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Equivalent formulations for Economic Lot-sizing Problem with Remanufacturing and Joint Setups

Sharifah Aishah Syed Ali[1], Latifah Sarah Supian[2], Sabarina Shafie[3]

[1]Department of Defence Science
Faculty of Defence Science and Technology
National Defence University of Malaysia
Kuala Lumpur, Malaysia

[2]Department of Electrical & Electronics Engineering
Faculty of Engineering
National Defence University of Malaysia
Kuala Lumpur, Malaysia

[3]Department of Mathematics
Faculty of Science and Mathematics
Sultan Idris Education University
Perak, Malaysia

email: aishah@upnm.edu.my, sarah@upnm.edu.my,
sabarina@fsmt.upsi.edu.my

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Abstract

In this paper, we investigate the economic lot-sizing problem with the remanufacturing and joint setups case (ELSRj), where remanufacturing and manufacturing processes share the same production line. This problem is modeled as a Mixed Integer Programming (MIP) formulation and is classified as an NP-hard problem. In order to

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strengthen this basic ELSRj model formulation, an extended reformulation, namely a shortest path (SP) reformulation and also (l, S, WW) inequalities are proposed for this problem. Then, these formulations are compared with another existing extended reformulation which is a facility location (FL) reformulation. The findings show that all proposed and existing formulations are proven to be theoretically equivalent. A computational analysis of average percentage of duality gap is then presented, which demonstrates the equivalence and the effectiveness of the proposed and existing formulations. These formulations outperform the basic formulation for all data instances tested.

1 Introduction

We study the lot-sizing problem with remanufacturing and joint setups, where remanufacturing operation is carried out in combination with forward production on a single production line. This problem requires high computational time and produces poor quality solutions. Some previous studies have proposed several solutions which are a shortest path (SP) reformulation [5], a facility location (FL) reformulation [1, 7], valid inequalities [1, 5, 7] and heuristics [8]. The main contribution of this paper is to examine theoretical properties and computational analysis concerning the equivalence of different mathematical programming techniques. First, the existing approach proposed in [7] which is a facility location reformulation is highlighted. Next, the valid inequalities and a shortest path (SP) reformulation developed by [5] are further improved and modified, respectively. The equivalence of these formulations will be theoretically proven and computationally tested.

The basic formulation of the ELSRj problem [1, 5, 8] is as a mixed integer programming (MIP) model. The primary objective of this problem is generally to minimize the total costs of setup, production and inventory. The model assumptions are both returns and demands which are known in advance and the quality of new and remanufactured products are not distinguished. Note that the serviceable products refer to both new and remanufactured products.

Let N be the number of periods, where $N = 1, ..., n$.. Let x_t^r and x_t^m represent the production variables for both remanufacturing and manufacturing in period t. The setup variable in period t is denoted by y_t. Moreover, the inventory variables for used products and serviceable products in period t are denoted by I_t^r and I_t^s, respectively. The parameter d_t is the demand for both products in period t, where $d_{t,t'}$ represents the total amount of demands between t and t' such that $d_{t,t'} = \sum_{i=t}^{t'} d_i$. The parameter r_t indicates the returns obtained to be remanufactured in period t, where $r_{t,t'}$ is the total amount of returns between t and t' such that $r_{t,t'} = \sum_{i=t}^{t'} r_i$.

The production cost coefficients for remanufacturing and manufacturing for each period t are denoted by p_t^r and p_t^m, respectively. The inventory cost coefficients for used and serviceable products for each period t are represented as h_t^r and h_t^s, respectively. Finally, let K_t denote the joint setup cost coefficient of both processes for each period t. Then, the basic formulation for the ELSRj problem is as follows:

$$Z^{js} = \min \sum_{t=1}^{n} (K_t y_t + p_t^r x_t^r + p_t^m x_t^m + h_t^r I_t^r + h_t^s I_t^s) \tag{1.1}$$

$$\text{s.t.} \quad I_t^r = I_{t-1}^r + r_t - x_t^r \qquad \forall t \in N \tag{1.2}$$

