International Journal of Mathematics and
Computer Science, **16**(2021), no. 3, 935–948

Auxiliary Information Based Variable Sampling Interval EWMA Chart for Process Mean Using Expected Average Time to Signal

Adamu Abubakar Umar[1,2], Michael Boon Chong Khoo[1], Sajal Saha[3], Zhi Lin Chong[1]

[1]School of Mathematical Sciences
Universiti Sains Malaysia
Pulau Pinang, Malaysia

[2]Department of Statistics
Faculty of Physical Sciences
Ahmadu Bello University
Zaria, Kaduna, Nigeria

[3]Department of Mathematics
International University of Business Agriculture and Technology
Dhaka, Bangladesh

email: aaumar4real@yahoo.com, mkbc@usm.my,
sajal.saha@iubat.edu, chongzl@usm.my

(Received February 16, 2021, Revised March 12, 2021,
Accepted May 11, 2021)

Abstract

An exponentially weighted moving average (denoted as EWMA) chart is quite effective to detect small to moderate shifts. The adoption of the variable sampling interval (denoted as VSI) technique into the auxiliary information based EWMA chart (denoted as AIB-VSIEWMA) significantly improves the efficiency of the auxiliary information based EWMA (denoted as AIB-EWMA) chart in detecting small and moderate shifts, using the average time to signal (ATS) criterion when the precise shift size which must be detected quickly can be specified. However, in many industrial processes, the aforementioned size of the shift cannot be specified, particularly when practitioners have no prior

Key words and phrases: Control charts, EWMA, AIB, VSI, ATS, EATS.
AMS (MOS) Subject Classifications: 62M02, 62M10, 62P30, 93E35.
ISSN 1814-0432, 2021, http://ijmcs.future-in-tech.net

knowledge about the process. This paper adopts the expected ATS (EATS) criterion based on the Markov chain approach to assess the efficiency of the AIB-VSIEWMA chart when the precise size of the shift cannot be specified. The AIB-VSIEWMA chart is compared with the AIB-EWMA chart using the ATS and EATS performance criteria. It is revealed that the AIB-VSIEWMA chart outperforms the AIB-EWMA chart as the former provides quicker detection of process shifts.

1 Introduction

Statistical process control (SPC) is a popular statistical technique commonly used for monitoring and improving the process quality in manufacturing industries. This is essential so that the final product meets consumers' needs and satisfaction. The control chart, which is SPC's most important tool, is adopted to reduce variation in manufacturing and service industries [1].

In 1924, the first control chart, known as the Shewhart $\bar{X}$ chart was introduced [1]. The Shewhart $\bar{X}$ chart is usually employed for detecting large process shifts, and it is less effective to detect small shifts compared to the memory charts, like the EWMA and cumulative sum (CUSUM) charts developed by [2] and [3]. The aforementioned memory control charts performs well in detecting small to moderate shifts.

An additional technique was recently introduced to enhance the accuracy of a chart by adopting the regression estimator. The auxiliary information Based (AIB) scheme is known as this technique. The AIB scheme can be integrated into the chart in monitoring the process mean or variance. [4] proposed an AIB-$\bar{X}$ chart that significantly enhances the Shewhart $\bar{X}$ chart's performance. An AIB-EWMA chart was developed by [5]. The AIB-EWMA chart outperforms the basic EWMA $\bar{X}$ chart to monitor process mean shift. [6] jointly adopted the AIB-EWMA and runs rules charts for detecting the mean shifts.

In [7], an AIB-DS chart was introduced to quickly detect the process mean shifts, while an AIB-VSSI chart was presented by [8] to effectively monitor the process mean. In most recent studies, [9] suggested suggested AIB-ACUSUM charts in detecting mean shifts, while in [10] the AIB-VSIDS chart was developed to monitor the process mean, where the aforementioned control charts were shown to outperforms the AIB competing control charts. The AIB-SSGR chart to detect process mean shifts was presented in [11].

A control chart's performance greatly affects the decision on selecting an appropriate control chart to be used in process monitoring. Thus, the efficiency of a control chart needs to be evaluated. The average time to signal (ATS) is one of the common performance measures of a control chart and it represents the average time to signal an out-of-control (OOC) shift when the precise shift size can be specified.

In practice, quality practitioners may not have historical information about a particular process. Therefore, practitioners cannot be able to ascertain the precise shift size where timely detection is required [12]. Because of this, the expected ATS (EATS) criterion is employed, where the EATS do not require the quality practitioners to ascertain the precise shift size for the process being monitored. The EATS compute the expected ATS value over the distribution function of the process shift size [13]. In [14], the control chart's performance was studied when the precise shift size cannot be specified. In [15] the DS based synthetic chart with parameter's estimation for the unknown shift sizes was examined. The VSSI chart based on the EATS criterion was introduced by [16].

