International Journal of Mathematics and
Computer Science, **16**(2021), no. 3, 921–934

Valid inequalities for the capacitated lot-sizing problem in a hybrid manufacturing and remanufacturing system

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(Received February 20, 2021, Revised March 15, 2021,
Accepted May 11, 2021)

Abstract

This paper investigates an existing model of capacitated lot-sizing problem in a hybrid manufacturing and remanufacturing system (HMRS) which is computationally hard to solve. Therefore, three classical families of valid inequalities are proposed to solve this problem.

Key words and phrases: Lot-sizing, integer programming, valid inequalities, production planning.
AMS (MOS) Subject Classifications: 90B30.
ISSN 1814-0432, 2021, http://ijmcs.future-in-tech.net

The efficiency of the proposed method are tested on numerical example available in the literature and compared with the existing method. The computational results show that the exact method provides better performance than the existing method in terms of LP gaps and the best feasible solution. Besides, the proposed method are also tested on a large number of test data sets, where the time-invariant cost parameters are considered. The findings demonstrate that these valid inequalities have significantly outperformed the original formulation with regard to LP gaps, solution times and the number of times the LP relaxation of a formulation found the integer optimal solution. Lastly, the concluding remarks and some future research directions are discussed.

1 Introduction

Due to the environmental concerns related to end-of-life products, many countries have enacted product take-back legislation that holds Original Equipment Manufacturers' (OEMs) financially responsible for the recovery of discarded products. One of the most powerful product recovery options is remanufacturing. Remanufacturing involves complex processes in which worn-out products are fully disassembled into several components or parts, which are cleaned, inspected, restored and rebuilt to the current standard of at least of its Original Equipment Manufacturers' (OEMs) performance specifications with a warranty [5, 9].

One of the challenges facing today's OEMs is to make the right decision, of whether to employ a dedicated model or a hybrid model. To be specific, OEMs can opt a dedicated model by outsourcing the remanufacturing process to third-party remanufacturers, which is commonly referred to as contract remanufacturers or independent remanufacturers. Otherwise, OEMs that adopt a hybrid model may combine both remanufacturing and manufacturing operations. This hybrid model is complex in nature as the production planning and control activities for both processes deal with uncertainty and risks issues related to the quality and quantity of returned-used products, demands for both products and limited resources. Additionally, the setup structure of this hybrid system further complicates the production planning activities.

Studies of deterministic models of the capacitated lot-sizing problem in a hybrid manufacturing and remanufacturing system (HMRS) with the consideration of a simple or a complex setup structure in the literature are scarce.

There are two important setup elements considered in the mathematical formulation, namely setup costs and setup times. It is important to address both setup elements in the problem as they directly affect the production planning activities. Generally, the setups are modeled as 0-1 binary variables, which makes the problem more complex. The problem involving the simple setup structure assumes that the sequence and the decision of the setup time and cost in the previous period do not depend on the current period in contrast to the complex setup structure.

With regard to the simple setup structure, [4] investigated the problem with product returns under substitutions and capacity restrictions, where batch manufacturing and batch remanufacturing, setup costs and emergency procurement are considered. First, they determined all periods required for production setup by a heuristic genetic algorithm (GA) and then used dynamic programming to determine the amount of remanufactured and manufactured products produced. The performance of this approach produced good and feasible solutions compared to the Branch-and-Bound algorithm implemented in the Lingo software.

In [11], a genetic algorithm heuristic approach was introduced to solve the problem in steel enterprise. They considered startup cost in the first period of a series of production periods, arbitrary and time-varying setup cost functions and limited production, remanufacturing and inventories. [12] collaborated with a paper product manufacturer to investigate the capacitated lot-sizing problem in a closed-loop supply chain with setup costs, product returns and remanufacturing. They solved the problem using the Lagrangian relaxation-based solution approach.

In [2], both simple setups elements were addressed namely setup cost and time in their model formulation as these setups are significant in the production environment. With the assumption that remanufactured and new parts have separate demands, they developed a solution procedure based on the Lagrangian decomposition. The procedure showed significant results in obtaining optimal solutions. [3] studied the production planning problems with back-orders and remanufacturing. They considered multiple factories; each has its own setup cost to produce new products or remanufactured products or both. They proposed three different models based on the scenarios and solved these models using self-adaptive genetic algorithm with population division (SAGA-PD).