$$I_t^s = I_{t-1}^s + x_t^r + x_t^m - d_t \qquad \forall t \in N \tag{1.3}$$

$$x_t^r + x_t^m \leq d_{t,n} y_t \qquad \forall t \in N \tag{1.4}$$

$$y_t \in \{0,1\}^n \qquad \forall t \in N \tag{1.5}$$

$$x_t^r, x_t^m, I_t^r, I_t^s \geq 0 \qquad \forall t \in N \tag{1.6}$$

$$I_0^r = I_0^s = 0 \tag{1.7}$$

The constraints (1.2) and (1.3) indicate inventory balance for used and serviceable products, respectively. Setup forcing constraint, given in constraint (1.4), is set to 1 if there is a positive production. Next, the integrality and nonnegativity requirements are stated in constraints (1.5) and (1.6), respectively. Finally, without loss of generality, there are no initial inventories for both used and serviceable products, indicated in constraint (1.7). The feasible region of the original formulation for the ELSRj problem as $X^{js} = \{(x^r, x^m, y, I^r, I^s) | (1.2) - (1.7)\}$ and the objective function $Z^{js} = \min \{(1.1) | (x^r, x^m, y, I^r, I^s) \in X^{js}\}$.

The rest of this paper is structured as follows: In Section 2, the existing and new formulations are discussed. In section 3, we provide theoretical and computational comparisons of all formulations. Finally, the conclusion and directions for future research are provided in Section 4.

2 Reformulations and Valid Inequalities

In this section, two reformulation techniques that solve the ELSRj problem are examined. The first technique is the facility location (FL) reformulation, which was originally developed in [1] for the ELSRj problem and then studied in [6]. Next, the new modified shortest path (SP) reformulation is introduced, initially developed in [5]. Finally, the valid inequalities, derived in [5], are strengthened further.

2.1 Facility location reformulation

In this section, we discuss the FL reformulation addressed in [6]. The decision variables are defined as follows. Let $v^{sr}_{t,t'}$ and $v^{sm}_{t,t'}$ be the amount of both products (remanufactured and new) produced in period t to meet the demand in period t', where $t' \geq t$. Then, $v^{r}_{t,t'}$ represents the amount of remanufactured products produced in period t' such that used products are returned in period t, where $t' \geq t$. Therefore, the mathematical formulation of FL is presented as follows:

$$x^r_t = \sum_{t'=1}^{t} v^r_{t',t} \qquad \forall t \in N \tag{2.8}$$

$$x^m_t = \sum_{t'=t}^{n} v^{sm}_{t,t'} \qquad \forall t \in N \tag{2.9}$$

$$v^{sr}_{t,t'} + v^{sm}_{t,t'} \leq d_{t'} y_t \qquad \forall t \in N, \qquad \forall t' \in [t, n] \tag{2.10}$$

$$v^r_{t',t} \leq r_{t'} y_t \qquad \forall t \in N, \qquad \forall t' \in [1, t] \tag{2.11}$$

$$\sum_{t'=1}^{t} \left(v^{sr}_{t',t} + v^{sm}_{t',t} \right) = d_t \qquad \forall t \in N \tag{2.12}$$

$$\sum_{t'=t}^{n} v^r_{t,t'} \leq r_t \qquad \forall t \in N \tag{2.13}$$

$$\sum_{t'=1}^{t} v^r_{t',t} = \sum_{t'=t}^{n} v^{sr}_{t,t'} \qquad \forall t \in N \tag{2.14}$$

$$v^{sr}, v^{sm}, v^r \geq 0 \tag{2.15}$$

The first two constraints (2.8) and (2.9) show the connections between original and new variables. The next two constraints (2.10) and (2.11) ensure positive productions of both operations. The demand of both products must be satisfied, indicated in constraint (2.12). Constraint (2.13) restricts the production of remanufactured products by the number of used products returned. Constraint (2.14) links $v^r_{t,t'}$ to the $v^{sr}_{t,t'}$ variables, which means that the total amount of returns retrieved at period 1 to t is remanufactured at period t to fulfill the total amount of demands from period t to n. Finally, constraint (2.15) indicates nonnegativity variables. From this, the feasible region of this FL formulation is $X^{js}_{FL} = \{(x^r, x^m, y, I^r, I^s, v^r, v^{sr}, v^{sm})|(1.2) - (2.15)\}$ and the objective function is given as $Z^{js}_{FL} = \min\{(1.1)|(x^r, x^m, y, I^r, I^s, v^r, v^{sr}, v^{sm}) \in X^{js}_{FL}\}$.