In [17], the known and unknown parameters based median EWMA chart was proposed and their chart on the basis of the precise and random shift sizes was evaluated. Readers can refer to [18], [19] and [20], to name a few, for more studies.

Recently, [21] introduced the AIB-VSIEWMA chart using only the ATS performance measure criterion and the findings revealed that the AIB-VSIEWMA chart is superior to the basic AIB-EWMA chart when the precise shift size can be specified. The remarkable performance of the AIB-VSIEWMA chart has motivated the authors of this article to investigate the aforementioned chart's performance when the precise shift size where a swift response is needed cannot be specified.

In this study, the optimal design for the AIB-VSIEWMA chart based on minimizing the EATS criterion when the underlying process is OOC is proposed. The AIB-VSIEWMA control chart's performance based on known and unknown shift sizes is extensively evaluated. The EATS efficiency of the AIB-VSIEWMA chart is compared to the AIB-EWMA chart.

The outline of this paper is as follows: In Section 2, a Background of the AIB-VSIEWMA chart, performance assessments and optimal designs of the aforementioned chart to minimize the OOC ATS(δ) and EATS($\delta_{\min}, \delta_{\max}$) values based on the Markov chain approach (MCA) are discussed. A numerical comparison of the AIB-VSIEWMA and AIB-EWMA charts when the precise shift sizes can be specified and when it cannot be specified is given in Section 3, while conclusions are drawn in Section 4.

2 Background of the AIB-VSIEWMA Chart

Assume that the study and auxiliary variables, (Y, X) joint distribution follow a bivariate normal distribution (BND) with charting statistics $\mu_Y, \mu_X, \sigma_Y^2, \sigma_X^2$

and ρ; i.e., $(Y, X) \sim N_2(\mu_{Y0} + \delta\sigma_Y, \mu_X, \sigma_Y^2, \sigma_X^2, \rho)$. Here, μ_Y and σ_Y^2 represents the mean and variance of the study variable Y; μ_X and σ_X^2 represents the mean and variance of the auxiliary variable X, while ρ denotes the linear relationship between Y and X. Also, $\delta = |\mu_{Y1} - \mu_{Y0}|/\sigma_Y$ denotes the size of the standardized mean shift of the Y, while μ_{Y0} and μ_{Y1} represents the in-control (IC) and OOC population means of the Y.

Let (Y_j, X_j) for $j(= 1, 2, \ldots, n_k)$, represent the k^{th} random size of sample n_k from a BND. Then the regression estimator of μ_{Y_k}, denoted as $\hat{\mu}^*_{Y_k}$, is given in [7] and [10]

$$\hat{\mu}^*_{Y_k} = \hat{\mu}_{Y_k} + \beta(\mu_X - \hat{\mu}_{X_k}), \tag{2.1}$$

where $\hat{\mu}_{Y_k}$ and $\hat{\mu}_{X_k}$ are the kth sample means of the Y and X; i.e., $\hat{\mu}_{Y_k} = \bar{Y}_k = \frac{1}{n_k}\sum_{j=1}^{n_k} Y_j$, $\hat{\mu}_{X_k} = \bar{X}_k = \frac{1}{n_k}\sum_{j=1}^{n_k} X_j$ and $\beta = \rho(\sigma_Y/\sigma_X)$. It is worth noting that $\hat{\mu}_{Y_k}$ is a special case of $\hat{\mu}^*_{Y_k}$ when ρ=0.

The respective mean and variance of $\hat{\mu}^*_{Y_k}$ in Eq. (2.1), denoted by $\mathrm{E}(\hat{\mu}^*_{Y_k})$ and $\mathrm{Var}(\hat{\mu}^*_{Y_k})$, respectively, are computed as follows: $\mathrm{E}(\hat{\mu}^*_{Y_k})$=$\mu_Y$ and $\mathrm{Var}(\hat{\mu}^*_{Y_k})$
$= \frac{\sigma_Y^2}{n_k}(1 - \rho^2)$.

As (Y, X) follows a BND, the estimator $\hat{\mu}^*_{Y_k}$ in Eq. (2.1) is a normal random variable (NRV) with the given distribution, $\hat{\mu}^*_{Y_k} \sim N(\mu_{Y0}+\delta\sigma_Y, \frac{\sigma_Y^2}{n_k}(1-\rho^2))$ ([7]). The AIB-VSIEWMA chart's sample statistic is

$$Z_k = \lambda\hat{\mu}^*_{Y_k} + (1 - \lambda)Z_{k-1}, \text{for } k\text{=1,2,}\ldots, \tag{2.2}$$

where $\lambda(0 < \lambda \leq 1)$ denote a smoothing constant parameter and $Z_0 = \mu_{Y0}$ (denote the IC population mean of Y). The lower control limit (LCL), lower warning limit (LWL), central line (CL), upper control limit (UCL) and upper warning limit (UWL) for the AIB-VSIEWMA chart are expressed as follows:

$$LCL = \mu_{Y0} - L\sigma_Y\sqrt{\frac{\lambda(1-\rho^2)}{n_k(2-\lambda)}}, LWL = \mu_{Y0} - W\sigma_Y\sqrt{\frac{\lambda(1-\rho^2)}{n_k(2-\lambda)}} \tag{2.3}$$

$$CL = \mu_{Y0} \tag{2.4}$$

$$UCL = \mu_{Y0} + L\sigma_Y\sqrt{\frac{\lambda(1-\rho^2)}{n_k(2-\lambda)}}, UWL = \mu_{Y0} + W\sigma_Y\sqrt{\frac{\lambda(1-\rho^2)}{n_k(2-\lambda)}}, \tag{2.5}$$

where n_k is the size of sample, while the design parameters L is the control's width and W is the warning limits for the optimal AIB-VSIEWMA chart. Interested readers can found the procedures for implementing the AIB-VSIEWMA chart in [21].

2.1 Performance Assessment

A control chart's performance is assessed by the speed at which an OOC process shift is detected. If the control chart of interest is faster than other control charts to detect a shift in a process when all the control charts under consideration are set to have equal IC efficiency, then the control chart of interest is said to outperform the competing control charts.

In this study, the EATS $(\delta_{\min}, \delta_{\max})$ performance measure is employed in assessing the AIB-VSIEWMA chart's performance for detecting unknown shift sizes. The AIB-VSIEWMA and AIB-EWMA charts are compared using the ATS(δ) and EATS($\delta_{\min}, \delta_{\max}$) criteria to monitor known and random shift sizes.

The ATS and EATS values of the AIB-VSIEWMA chart are obtained by adopting the MCA. Suppose that a Markov chain model (MCM) having c+1 state is observed, where states $(1, 2, \ldots, c)$ represents the transient, while state (c+1) represent the absorbing state. The transition probability matrix (TPM), $\mathbf{R}$ of the MCM is given in [22] and [21]

$$\mathbf{R} = \begin{pmatrix} \mathbf{J} & \mathbf{r} \\ \mathbf{0}^{\mathrm{T}} & 1 \end{pmatrix} = \begin{pmatrix} J_{1,1} & J_{1,2} & \cdots & J_{1,c} & r_1 \\ J_{2,1} & J_{2,2} & \cdots & J_{2,c} & r_2 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ J_{c,1} & J_{c,2} & \cdots & J_{c,c} & r_c \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix}, \tag{2.6}$$

where $\mathbf{J}$ is a $c \times c$ matrix of the transition probabilities (TP) for the transient states of the AIB-VSIEWMA chart, while $\mathbf{r}$ denotes a $c \times 1$ vector that satisfies the condition $\mathbf{r} = \mathbf{1} - \mathbf{J}\mathbf{1}$. The procedure for obtaining the respective TP, $J_{i,t}$ in the TPM $\mathbf{J}$ involves to divide the LCL and UCL intervals into an odd quantity $c = 2w + 1$ sub intervals, each having width $2\Delta = (UCL - LCL)/c$.

Let H_t denote the midpoint of the t^{th} sub interval; i.e., $H_t = \frac{(LCL+UCL)}{2} + 2t\Delta$, for $t = \{-w, -(w-1), ..., -1, 0, 1, ..., w-1, w\}$. Thus, the entries of the TP, $J_{i,t}$, for $i, t = \{-w, -(w-1), ..., -1, 0, 1, ..., w-1, w\}$, of matrix $\mathbf{J}$ are obtained as ([22] and [21])

$$\begin{aligned} J_{i,t} = \Phi((\frac{H_t + \Delta - (1-\lambda)H_i}{\lambda} - \delta)\sqrt{\frac{n}{(1-\rho^2)}}) \\ -\Phi((\frac{H_t - \Delta - (1-\lambda)H_i}{\lambda} - \delta)\sqrt{\frac{n}{(1-\rho^2)}}), \end{aligned} \tag{2.7}$$

where the quantity δ is the standardized shift size and $\Phi(\cdot)$ is the cumulative distribution function (cdf) of the standard NRV.

The ATS criterion for the AIB-VSIEWMA chart to detect a known shift size δ, in the study variable, Y is computed by

$$\mathrm{ATS}(\delta) = \mathbf{q}^{\mathrm{T}}(\mathbf{I}-\mathbf{J})^{-1}\mathbf{h}, \tag{2.8}$$

where $\mathbf{q}$ represents a $c \times 1$ vector of initial probabilities which consists the probabilities corresponding to the restarting state having the $(w+1)^{\text{th}}$ entry equal to one and zeros otherwise, $\mathbf{h}$ is a $c \times 1$ vector of sampling intervals that contains the element h_2 (long sampling interval) or h_1 (short sampling interval) and $\mathbf{I}$ is an identity matrix of order $c \times c$.