In short, all previous studies solved the capacitated lot-sizing problem in a HMRS by using heuristic approaches. Their approaches can produce solutions quickly but are often low in quality. In this paper, our main aim is to investigate the capacitated lot-sizing problem with a simple setup structure of setup cost and time in a HMRS, motivated by the study of [2]. Unlike their study, an efficient exact method is introduced to tackle this problem. This approach will be able to produce high-quality solutions in a reasonable computation time.

2 Problem Formulation

In this section, the original formulation of HMRS problem of [2] is addressed. The problem is to find the most cost-efficient production plans that meet demands for manufactured and remanufactured parts on time as well as to minimize total costs which are production costs for new and remanufactured parts, setup costs for new and remanufactured, inventory costs for new and remanufactured parts and the acquisition, disassembly, setup and inventory costs of returned-used products. This problem is formulated as a Mixed Integer Linear Programming (MILP). This model assumes that remanufacturing and manufacturing have separate production setups and separate customer demands.

First, the decision variables are described. Let T, I, J be the number of periods, parts and returned-used products such that $NT = 1, ..., T$, $NI = 1, ..., I$ and $NJ = 1, ..., J$, respectively. Suppose $x_{i,t}$ and $\bar{x}_{i,t}$ are the amount of new part i and remanufactured part i produced in period t, respectively. The inventories of new part i, remanufactured part i and returned-used product j at the end of period t are indicated by $e_{i,t}$, $\bar{e}_{i,t}$ and $f_{j,t}$, respectively. The amount of returned-used products j to be disassembled and acquired in period t are denoted as $d_{j,t}$ and $r_{j,t}$, respectively. Next, let $y_{i,t}$, $\bar{y}_{i,t}$ and $\hat{y}_{j,t}$ indicate setup variables for new part i, remanufacturing part i and returned-used product j in period t, respectively.

In addition, the parameters $P_{i,t}$, $S_{i,t}$ and $V_{i,t}$ indicate the production, setup and inventory costs for new part i in period t, respectively. Likewise, $p_{i,t}$, $s_{i,t}$ and $v_{i,t}$ are the production, setup and inventory costs for remanufactured part i in period t. The parameter $D_{i,t}$ represents the demands for new part i in period t, where $DM^i_{t',l} = \sum_{t=t'}^{l} D_{i,t}$. Next, $\bar{D}_{i,t}$ indicates the demands for remanufactured part i in period t, where $DR^i_{t',l} = \sum_{t=t'}^{l} \bar{D}_{i,t}$. Then, $AQ_{j,t}$ and $RD_{j,t}$ denote the acquisition and disassembly costs of returned-used product j in period t, respectively. Setup cost for dismantling returned-used product j in period t is presented by $SD_{j,t}$ and $IN_{j,t}$ is the inventory cost of returned-used product j in period t. The parameters

AST_i and ST_i indicate the production and setup times for manufacturing new part i, respectively. Meanwhile, ASR_i and SR_i represent the production and setup times for remanufacturing used part i. In addition, the parameter $B_{i,j}$ indicates the amount of part i included in product j. The average rate for recovered part i from all returned-used products is presented by the parameter UR_i. The parameter $ACAP_t$ indicates the available production time in period t and lastly, M is a very large positive number. The formulation of the basic HMRS model follows:

$$\begin{aligned} \text{Z}c = \min \sum_{t=1}^{T}\sum_{i=1}^{I}(P_{i,t}x_{i,t} + S_{i,t}y_{i,t} + V_{i,t}e_{i,t} + p_{i,t}\bar{x}_{i,t} + s_{i,t}\bar{y}_{i,t} + v_{i,t}\bar{e}_{i,t}) \\ + \sum_{t=1}^{T}\sum_{j=1}^{J}(AQ_{j,t}r_{j,t} + SD_{j,t}\hat{y}_{j,t} + RD_{j,t}d_{j,t} + IN_{j,t}f_{j,t}) \quad (2.1) \end{aligned}$$