2.2 Shortest path reformulation

With respect to the ELSRj problem, we propose an alternative formulation for SP which was developed in [5]. The first new decision variable is $g^{sp}_{t,t'}$ which indicates that the value is 1 if the remanufacturing and manufacturing operations happen in period t to meet all the demands in periods $t, ..., t'$ and 0 otherwise. Then, $g^r_{t,t'}$ is 1 if used products retrieved in periods $t, .., t'$ to be remanufactured in period t' and 0 otherwise. As mentioned in [5], since not all used products are required to be remanufactured within the planning period, there is a possibility to have the final inventory of used products. Therefore, z_t is defined as the fraction of return in each of the periods t until n that is added to the final inventory of used products at the end of period n, where $I^r_t = \sum_{t=1}^{n} r_{t,n} z_t$.

In the objective function (2.16), the formulations for setup and production costs of both processes remain the same as in the original formulation (1.1). However, the formulation for inventory costs for used and serviceable products are redefined. Then, the objective function of SP formulation is given as:

$$x_t^r = \sum_{i=1}^{t} r_{i,t} g_{i,t}^{sr} \qquad t \in N \tag{2.16}$$

$$x_t^m = \sum_{i=t}^{n} d_{t,i} g_{t,i}^{sm} \qquad t \in N \tag{2.17}$$

$$\sum_{t'=t:d_{t,t'}>0}^{n} g_{t,t'}^{sp} \leq y_t \qquad t \in N \tag{2.18}$$

$$\sum_{t'=1:r_{t',t}\geq 0}^{t} g_{t',t}^{r} \leq y_t \qquad t \in N \tag{2.19}$$

$$\sum_{t=1}^{n} g_{t,n}^{sp} = 1 \tag{2.20}$$

$$\sum_{t=1}^{n} g_{1,t}^{sp} = -1 \tag{2.21}$$

$$\sum_{t=1}^{n} g_{t,n}^{r} + z_t = 1 \qquad t \in N \tag{2.22}$$

$$-\sum_{t=1}^{n} g_{1,t}^{r} - z_1 = -1 \tag{2.23}$$

$$\sum_{t=1}^{t'} g_{t,t'}^{r} = \sum_{t=t'+1}^{n} g_{t'+1,t}^{r} + z_{t'+1} \qquad t' \in [1, n-1] \tag{2.24}$$

$$\sum_{t=1}^{t'} g_{t,t'}^{sp} = \sum_{t=t'+1}^{n} g_{t'+1,t}^{sp} \qquad t' \in [1, n-1] \tag{2.25}$$

$$\sum_{t=1}^{t'} r_{t,t'} g_{t,t'}^{r} \leq \sum_{t=t'}^{n} d_{t',t} g_{t',t}^{sp} \qquad t' \in N \tag{2.26}$$

$$g^{sp}, g^r \geq 0 \tag{2.27}$$

The constraints (2.16) and (2.17) are added into SP reformulation to link the original and new decision variables. Setup forcing constraints for production of both products and acquisition process of used products are stated in constraints (2.18) and (2.19), respectively. This is followed by inventory flow balance constraints which are constraints (2.20)–(2.25). The relationship between g^r and g^{sp} is represented in constraint (2.26). Finally, constraint (2.27) indicates nonnegativity constraints. Then, the feasible region of SP formulation is given by $X^{js}_{SP} = \{(x^r, x^m, y, g^r, g^{sp})|(1.5), (2.16) - (2.27)\}$ and the objective function is $Z^{js}_{SP} = \min \left\{(\textbf{??})|(x^r, x^m, y, g^r, g^{sp}) \in X^{js}_{SP}\right\}$.