It worth noting that the computation of the ATS needs the quality practitioners to specify the shift size in advance. However, in practical applications, the shift size where a quick detection ability is needed cannot be specified, as quality practitioners do not have prior information of the overall process to specify the precise size of the shift. To address this setback, the EATS criterion should be considered as the performance assessment for the overall shifts interval $(\delta_{\min}, \delta_{\max})$, where $\delta_{\min}$ represent the lower bound while $\delta_{\max}$ represent upper bound for the shift size. The EATS criterion of the AIB-VSIEWMA chart, for the shift interval $(\delta_{\min}, \delta_{\max})$ is expressed as follows:

$$\mathrm{EATS}(\delta_{\min}, \delta_{\max}) = \int_{\delta_{\min}}^{\delta_{\max}} \mathrm{ATS}(\delta)\mathrm{g}_{\delta}(\delta)\mathrm{d}\delta, \tag{2.9}$$

where $\mathrm{g}_{\delta}(\delta)$ in Eq. (2.9) is the probability density function (pdf) of the magnitude of the shift size, δ and $\mathrm{ATS}(\delta)$ is adopted from Eq. (2.8). According to [23] and [14], as the exact shape of $\mathrm{g}_{\delta}(\delta)$ is typically unknown, each shift size, δ value in the interval $(\delta_{\min}, \delta_{\max})$ is presumed to exist with an equal probability of occurrence. Then δ is assumed to follow a uniform distribution on the interval of shift $(\delta_{\min}, \delta_{\max})$. Therefore, Eq. (2.9) becomes

$$\mathrm{EATS}(\delta_{\min}, \delta_{\max}) = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \mathrm{ATS}(\delta)\mathrm{d}\delta. \tag{2.10}$$

Remember that in Eq. (2.10), the Gau.ss-Legendre quadrature is adopted in approximating the corresponding integral. From Eqs. (2.8) and (2.10), the IC ATS (say ATS(0)) and the OOC ATS (say $\mathrm{ATS}(\delta)$) represents the ATS (as defined in Section 1) when the process is IC and OOC, respectively. Additionally, the IC and OOC EATSs are represented by EATS(0) and $\mathrm{EATS}(\delta_{\min}, \delta_{\max})$. In this study, both the ATS and EATS criteria are used in assessing the performance of the AIB-VSIEWMA chart when the process mean shift size is known and unknown.

2.2 Optimal Designs

The optimization procedure for computing the optimal parameter combinations (λ, L, W) of the AIB-VSIEWMA chart that minimizes ATS(δ) or EATS($\delta_{\min}, \delta_{\max}$) value is written as follows:

$$\underset{(\lambda,L,W)}{\text{Minimize}}[\text{ATS}(\delta)] \tag{2.11}$$

or

$$\underset{(\lambda,L,W)}{\text{Minimize}}[\text{EATS}(\delta_{\min}, \delta_{\max})], \tag{2.12}$$

s.t. ATS(0) (or EATS(0))=τ, ASI(0) $=h_0$ and $h_1 < h_0 < h_2$. Here, ASI(0) represent the IC average sampling interval.

The MATLAB optimization programs are used in computing the optimal charting parameter combination (λ, L, W) of the AIB-VSIEWMA chart which minimizes the ATS(δ) or EATS($\delta_{\min}, \delta_{\max}$) value.

3 Numerical Comparison

In practical situation, the precise shift size in a process is usually unknown. Therefore, if the optimal charting parameters corresponding to a particular shift size are used, the control chart's performance will be greatly affected if the size of the actual shift is different. It is therefore crucial to evaluate the AIB-VSIEWMA chart's performance using the EATS($\delta_{\min}, \delta_{\max}$) criterion as an alternative to the ATS(δ) performance measure.

For the purpose of illustration in this study, $\tau = 370$, $h_0 = 1$, $h_1 = 0.1$, $h_2 = 1.9$, $n_0 \in \{5, 7, 10\}$ and $\rho \in \{0.00, 0.20, 0.50, 0.75\}$ are considered. Then, the charting parameter combinations of the optimal AIB-VSIEWMA chart are computed by minimizing ATS(δ) and EATS($\delta_{\min}, \delta_{\max}$), for $\delta \in \{0.1, 0.3, 0.5, 1, 1.5\}$ and $(\delta_{\min}, \delta_{\max}) \in \{(0.1, 1.0), (1.0, 1.5)\}$. These corresponding δ and pairs $(\delta_{\min}, \delta_{\max})$ values are the precise sizes of the shift and the intervals of the shift, where swift detection by the AIB-VSIEWMA chart is of great importance.