$$\begin{aligned}
\text{s.t.} \quad & e_{i,t-1} + x_{i,t} - e_{i,t} = D_{i,t} && \forall i \in NI, \forall t \in NT && (2.2)\\
& \bar{e}_{i,t-1} + \bar{x}_{i,t} - \bar{e}_{i,t} = \bar{D}_{i,t} && \forall i \in NI, \forall t \in NT && (2.3)\\
& f_{j,t} + d_{j,t} - f_{j,t-1} = r_{j,t} && \forall j \in NJ, \forall t \in NT && (2.4)\\
& x_{i,t} \le My_{i,t} && \forall i \in NI, \forall t \in NT && (2.5)\\
& \bar{x}_{i,t} \le M\bar{y}_{i,t} && \forall i \in NI, \forall t \in NT && (2.6)\\
& d_{j,t} \le M\hat{y}_{j,t} && \forall j \in NJ, \forall t \in NT && (2.7)\\
& \sum_{i=1}^{I}(AST_i x_{i,t} + ST_i y_{i,t} + ASR_i \bar{x}_{i,t} + SR_i \bar{y}_{i,t}) \le ACAP_t && \forall t \in NT && (2.8)\\
& \bar{x}_{i,t} \le UR_i \sum_{j=1}^{J} B_{i,j} d_{j,t} && \forall i \in NI, \forall t \in NT && (2.9)\\
& e_{i,0} = \bar{e}_{i,0} = f_{j,0} = 0 && \forall i \in NI, \forall j \in NJ && (2.10)\\
& x_{i,t}, \bar{x}_{i,t}, r_{j,t}, d_{j,t}, e_{i,t}, \bar{e}_{i,t}, f_{j,t} \ge 0 && \forall i \in NI, \forall j \in NJ, \forall t \in NT && (2.11)\\
& y_{i,t}, \bar{y}_{i,t}, \hat{y}_{j,t} = \{0,1\} && \forall i \in NI, \forall j \in NJ, \forall t \in NT && (2.12)
\end{aligned}$$

Constraints (2.2) - (2.4) are flow conservations (inventory balances) for new part, remanufactured part and returned-used product, respectively. Constraints (2.5) - (2.7) guarantee that the setup variables are forced to be 1 if manufacturing, remanufacturing and disassembly processes take place, respectively. Constraint (2.8) is the capacity constraints for both processes. Constraint (2.9) restricts the amount of parts recovered from the returned-used products. Without loss of generality, there are no initial inventories for new part, remanufactured part and returned-used product as stated in constraint (2.10). Constraint (2.11) represents nonnegativity restrictions of new part, remanufactured part, returned-used product, and returned-used product to be disassembled and inventory variables of new part, remanufactured part and returned-used product, respectively. Lastly, constraint (2.12) provides the integrality of manufacturing, remanufacturing and disassembly processes.

Since the manufacturing and remanufacturing operation depend on the amount of demands, the following variable upper bounds on $x_{i,t}$ and $\bar{x}_{i,t}$ are given as follows.

$$x_{i,t} \leq DM^{i}_{t,T} y_{i,t} \qquad \forall i \in NI, \forall t \in NT \qquad (2.13)$$

$$\bar{x}_{i,t} \leq DR^{i}_{t,T} \bar{y}_{i,t} \qquad \forall i \in NI, \forall t \in NT \qquad (2.14)$$

These new valid upper bounds (2.13) on $x_{i,t}$ and (2.14) on $\bar{x}_{i,t}$ indicate that the amount of new and remanufactured parts are restricted to the total amount of demands from period t to T. From this, the feasible region of this HMRS problem is defined as $X^c = \{(x, \bar{x}, y, \bar{y}, \hat{y}, e, \bar{e}, f, d, r) | (2.2) - (2.4), (2.7) - (2.14)\}$ and its objective function is $Z^c = \min \{(2.1) | (x, \bar{x}, y, \bar{y}, \hat{y}, e, \bar{e}, f, d, r) \in X^c\}$. In the next section, an exact method which is valid inequalities is discussed in detail.

3 Valid Inequalities

This section addresses three families of valid inequalities, so called (ℓ, S) inequalities for the capacitated lot-sizing problem with a simple setup structure in the HMRS setting. This approach was originally proposed by [1] for the classical lot-sizing problem. Their approach is adapted by adding some valid inequalities to the basic formulation in order to strengthen the formulation and to minimize the total branch-and-bound computation time.