2.3 Valid inequalities

This section discusses two existing families of valid inequalities which are (ℓ, S, WW) inequalities, proposed in [5], for the ELSRj problem. Observe that their first families of valid inequalities do not include the period of 1. The inclusion of the first period in the model formulation is important since there are no initial inventories of used and serviceable products at the beginning of period 1. Thus, the remanufactured or new products should be produced at period 1 in order to meet the demands at that period.

Assume that $1 \leq j \leq \ell \leq n$. Then, the following inequalities (ℓ, S, WW) are valid for X^{js}:

$$I^r_\ell + \sum_{i=k}^{\ell} r_{k,i} y_i \geq r_{k,\ell} \tag{2.28}$$

$$I^s_{k-1} + \sum_{i=k}^{\ell} d_{i,\ell} y_i \geq d_{k,\ell} \tag{2.29}$$

The feasible region and objective function of these new families of valid inequalities for the ELSRj problem are $X^{js}_{lsww} = \{(x^r, x^m, y, I^r, I^s)|(1.2) - (1.7), (2.28), (2.29)\}$ and $Z^{js}_{lsww} = \min \left\{(1.1)|(x^r, x^m, y^r, y^m, I^r, I^s) \in X^{js}_{lsww}\right\}$, respectively.

3 Theoretical and Computational Results

In this section, we investigate the theoretical property of the solution approaches and further analyses of their computational effectiveness.

3.1 The equivalence of formulations

First of all, it should be noted that the setup binary variable y can only take two integer values 0 or 1 and if this binary variable y is relaxed to be continuous in the interval $0 \leq y \leq 1$, then this is called an LP relaxation. For example, Z_{FL}^{LPj} is the LP relaxation of a FL formulation.

Proposition 1. $Z_{FL}^{LPj} = Z_{SP}^{LPj}$ *shows that the facility location reformulation and the shortest path reformulation are identical and provides the equivalent lower bounds for the basic ELSRj problem.*

Proof. The FL and SP reformulations are theoretically equivalent, $Z_{FL}^{LPj} = Z_{SP}^{LPj}$ such that $v_{t-1,t'}^{r} \geq v_{t,t'}^{r}$ for any $1 < t \leq t' \leq n$; and $v_{t,t'}^{sr} + v_{t,t'}^{sm} \geq v_{t,t'+1}^{sr} + v_{t,t'+1}^{sm}$ for any $1 \leq t \leq t' < n$ and using the substitution of variables changes $g_{t,t'}^{r} = v_{t-1,t'}^{r} - v_{t,t'}^{r}$ for any $1 < t \leq t' \leq n$ and $g_{t,t'}^{sp} = v_{t,t'}^{sr} + v_{t,t'}^{sm} - v_{t,t'+1}^{sr} - v_{t,t'+1}^{sm}$ for any $1 \leq t \leq t' \leq n$. This demonstrates the equivalence of three reformulation techniques. This completes the proof (Interested readers can refer to [2] to see the proof of the simple lot-sizing problem). □

Proposition 2. $Z_{lsww}^{LPj} = Z_{FL}^{LPj}$ *indicates that the LP relaxation obtained by* (ℓ, S, WW) *inequalities is identical with the facility location reformulation for ELSRj. Note that all reformulation techniques are equivalent; therefore, only the facility location reformulation is considered in this proof.*

Proof. In this paper, the (ℓ, S, WW) inequalities and the FL reformulation are proven to be identical. The (ℓ, S, WW) inequalities can be used to derive an extended formulation of ELSRj. This work is similar to that in [3] where the classical uncapacitated lot-sizing problem was solved. Note that the objective function of these two formulations is the same. The proofs begin by eliminating the inventory variables from the constraints (1.2) and (1.3) of the original formulation. The equivalent formulation is shown as follows.