The charting parameters combination (λ, L, W) of the AIB-VSIEWMA chart that minimize ATS(δ) are presented in Table 1. For instance, when the AIB-VSIEWMA chart is optimally designed in minimizing ATS(0.5), for ATS(0) $= 370$, $h_1 = 0.1$, $h_2 = 1.9$ and $\rho = 0.75$, the charting parameters computed are $(\lambda, L, W) \in \{(0.41, 2.9623, 0.6649), (0.52, 2.9810, 0.6691),$ $(0.62, 2.9903, 0.6712)\}$, for $n_0 =$ (5, 7 and 10).

Similarly, the charting parameters of the AIB-VSIEWMA chart that minimize EATS($\delta_{\min}, \delta_{\max}$) are provided in Table 2. For instance, when

Table 1: Optimal charting parameters (λ, L, W) of the AIB-VSIEWMA chart to minimize ATS(δ) when ATS(0)=370 and $h_0 = 1$.

		n_0=5			n_0=7			n_0=10		
ρ	δ	λ	L	W	λ	L	W	λ	L	W
0.00	0.1	0.02	2.1728	0.5960	0.02	2.1728	0.5960	0.03	2.3285	0.6155
	0.3	0.10	2.7091	0.6486	0.12	2.7545	0.6594	0.16	2.8190	0.6608
	0.5	0.21	2.8714	0.6588	0.28	2.9173	0.6693	0.37	2.9521	0.6626
	1.0	0.58	2.9872	0.6705	0.68	2.9938	0.6719	0.79	2.9976	0.6728
	1.5	0.82	2.9982	0.6729	0.91	2.9993	0.6732	0.96	2.9996	0.6732
0.20	0.1	0.02	2.1728	0.5960	0.02	2.1728	0.596	0.03	2.3285	0.6155
	0.3	0.10	2.7091	0.6486	0.12	2.7545	0.6594	0.17	2.8314	0.6637
	0.5	0.22	2.8795	0.6606	0.29	2.9222	0.6704	0.38	2.9548	0.6632
	1.0	0.6	2.9889	0.6708	0.69	2.9943	0.6720	0.79	2.9976	0.6728
	1.5	0.83	2.9984	0.673	0.92	2.9994	0.6732	0.96	2.9996	0.6732
0.50	0.1	0.02	2.1728	0.5960	0.03	2.3285	0.6155	0.03	2.3285	0.6155
	0.3	0.11	2.7333	0.6544	0.15	2.8053	0.6576	0.20	2.8626	0.6568
	0.5	0.27	2.9121	0.6681	0.35	2.9459	0.6612	0.46	2.9723	0.6671
	1.0	0.67	2.9934	0.6718	0.76	2.9968	0.6726	0.87	2.9989	0.6731
	1.5	0.90	2.9992	0.6731	0.96	2.9996	0.6732	0.4	2.9599	0.6643
0.75	0.1	0.03	2.3285	0.6155	0.04	2.4313	0.6306	0.06	2.5640	0.6394
	0.3	0.18	2.8428	0.6664	0.24	2.8939	0.6639	0.32	2.9351	0.6734
	0.5	0.41	2.9623	0.6649	0.52	2.9810	0.6691	0.62	2.9903	0.6712
	1.0	0.82	2.9982	0.6729	0.91	2.9993	0.6732	0.96	2.9996	0.6732
	1.5	0.94	2.9995	0.6732	0.24	2.8939	0.6639	0.12	2.7545	0.6594

the AIB-VSIEWMA chart is optimally designed in minimizing EATS(1.0, 1.5); i.e., $\delta_{\min}$= 1.0 and $\delta_{\max}$= 1.5, for ATS(0) = 370, h_1= 0.1, h_2= 1.9 and ρ= 0.50, the charting parameters obtained are given as $(\lambda, L, W) \in \{(0.75, 2.9966, 0.6726), (0.85, 2.9987, 0.6730), (0.92, 2.9994, 0.6732)\}$, for n_0= (5, 7 and 10).

Table 2: Optimal charting parameters (λ, L, W) of the AIB-VSIEWMA chart to minimize EATS($\delta_{\min}, \delta_{\max}$) when ATS(0)=370 and $h_0 = 1$.

			n_0=5			n_0=7			n_0=10		
ρ	$\delta_{\min}$	$\delta_{\max}$	λ	L	W	λ	L	W	λ	L	W
0.20	0.1	1.0	0.06	2.5643	0.6395	0.06	2.5643	0.6395	0.08	2.6491	0.6474
	1.0	1.5	0.68	2.9938	0.6719	0.77	2.9971	0.6727	0.87	2.9989	0.6731
0.50	0.1	1.0	0.06	2.5643	0.6395	0.08	2.6491	0.6474	0.10	2.7093	0.6486
	1.0	1.5	0.75	2.9966	0.6726	0.85	2.9987	0.6730	0.92	2.9994	0.6732
0.75	0.1	1.0	0.10	2.7093	0.6486	0.11	2.7334	0.6544	0.14	2.7903	0.6541
	1.0	1.5	0.89	2.9991	0.6731	0.94	2.9995	0.6732	0.26	2.9065	0.6668

