)} \neq \emptyset \text{ for each } U \in \Lambda_{sp}O(X, \tau) \text{ containing } x\}.$$

A subset A of a topological space (X, τ) is said to be $\theta(\Lambda, sp)$-*closed* [3] if $A = A^{\theta(\Lambda,sp)}$. The complement of a $\theta(\Lambda, sp)$-closed set is said to be $\theta(\Lambda, sp)$-*open*.

Lemma 2.2. [3] *For a subset A of a topological space (X, τ), the following properties hold:*

(1) If A is (Λ, sp)-open in X, then $A^{(\Lambda,sp)} = A^{\theta(\Lambda,sp)}$.

(2) $A^{\theta(\Lambda,sp)}$ is (Λ, sp)-closed.

Throughout this paper, the spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces and $F : X \to Y$ (resp. $f : X \to Y$) presents a multivalued (resp. single valued) function. Following [2], for a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively; that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular,

$$F^-(y) = \{x \in X \mid y \in F(x)\}$$

for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$. A multifunction $F : X \to Y$ is said to be *injective* if $x \neq y$ implies that $F(x) \cap F(y) = \emptyset$.

3 Characterizations

We begin this section by introducing the concepts of upper and lower faintly (Λ, sp)-continuous multifunctions.

Definition 3.1. *A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to be:*

(i) upper faintly (Λ, sp)-continuous if, for each $x \in X$ and each $\theta(\Lambda, sp)$-open set V of Y containing $F(x)$, there exists a (Λ, sp)-open set U of X containing x such that $F(U) \subseteq V$;

(ii) lower faintly (Λ, sp)-continuous if, for each $x \in X$ and each $\theta(\Lambda, sp)$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a (Λ, sp)-open set U of X containing x such that $F(z) \cap V \neq \emptyset$ for each $z \in U$.

Theorem 3.2. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is upper faintly (Λ, sp)-continuous;

(2) $F^+(V)$ is (Λ, sp)-open in X for every $\theta(\Lambda, sp)$-open set V of Y;

(3) $F^-(K)$ is (Λ, sp)-closed in X for every $\theta(\Lambda, sp)$-closed set K of Y.

Proof. (1) ⇒ (2): Let V be any $\theta(\Lambda, sp)$-open set of Y and let $x \in F^+(V)$. Then, $F(x) \subseteq V$. Since F is upper faintly (Λ, sp)-continuous, there exists $U_x \in \Lambda_{sp}O(X, \tau)$ containing x such that $F(U_x) \subseteq V$. Thus, $U_x \subseteq F^+(V)$. It follows that $F^+(V) = \cup_{x \in F^+(V)} U_x$ and hence $F^+(V)$ is (Λ, sp)-open in X.

(2) ⇒ (1): Let $x \in X$ and let V be any $\theta(\Lambda, sp)$-open set of Y containing $F(x)$. By (2), we have $F^+(V)$ is (Λ, sp)-open in X. Put $U = F^+(V)$, then U is a (Λ, sp)-open set of X containing x such that $F(U) \subseteq V$. This shows that F is upper faintly (Λ, sp)-continuous.

(2) ⇔ (3): Since $F^+(Y - B) = X - F^-(B)$ and $F^-(Y - B) = X - F^+(B)$ for every subset B of Y, the proof is complete. □

Theorem 3.3. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is lower faintly (Λ, sp)-continuous;

(2) $F^{-}(V)$ *is* (Λ, sp)*-open in* X *for every* $\theta(\Lambda, sp)$*-open set* V *of* Y*;*

(3) $F^{+}(K)$ *is* (Λ, sp)*-closed in* X *for every* $\theta(\Lambda, sp)$*-closed set* K *of* Y*.*

Proof. The proof is similar to that of Theorem 3.2. □

Definition 3.4. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be faintly* (Λ, sp)*-continuous if, for each* $x \in X$ *and each* $\theta(\Lambda, sp)$*-open set* V *of* Y *containing* $f(x)$*, there exists a* (Λ, sp)*-open set* U *of* X *containing* x *such that* $f(U) \subseteq V$*.*

Corollary 3.5. *For a function* $f : (X, \tau) \to (Y, \sigma)$*, the following properties are equivalent:*

(1) f *is faintly* (Λ, sp)*-continuous;*

(2) $f^{-1}(V)$ *is* (Λ, sp)*-open in* X *for every* $\theta(\Lambda, sp)$*-open set* V *of* Y*;*

(3) $f^{-1}(K)$ *is* (Λ, sp)*-closed in* X *for every* $\theta(\Lambda, sp)$*-closed set* K *of* Y*.*

Definition 3.6. *A multifunction* $(X, \tau) \to (Y, \sigma)$ *is called punctually* $\theta(\Lambda, sp)$*-closed if, for each point* $x \in X$*,* $F(x)$ *is* $\theta(\Lambda, sp)$*-closed.*

Definition 3.7. *A topological space* (X, τ) *is called* Λ_{sp}*-*T_2 *if, for each distinct points* x *and* y *in* X*, there exist* $U, V \in \Lambda_{sp}O(X, \tau)$ *containing* x *and* y*, respectively, such that* $U \cap V = \emptyset$*.*

Definition 3.8. *A topological space* (X, τ) *is said to be* $\theta(\Lambda, sp)$*-normal if, for any disjoint* $\theta(\Lambda, sp)$*-closed sets* K_1, K_2 *of* X*, there exist disjoint* $\theta(\Lambda, sp)$*-open sets* V_1, V_2 *such that* $K_1 \subseteq V_1$ *and* $K_2 \subseteq V_2$*.*

Theorem 3.9. *Let* $F : (X, \tau) \to (Y, \sigma)$ *be an injective upper faintly* (Λ, sp)*-continuous multifunction. If* (Y, σ) *is* $\theta(\Lambda, sp)$*-normal and* F *is punctually* $\theta(\Lambda, sp)$*-closed, then* (X, τ) *is* Λ_{sp}*-*T_2*.*

Proof. Let x and y be any distinct points of X. Since F is injective, $F(x) \cap F(y) = \emptyset$. Since (Y, σ) is $\theta(\Lambda, sp)$-normal, there exist disjoint $\theta(\Lambda, sp)$-open sets V_x and V_y containing $F(x)$ and $F(y)$, respectively. Since F is upper faintly (Λ, sp)-continuous, there exist (Λ, sp)-open sets U_x and U_y containing x and y, respectively, such that $F(U_x) \subseteq V_x$ and $F(U_y) \subseteq V_y$. Since V_x and V_y are disjoint, U_x and U_y are disjoint. Thus, (X, τ) is Λ_{sp}-T_2. □

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