$$r_{1,t-1} + \sum_{j=t}^{\ell} r_{t,j} y_j \geq \sum_{j=1}^{\ell} x_j^r \qquad 1 \leq t \leq \ell \leq n \tag{3.30}$$

$$\sum_{j=1}^{t-1} \left(x_j^r + x_j^m\right) + \sum_{j=t}^{\ell} d_{j,\ell} y_j \geq d_{1,\ell} \qquad 1 \leq t \leq \ell \leq n \tag{3.31}$$

$$y_1 = 1 \tag{3.32}$$

$$x_j^r, x_j^m \geq 0,\ 0 \leq y_j \leq 1 \qquad \forall j \tag{3.33}$$

The inequalities $I^r_\ell = r_{1,\ell} - \sum_{j=1}^{\ell} x^r_j \geq 0$ and $I^s_\ell = \sum_{j=1}^{\ell}(x^r_j + x^m_j) - d_{1,\ell} \geq 0$ with $y_j = 0$, $\forall j \in \{t, ..., \ell\}$ correspond to (3.30) and (3.31), respectively. Then, $y_1 = 1$ as presented in (3.32) comes from $x^r_1 + x^m_1 = d_1 + I^s_1 \geq d_1 > 0$. Finally, (3.33) ensures nonnegativity and integrality, respectively.

Now, the relationship between these (ℓ, S, WW) inequalities and the facility location reformulation is established. The new variables, $\pi^{sr}_{j,\ell}$ and $\pi^{sm}_{j,\ell}$ are introduced to represent the production of remanufactured and new products in period j for periods j up to ℓ, respectively. The variable, $\pi^r_{j,\ell}$ is also considered to represent the number of returns in periods j up to ℓ, where at period ℓ the production of remanufactured products will occur. This variable is used as a linking variable to the variables, $\pi^{sr}_{j,\ell}$. Then, we obtain the following formulation Q :

$$\sum_{j=1}^{\ell} \pi^r_{j,\ell} \geq \sum_{j=1}^{\ell} x^r_j \qquad 1 \leq \ell \leq n \tag{3.34}$$

$$\pi^r_{j,\ell} \leq r_j \qquad 1 \leq j \leq \ell \leq n \tag{3.35}$$

$$\pi^r_{j,\ell} \leq r_j y_\ell \qquad 1 \leq j \leq \ell \leq n \tag{3.36}$$

$$\sum_{j=1}^{\ell} \left(\pi^{sr}_j + \pi^{sm}_j\right) \geq d_{1,\ell} \qquad 1 \leq \ell \leq n \tag{3.37}$$

$$\pi^{sr}_{j,\ell} + \pi^{sm}_{j,\ell} \leq x^r_j + x^m_j \qquad 1 \leq j \leq \ell \leq n \tag{3.38}$$

$$\pi^{sr}_{j,\ell} + \pi^{sm}_{j,\ell} \leq d_{j,\ell} y_j \qquad 1 \leq j \leq \ell \leq n \tag{3.39}$$

$$\pi^r_{1,j} = \pi^{sr}_{j,n} \qquad 1 \leq j \leq n \tag{3.40}$$

$$\pi^r_{j,\ell}, \pi^{sr}_{j,\ell}, \pi^{sm}_{j,\ell} \geq 0 \qquad 1 \leq j \leq \ell \leq n \tag{3.41}$$

$$0 \leq y_j \leq 1 \qquad 1 \leq j \leq \ell \leq n \tag{3.42}$$

The feasible region of this extended formulation for ELSRj is $(x^r, x^m, y, \pi^r, \pi^{sr}, \pi^{sm}) \in Q$ with its objective function $\min\{(1.1)|(x^r, x^m, y, \pi^r, \pi^{sr}, \pi^{sm}) \in Q\}$.