The aforementioned optimal charting parameters described above yield the smallest ATS(0.5)$\in \{1.9507, 1.5689, 1.2988\}$ and EATS(1.0, 1.5)$\in \{1.0972, 1.0443, 1.0197\}$, for n_0= (5, 7 and 10) (see Tables 3 and 4, respectively). For the purpose of comparison, $\delta \in \{0.1, 0.3, 0.5, 1, 1.5\}$ are considered. Here, $\delta \in \{0.1, 0.3, 0.5\}$ and $\delta \in \{1, 1.5\}$ fall in $(\delta_{\min}, \delta_{\max})$=(0.1, 1.0) and $(\delta_{\min}, \delta_{\max})$= (1.0, 1.5). In Table 1, when δ= 0.3, n_0= 5, ρ= 0.5, h_1= 0.1 and h_2= 1.9, the optimal parameters (λ, L, W) that minimize ATS(0.3) are (0.11, 2.7333, 0.6544). These computed optimal parameters yield the smallest ATS(δ) value; i.e., ATS(0.3) = 6.7292 (see Table 3), while attaining the desired ATS(0) = 370.

On similar lines, for n_0= 5, ρ= 0.5, h_1= 0.1, h_2= 1.9, $\delta_{\min}$= 0.1 and $\delta_{\max}$= 1.0, the parameters that minimize EATS(0.1, 1.0) are (λ, L, W)=(0.06, 2.5643, 0.6395). These optimal charting parameters yield the smallest EATS($\delta_{\min}, \delta_{\max}$) value; i.e., EATS(0.1, 1.0) = 6.7039 (see Table 4), while attaining the desired ATS(0) = 370 value.

By considering δ= 0.3 (i.e. $\delta \in (\delta_{\min}, \delta_{\max}) = (0.1, 1.0)$) for the same combination (n_0, ρ, h_1 and h_2), ATS(0.3) = 6.7292 and EATS(0.1, 1.0) = 6.7039, respectively (see the discussions in the previous paragraphs). This findings indicate that the ATS(0.3) and EATS(0.1, 1.0) values are almost similar. Therefore, the optimal charting parameters computed based on minimizing EATS($\delta_{\min}, \delta_{\max}$) can be used effectively as long as the shift size, $\delta \in (\delta_{\min}, \delta_{\max})$, when there is no historical information for quality practitioners to identify the precise shift size.

Table 3: ATS(δ) values of the AIB-VSIEWMA and AIB-EWMA charts when ATS(0)=370 and $h_0 = 1$.

		n_0=5		n_0=7		n_0=10	
δ	ρ	**AIB-EWMA**	**AIB-VSIEWMA**	**AIB-EWMA**	**AIB-VSIEWMA**	**AIB-EWMA**	**AIB-VSIEWMA**
0.1	0.00	88.2755	52.309	68.7244	40.1567	52.2593	30.1405
	0.20	85.6881	50.642	66.6244	38.9114	50.6342	29.1464
	0.50	71.3104	41.706	55.1220	31.9152	41.8328	23.9279
	0.75	47.1274	27.034	36.3635	20.5585	27.7863	15.159
0.3	0.00	16.9978	8.5017	13.2699	6.4740	10.1710	4.7965
	0.20	16.4980	8.2128	12.8769	6.2661	9.8658	4.6462
	0.50	13.7595	6.7292	10.7126	5.0643	8.1906	3.7659
	0.75	9.2011	4.3200	7.1363	3.2736	5.4446	2.5070
0.5	0.00	7.9421	3.6487	6.1554	2.8256	4.6966	2.1410
	0.20	7.7003	3.5358	5.9675	2.7409	4.5533	2.0803
	0.50	6.3879	2.9308	4.9484	2.2488	3.7781	1.7592
	0.75	4.2448	1.9507	3.2934	1.5689	2.5204	1.2988
1.0	0.00	2.7858	1.3850	2.1624	1.1979	1.6394	1.0898
	0.20	2.7018	1.3569	2.0961	1.1815	1.5881	1.0816
	0.50	2.2440	1.2191	1.7305	1.1052	1.3231	1.0451
	0.75	1.4791	1.0656	1.1831	1.0288	1.0388	1.0135
1.5	0.00	1.4968	1.0681	1.1932	1.0299	1.0422	1.0138
	0.20	1.4514	1.0617	1.1676	1.0271	1.0338	1.0130
	0.50	1.2270	1.0337	1.0599	1.0156	1.0067	1.0095
	0.75	1.0195	1.0116	1.0014	1.0184	1.0000	1.0272

