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Embedding of Recursive Circulant $RC(2^n, 4)$ into Circular Necklace

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Abstract

In this paper, we compute the wirelength of embedding Recursive Circulants $RC(2^n, 4)$ into Circular Necklace. Further, we identify a set of edges S in $RC(2^n, 4)$ such that the wirelength of embedding $RC(2^n, 4)\backslash S$ into circular necklace is minimum.

1 Introduction

The problem of simulating one network by another is modeled as a graph embedding problem. Embedding is a function f that maps the vertex set of G (guest graph) to the vertex set of H (host graph) such that every edge in G is mapped to a path in H [1]. Park and Chwa [2] initiated the study of recursive circulants in 1994. The recursive circulant graphs $RC(2^n, 4)$ is a family of circulant graphs. It is n-regular, vertex symmetric and hamiltonian with a high recursive structure and maximum connectivity. Yang et al., solved the maximum subgraph problem for $RC(2^n, 4)$ in 2005 [3]. In this paper, we obtain the minimum wirelength for embedding $RC(2^n, 4)$ into the circular necklace $CN(K_{2^{n-2}}, C_4)$. We also identify a set of edges S in $RC(2^n, 4)$ such that the wirelength of embedding $RC(2^n, 4)\backslash S$ into $CN(K_{2^{n-2}}, C_4)$ is minimum.

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Lemma 1.1. Congestion Lemma [4] *Let f be an embedding of a graph G into H with same order. Let S be an edge cut of H such that the removal of edges of S separates H into exactly two connected components H_1 and H_2 and let $G_1 = f^{-1}(H_1)$ and $G_2 = f^{-1}(H_2)$. Suppose G_1 and G_2 are maximum subgraphs of G and $P_f(u,v)$ with $u \in G_1$ and $v \in G_2$ contains exactly one edge in S for $(u,v) \in G$, then $EC_f(S) = \sum_{u \in V(G_1)} deg_G(u) - 2|E(G_1)| = \sum_{v \in V(G_2)} deg_G(v) - 2|E(G_2)|$*

Lemma 1.2. ***k-Partition Lemma*** [4] *Let $f : G \to H$ be an embedding. Let $E^k(H)$ denote a collection of edges of H with each edge in H repeated exactly k times. Let $\{S_1, S_2, \ldots, S_p\}$ be a partition of $E^k(H)$ such that each S_i is an edge cut of H. Then $WL_f(G,H) = \frac{1}{k}\sum_{i=1}^{p} EC_f(S_i)$.*

2 Recursive Circulant Graph

Definition 2.1. [2] *For any positive integer n, the recursive circulant $RC(2^n, 4)$ is defined by $V(RC(2^n, 4)) = \{0, 1, 2, \ldots, 2^n - 1\}$ and $E(RC(2^n, 4)) = \{(p, q) : p - q \equiv 4^i (mod\ 2^n)$ or $q - p \equiv 4^i (mod\ 2^n)$ for some $0 \le i \le \lceil n/2 \rceil - 1\}$.*

Consider the recursive circulant $RC(2^n, 4)$, where $n \ge 2$ and n even. Let the vertices of the outer cycle of $RC(2^n, 4)$ be $v_0, v_1, \ldots, v_{2^n-1}$ in the clockwise sense. There are four decks in $RC(2^n, 4)$ with 2^{n-2} vertices in each deck. There are 2^{n-2} number of decks each inducing a 4-cycle. We label the outer cycle of $RC(2^n, 4)$ in the clockwise sense as $l(v_i) = \lfloor \frac{i}{4} \rfloor + (i\ mod\ 4)2^{n-2}, 0 \le i \le 2^n - 1$. See Figure 1(a). The circular and the recursive view of $RC(2^4, 4)$ are shown in Figure 1 (a) and (b).

Lemma 2.2. [3] *Let $I_G(m)$ denote the maximum number of edges in a subgraph of graph G induced by m vertices. Then $I_G(m) = \sum_{i=0}^{r}(p_i/2 + i)2^{p_i}$ where $p_0 > p_1 > \ldots > p_r \ge 0$ are non-negative integers defined by $m = \sum_{i=0}^{r} 2^{p_i}$.*

Remark 2.3. *By Lemma 2.2, the label $L_i = \{0, 1, 2, \ldots, i-1\}$ induces a maximum subgraph of $RC(2^n, 4)$ on i vertices, $0 \le i \le 2^n - 1$.*

3 Embedding of $RC(2^n, 4)$ into $CN(K_{2^{n-2}}, C_4)$

Consider a complete graph $K_{2^{n-2}}$ on 2^{n-2} vertices. With each of its vertices attach a 4-cycle C_4. The resulting graph is a circular necklace denoted by $CN(K_{2^{n-2}}, C_4)$.

Embedding Algorithm A

Input: The n-regular Recursive Circulant $RC(2^n, 4)$ and the circular necklace $CN(K_{2^{n-2}}, C_4)$, where $n \geq 2$ and n even.