In regard to the relationship between Q and the facility location (FL) reformulation, we consider the definitions $\pi^r_{j,\ell} = \sum_{t=j}^{\ell} v^r_{j,t}$, $\pi^{sr}_{j,\ell} = \sum_{t=j}^{\ell} v^{sr}_{j,t}$ and $\pi^{sm}_{j,\ell} = \sum_{t=j}^{\ell} \sum v^{sm}_{j,t}$. Using these definitions of variable changes, it suffices to show that any solution $(x^r, x^m, y, v^r, v^{sr}, v^{sm}) \in X^{js}_{FL}$ of the linear programming relaxation of FL, the constraints of (2.8) - (2.15) correlate to a point $(x^r, x^m, y, \pi^r, \pi^{sr}, \pi^{sm}) \in Q$ with the same objective function value.

Suppose that any $(x^r, x^m, y, v^r, v^{sr}, v^{sm})$ satisfies the constraints of (2.8)–(2.15). Then the point $(x^r, x^m, y, \pi^r, \pi^{sr}, \pi^{sm})$ is checked whether it belongs to Q. First, constraints (2.8) and (2.9), $x^r_t = \sum_{t=j}^{n} v^{sr}_{j,t} \geq \sum_{t=j}^{\ell} v^{sr}_{j,t} = \pi^{sr}_{j,\ell}$ and $x^m_t = \sum_{t=j}^{n} v^{sm}_{j,t} \geq \sum_{t=j}^{\ell} v^{sm}_{j,t} = \pi^{sm}_{j,\ell}$ for all $1 \leq j \leq \ell \leq n$. Then, summing the constraint (2.10) over $t = 1, ..., \ell$ gives $\sum_{t=1}^{\ell} \sum_{j=1}^{t} \left(v^{sr}_{j,t} + v^{sm}_{j,t}\right) = \sum_{j=1}^{\ell} \sum_{t=j}^{\ell} \left(v^{sr}_{j,t} + v^{sm}_{j,t}\right) = \sum_{j=1}^{\ell} \left(\pi^{sr}_{j,\ell} + \pi^{sm}_{j,\ell}\right) = d_{1,\ell}$ for all $\ell = 1, ..., n$. Next, for constraint (2.11), let $\ell = n$. Then $\sum_{j=t}^{n} v^r_{t,j} = \pi^r_{t,n} \leq r_t$ for all $t = 1, ..., n$. As for constraint (2.12), since $\pi^r_{1,j} = \pi^{sr}_{j,n}$, $\sum_{j=1}^{t} v^r_{j,t} = \sum_{j=t}^{n} v^{sr}_{t,j}$. Moreover, summing the constraint (2.13) over $t = j, ..., \ell$, we have $\sum_{t=j}^{\ell} \left(v^{sr}_{j,t} + v^{sm}_{j,t}\right) = \pi^{sr}_{j,\ell} + \pi^{sm}_{j,\ell} \leq d_{j,\ell} y_j = d_t y_j$ for all $t = j, ..., \ell$. Finally, we have the constraint (2.14) $v^r_{t,j} = \pi^r_{t,j} \leq r_t y_j$, for all $1 \leq t \leq j \leq n$. This completes the proof. □

The following section provides numerical tests of the ELSRj problem, where different solution techniques are tested using a variety of test instances available in the literature.

3.2 Numerical tests

The main goal of this section is to computationally test the proven theoretical results. In this study, 360 test instances of [5] are run on a PC with an Intel®Core™ i7 2.40GHz processor with 8GB of RAM and solved by using Xpress Mosel version 7.9 without any default cuts. The default time is set to a maximum of 600 seconds (10 minutes) for each test instance. The solution time is counted as a maximum time limit if a test instance does not reach optimal solution within the given time. The planning periods are tested at 25, 50 and 75 periods. The demands and returns are nonnegativity and normally distributed. There are three return scenarios considered which are low, medium, and high returns. In addition, all cost parameters are assumed to be time-invariant except for setup costs for both remanufacturing and manufacturing, which are set to be 125, 250, 500 and 1000. For interested readers, the details of data assumptions can be seen in [5]. In Table 1, we report the average duality gaps (%) for test instances after the default time, where "All" represents all formulations as FL, SP and (ℓ, S, WW) inequalities that have provided the same lower bounds. Note that the duality gaps are also known as LP gaps. In order to provide clear illustrations for readers, the best result is highlighted in bold-face. As discussed earlier, all existing and proposed formulations have been theoretically proven to be equivalent. These findings are supported by computational tests and are also in line with the findings of the uncapacitated single-item lot-sizing problem addressed in [4].