Table 4: EATS($\delta_{\min}, \delta_{\max}$) values of the AIB-VSIEWMA and AIB-EWMA charts when ATS(0)=370 and $h_0 = 1$.

			n_0=5		n_0=7		n_0=10	
$\delta_{\min}$	$\delta_{\max}$	ρ	**AIB-EWMA**	**AIB-VSIEWMA**	**AIB-EWMA**	**AIB-VSIEWMA**	**AIB-EWMA**	**AIB-VSIEWMA**
0.20	0.1	1.0	13.8744	8.0777	11.0188	6.2800	8.5945	4.7633
	1.0	1.5	1.9973	1.1667	1.5452	1.0792	1.2217	1.0341
0.50	0.1	1.0	11.7166	6.7039	9.2787	5.1625	7.2255	3.9197
	1.0	1.5	1.6530	1.0972	1.3027	1.0443	1.0962	1.0197
0.75	0.1	1.0	8.0518	4.3947	6.3556	3.4425	4.9441	2.6441
	1.0	1.5	1.1658	1.0275	1.0451	1.0142	1.0064	1.0225

Additionally, the optimal AIB-VSIEWMA and basic AIB-EWMA charts are compared for detecting the process mean shifts. To guarantee a fair assessment between the aforementioned charts, the fixed sampling interval (FSI) of the basic AIB-EWMA chart is set similar to the h_0 for the optimal AIB-VSIEWMA chart.

According to Tables 3 and 4, the ATS(δ) and EATS($\delta_{\min}, \delta_{\max}$) performances for the AIB-VSIEWMA chart surpass the AIB-EWMA chart, when considered all the precise shifts (δ) and intervals of shift ($\delta_{\min}, \delta_{\max}$), as the AIB-VSIEWMA chart has smaller ATS(δ) and EATS($\delta_{\min}, \delta_{\max}$) values compared with the AIB-EWMA chart, for all combinations ($n_0, \rho, \delta, \delta_{\min}$ and $\delta_{\max}$). For instance, as n_0= 5, $\delta_{\min}$= 0.1 and $\delta_{\max}$= 1.0, the values of EATS($\delta_{\min}, \delta_{\max}$) criterion of the AIB-VSIEWMA chart for ρ= {0.2, 0.5, 0.75} are {8.0777, 6.7039, 4.3947}, and that of the AIB-EWMA chart are {13.8744, 11.7166, 8.0518} (see Table 4). This shows that the VSI technique improves the control chart's efficiency for a quicker detection of the process shifts.

4 Conclusion

In this study, the AIB-VSIEWMA chart's sensitivity is evaluated using the ATS and EATS criteria when the precise shift size can be specified and when it cannot be specified, respectively. There are many situations in real-life applications in which quality practitioners have no prior knowledge of the overall process and this makes specifying the precise shift size in which a quick detection is required difficult. In such situation, if a quality practitioner uses the optimal parameters corresponding to the precise shift size, the AIB-VSIEWMA chart's sensitivity will be considerably deteriorated if a different size of the shift occurs in reality. Therefore, the EATS performance measure is adopted in assessing the AIB-VSIEWMA chart's performance when dealing with unknown shift sizes. The findings indicate that adopting the optimal charting parameters based on minimizing EATS($\delta_{\min}, \delta_{\max}$) is reliable, as the chart's performance is almost the same as the case in adopting the optimal charting parameters based on minimizing ATS(δ). It is shown that the optimal parameters in minimizing EATS($\delta_{\min}, \delta_{\max}$) could be used as long as the shift size, $\delta \in (\delta_{\min}, \delta_{\max})$.

Since this study deals with the univariate AIB-VSIEWMA chart with known process parameters, future researches may focus on the development of the univeriate AIB-VSIEWMA chart when parameters are estimated as well as the multivariate AIB-VSIEWMA chart with known and estimated process parameters as many real-life situations involve multivariate data. Further research can also be made in evaluating the performance of other advanced control charts, such as the control charts for the EWMA t and CUSUM with auxiliary information technique when the precise shift size is random.

Acknowledgment. This wok was supported by the Kementerian Pendidikan Malaysia, Fundamental Research Grant Scheme (FRGS), number 203.PMATHS.6711603.

References

[1] D. C. Montgomery, Introduction to Statistical Quality Control, Wiley and Sons, New York, 2012.

[2] E. Page, Continuous Inspection Schemes, Biometrika, **41,** nos. 1-2, (1954), 100–115.

[3] S. W. Roberts, Control Chart Tests Based on Geometric Moving Averages, Technometrics, **1,** no. 3, (1959), 239–250.