Alogrithm: Label the 2^{n-2} number of decks on 4 vertices of $RC(2^n, 4)$ sequentially as $0, 1, \ldots, 2^n - 1$. Label the vertices of $K_{2^{n-2}}$ in $CN(K_{2^{n-2}}, C_4)$ as $4i$, $0 \leq i \leq 2^{n-2} - 1$ and vertices of C_4 in $CN(K_{2^{n-2}}, C_4)$ corresponding to i^{th} vertex as $4i, 4i+1, 4i+2, 4i+3$, $0 \leq i \leq 2^{n-2} - 1$. Let $f(x) = x$ for all $0 \leq x \leq 2^n - 1$ and for $(a, b) \in E(G)$, let $P_f(a, b)$ be a shortest path between $f(a)$ and $f(b)$ in H.

Output: An embedding f of $RC(2^n, 4)$ into $CN(K_{2^{n-2}}, C_4)$ given by $f(x) = x$ yielding minimum wirelength $= (3n - 4)2^{n-1}$.

Proof of correctness: Let S_i denote the set of all edges of $K_{2^{n-2}}$ incident at the i^{th} vertex of a hamiltonian cycle in $K_{2^{n-2}}$, taken in the clockwise sense, $1 \leq i \leq 2^{n-2}$. The removal of S_i leaves $CN(K_{2^{n-2}}, C_4)$ into two components H_{i_1} and H_{i_2} such that the inverse images $G_{i_1} = f^{-1}(H_{i_1})$ and $G_{i_2} = f^{-1}(H_{i_2})$ are maximum subgraph of $RC(2^n, 4)$. Let D_{i_1} and D_{i_2} denote diametrically opposite edges of the 4-cycle attached at the i^{th} vertex of the hamiltonian cycle. See Figure 1 (c). The two components obtained by deleting each of these cut sets satisfy the condition of Congestion Lemma. Moreover each edge of $CN(K_{2^{n-2}}, C_4)$ is cut exactly twice. By Congestion Lemma and 2-Partition Lemma the wirelength is minimum. Further, $WL_f(RC(2^n, 4),\ CN(K_{2^{n-2}}, C_4)) = \sum_{i=1}^{2^{n-2}} EC_f(S_i) + \sum_{i=1}^{2^{n-2}} EC_f(D_{i_1}) + \sum_{i=1}^{2^{n-2}} EC_f(D_{i_2}) = (3n - 4)2^{n-1}$.

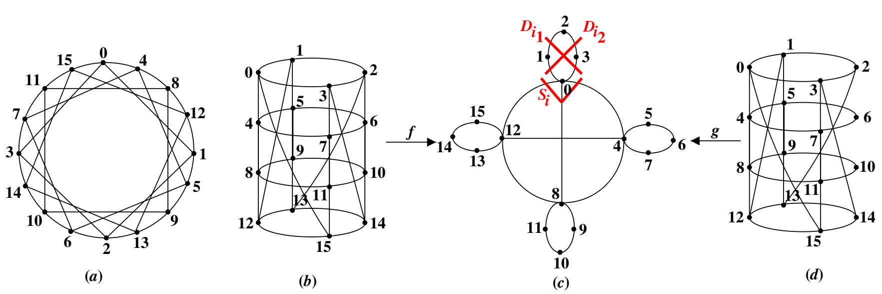

Figure 1: Recursive Circulant $RC(2^n, 4)$ (a). Circular view; (b). recursive view; (b) & (c). Embedding of $RC(2^n, 4)$ into $CN(K_{2^{n-2}}, C_4)$; (c) & (d). Embedding of $RC(2^n, 4) \backslash S$ into $CN(K_{2^{n-2}}, C_4)$

Thus, we arrive at the following theorem.

Theorem 3.1. *The exact wirelength of embedding $RC(2^n, 4)$ into $CN(K_{2^{n-2}}, C_4)$ is given by $WL_f(RC(2^n, 4), CN(K_{2^{n-2}}, C_4)) = (3n - 4)2^{n-1}$.*

Theorem 3.2. *Let $f : G \to H$ be an embedding such that $WL(G,H) = WL_f(G,H)$. Let $e_1, e_2, \ldots, e_k$ be edges in G that have maximum dilation d in H. Then, $WL(G - \{e_1, e_2, \ldots, e_k\}, H) = WL(G,H) - kd$.*

Remark 3.3. *The diameter of $CN(K_{2^{n-2}}, C_4)$ is 5. The embedding f given in the Embedding Algorithm yields a set S of $3 \times 2^{n-2}$ number of edges, each mapped to a path of dilation 5. These are the edges $(4i+2, 4i+6), i = 0, 1, 2, 3, 4, 5, \ldots, 2^{n-1}-4, 2^{n-1}-3, 2^{n-1}-2$. See Figure 2(b). Deletion of these edges gives a non-regular subgraph of $RC(2^n, 4)$ for which the wirelength of embedding $RC(2^n, 4)\backslash S$ into $CN(K_{2^{n-2}}, C_4)$ is $2^{n-1}(3n-4) - 3 \times 2^{n-2} \times 2 = (3n-7)2^{n-1}$.*

4 Conclusion

In this paper, we have determined the minimum wirelength of $RC(2^n, 4)$ into $CN(K_{2^{n-2}}, C_4)$ and a set S of edges in $RC(2^n, 4)$ such that the wirelength of embedding $RC(2^n, 4)\backslash S$ into circular necklace is minimum.

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