Table 1: Average duality gaps (%) of different formulations ($N = 25, 50, 75$)

Returns Scenario	Setup cost	$NT = 25$ Basic	$NT = 25$ All	$NT = 50$ Basic	$NT = 50$ All	$NT = 75$ Basic	$NT = 75$ All
Low returns	125	81.52	**0**	88.79	**0.01**	92.54	**0**
	250	77.73	**0**	86.56	**0**	91.25	**0**
	500	72.85	**0**	83.51	**0**	89.22	**0**
	1000	67.09	**0**	79.63	**0**	86.59	**0**
Medium returns	125	77.19	**0.27**	84.61	**1.03**	85.58	**1.04**
	250	76.93	**0.26**	84.87	**0.48**	87.47	**0.42**
	500	74.17	**0.03**	83.33	**0.11**	87.22	**0.22**
	1000	69.89	**0.04**	80.51	**0.01**	85.83	**0.08**
High returns	125	42.08	**4.53**	41.73	**3.26**	43.19	**3.42**
	250	52.33	**3.64**	52.65	**3.51**	54.98	**3.32**
	500	59.47	**3.16**	61.14	**3.07**	64.35	**3.65**
	1000	61.92	**2.04**	66.33	**1.95**	70.70	**2.31**

4 Conclusions

We studied the lot-sizing problem with the remanufacturing and joint setups case (ELSRj), where both processes were carried out on the same production line. In general, this problem is NP-hard which provided the motivation to further study the existing solution techniques and develop other techniques that can solve the problem effectively and efficiently. The proposed formulations were the modified shortest path (SP) reformulation and improved (ℓ, S, WW) inequalities. The existing formulation and the facility location (FL) reformulation were also discussed and we compared the results. Interestingly, these formulations were theoretically proven to be identical. The computational testing have also shown the same findings. For future research, the inclusion of capacity restrictions for both operations or several important remanufacturing elements or processes into the basic formulation should be an interesting topic to explore.

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References

[1] Jesus O. Cunha, Rafael A. Melo, A computational comparison of formulations for the economic lot-sizing with remanufacturing, Computers & Industrial Engineering, **92,** (2016), 72–81.

[2] Y. Pochet, L. A. Wolsey, Lot-size models with backlogging: Strong reformulations and cutting planes, Mathematical Programming, **40,** nos. 1-3, (1988), 317–335.

[3] Yves Pochet, Laurence A. Wolsey et al., Algorithms and reformulations for lot-sizing problems, DIMACS Series in Discrete Mathematics and Theoretical Computer Science, **20,** (1995), 245–293.

[4] Pochet, Yves Pochet, Laurence A. Wolsey, Production planning by mixed integer programming, 2006.

[5] Helmrich Retel, J. Mathijn, Raf Jans, Wilco van den Heuvel, Albert P. M. Wagelmans, Economic lot-sizing with remanufacturing: complexity and efficient formulations, IIE Transactions, **46,** no. 1, (2014), 67–86.

[6] Sharifah Aishah Syed Ali, Mahdi Doostmohammadi, Kerem Akartunalı, Robert van der Meer, A theoretical and computational analysis of lot-sizing in remanufacturing with separate setups, International Journal of Production Economics, **203,** (2018), 276–285.

[7] Sharifah Aishah Syed Ali, Mohd Syazwan Mohamad Anuar, Khairani Abdul Majid, A computational study of lower bounds for the economic lot-sizing with remanufacturing and joint setups, AIP Conference Proceedings, **2184,** no. 1, (2019), 040002.

[8] Ruud H. Teunter, Z. Pelin Bayindir, Wilco Van Den Heuvel, Dynamic lot-sizing with product returns and remanufacturing, International Journal of Production Research, **44,**, no. 20, (2006), 4377–4400.