[4] M. Riaz, Monitoring Process Mean Level Using Auxiliary Information, Statistica Neerlandica, **62,** no. 4, (2008), 458–481.

[5] N. Abbas, M. Riaz, R. J. M. M. Does, An EWMA-type Control Chart for Monitoring the Process Mean Using Auxiliary Information, Communications in Statistics-Theory and Methods, **43,** no. 16, (2014), 3485–3498.

[6] W. Arshad, N. Abbas, M. Riaz, Simultaneous Use of Runs Rules and Auxiliary Information with Exponentially Weighted Moving Average Control Charts, Quality and Reliability Engineering International, **32,** no. 2, (2017), 323–336.

[7] A. Haq, M. B. C Khoo, A New Double Sampling Control Chart for Monitoring Process Mean Using Auxiliary Information, Journal of Statistical Computation and Simulation, **88,** no. 5, (2018), 869–899.

[8] S. Saha, M. B. C. Khoo, M. H. Lee, A Variable Sample Size and Sampling Interval Control Chart for Monitoring the Process Mean Using Auxiliary Information, Quality Technology and Qualitative Management, **16,** no. 4, (2019a), 389–406.

[9] S. Abbasi, A. Haq, Optimal CUSUM and Adaptive CUSUM Charts with Auxiliary Information for Process Mean, Journal of Statistical Computation and Simulation, **89,** no. 2, (2019), 337–361.

[10] A. A. Umar, M. B. C. Khoo, S. Saha, A. Haq, A Combined Variable Sampling Interval and Double Sampling Control Chart with Auxiliary Variable for the Process Mean, Transactions of the Institute of Measurement and Control, **42,** no. 6, (2020), 1151–1165.

[11] S. Saha, M. B. C. Khoo, A. A. Umar, P. S. Ng, Side-Sensitive Group Runs Chart for Detecting Mean Shifts Using Auxiliary Information, Proceeding of the International Conference on Mathematical Sciences and Technology (MathTech 2018), AIP Conf. Proc., (2019b), 2184, 050010-1-050010-7.

[12] G. Celano, Robust Design of Adaptive Control Charts for Manual Manufacturing/Inspection Workstations, Journal of Applied Statistics, **36,** no. 2, (2009), 181–203.

[13] H. W. You, M. B. C. Khoo, Z. L. Chong, Performance of Side Sensitive Group Runs Chart Based on Exact and Random Process Shifts, Austrian Journal of Statistics, **47,** (2018a), 72–78.

[14] P. Castagliola, G. Celano, S. Psarakis, Monitoring the Coefficient of Variation Using EWMA Charts, Journal of Quality Technology, **43,** no. 3, (2011), 249–265.

[15] H. W. You, Performance of Synthetic Double Sampling Chart with Estimated Parameters Based on Expected Average Run Length, Journal of Probability and Statistics, (2018b), 7583610, 1–6.

[16] K. W. Khaw, X. Y. Chew, S. Y. Teh, W. C. Yeong, Optimal Variable Sample Size and Sampling Interval Control Chart for the Process Mean Based on Expected Average Time to Signal, International Journal of Machine Learning and Computing, **9,** no. 6, (2019), 880–885.

[17] H. W. You, M. B. C. Khoo, Z. L. Chong, The Expected Average Run Length of the EWMA Median Chart with Estimated Process Parameters, Austrian Journal of Statistics, **49,** (2020), 19–24.

[18] M. Schoonhoven, On the Use of the Expected ARL Metric for Control Charts Based on Estimated Parameters, Quality and Reliability Engineering International, **33,** no. 4, (2017), 745–749.

[19] P. S. Ng, M. B. C. Khoo, W. C. Yeong, S. L. Lim, A Variable Sampling Interval EWMA t Control Chart with Unknown Shift Size. Journal of Fundamental and Applied Sciences, **9,** no. 5S, (2017), 542–555.

[20] H. W. You, Design of the Side Sensitive Group Runs Chart with Estimated Parameters Based on Expected Average Run Length. Pertanika Journal of Science and Technology, **26,** no. 3, (2018c), 1231–1240.

[21] P. S. Ng, M. B. C. Khoo, W. C. Yeong, S. L. Lim, A Variable Sampling Interval EWMA $\bar{X}$ Chart for the Mean with Auxiliary Information, 6th ICCST Kota Kinabalu, Malaysia, Computational Science and Technology, **2019,** 603, (2020), 113–122.

[22] L. Zhang, G. Chen, P. Castagliola, On t and EWMA t Charts for Monitoring Changes in the Process Mean, Quality and Reliability Engineering International, **25,** no. 8, (2009), 933–945.

[23] R. S. Sparks, CUSUM Charts for Signalling Varying Location Shifts, Journal of Quality Technology, **32,** no. 2, (2000), 157–171.