Proposition 1. *The following families of valid inequalities are valid for X^c. For any $1 \leq i \leq I$ and $1 \leq u \leq \ell \leq T$, let $L = \{u, .., \ell\}$ and $S \subseteq L$, then*

$$\sum_{t \in S} x_{i,t} \leq \sum_{t \in S} DM^i_{t,\ell} y_{i,t} + e_{i,\ell} \tag{3.15}$$

$$\sum_{t \in S} \bar{x_{i,t}} \leq \sum_{t \in S} DR^i_{t,\ell} \bar{y}_{i,t} + \bar{e}_{i,\ell} \tag{3.16}$$

Proof. The proving for both simple valid inequalities are straightforward as discussed in [6]. These inequalities are similar to those in single-item uncapacitated lot-sizing problem. As mentioned by [6], the single-item uncapacitated lot-sizing problem give a complete linear inequality description of convex hull of solutions that can be solved in polynomial time. □

Proposition 2. *The following family of valid inequalities is also valid for X^c. Suppose that $1 \leq j \leq J$ and $1 \leq u \leq \ell \leq T$, let $L = \{u, .., \ell\}$ and $S \subseteq L$, then*

$$\sum_{t \in S} d_{j,t} \leq \sum_{i \in S} M \hat{y}_{j,t} + f_{j,u-1} \tag{3.17}$$

Proof. The proving for this type of valid inequalities is straightforward and is similar to [8]. The interested reader can refer to the cited paper. □

From this, the feasible region of (ℓ, S) formulation is $X^c_{ls} = \{(x, \bar{x}, y, \bar{y}, \hat{y}, e, \bar{e}, f, d, r)|(2.2) - (2.4), (2.7) - (3.17)\}$ and its objective function is $Z^c_{ls} = \min\{(2.1)|(x, \bar{x}, y, \bar{y}, \hat{y}, e, \bar{e}, f, d, r) \in X^c_{ls}\}$. This formulation consists of an exponential number of these inequalities as a result of adding these three families of valid inequalities into the original formulation. A separation algorithm is suggested in this paper so that only the significant violated inequalities can be added into the original formulation. The algorithm is presented in Algorithm 1. Next, in the following section, computational tests of all formulations are presented and analyzed.

4 Computational Tests

This section examines the strength or effectiveness of all the formulations. First, the test set-up of the computational experiments is elaborately discussed.

Algorithm 1: Separation algorithm for the capacitated lot-sizing problem in the HMRS

Require: Given LP relaxation solution $(x^*, \bar{x}^*, y^*, \bar{y}^*, \hat{y}^*, e^*, \bar{e}^*, f^*, d^*, r^*)$
Ensure: Generated violated inequalities

for all $i = 1$ to I, $\ell = NT$ **do**
    for all $u = 1$ to ℓ **do**
        for all $t = u$ to ℓ **do**
            Set $S \leftarrow S \cup \{\}$
            if $x^*_{i,t}$¿$DM^i_{k,t} y^*_{i,t}$ or $\bar{x}^*_{i,t}$¿$DR^i_{k,t} \bar{y}^*_{i,t}$ **then**
                $S \leftarrow S \cup \{i, t\}$
            end if
        end for
        if $\sum_{t \in S} x^*_{i,t} > \sum_{t \in S} DM^i_{t,\ell} y^*_{i,t} + e^*_{i,\ell}$ **then**
            Add first violated inequality
        end if
        if $\sum_{t \in S} \bar{x}^*_{i,t} > \sum_{t \in S} DR^i_{t,\ell} \bar{y}^*_{i,t} + \bar{e}^*_{i,\ell}$ **then**
            Add second violated inequality
        end if
    end for
end for
for all $j = 1$ to J, $\ell = NT$ **do**
    for all $u = 1$ to ℓ **do**
        for all $t = u$ to ℓ **do**
            Set $S \leftarrow S \cup \{\}$
            if $d^*_{j,t}$¿$M \hat{y}^*_{j,t}$ **then**
                $S \leftarrow S \cup \{j, t\}$
            end if
        end for
        if $\sum_{t \in S} d^*_{j,t} > \sum_{i \in S} M \hat{y}^*_{j,t} + f^*_{j,u-1}$ **then**
            Add third violated inequality
        end if
    end for
end for

4.1 Test set-up

The performance analysis of the original formulation and our valid inequalities are tested using various test instances. In order to provide diversified results, two types of data sets are considered in this paper.

Type 1 data sets considers the planning horizon of 6 periods. The rest of the details of the data sets can be referred to [2]. All cost and time parameters are time-variant except for setup and inventory costs for both manufacturing and remanufacturing and inventory cost for returned-used products.

For type 2 data sets, the planning horizon are 25 and 100 periods. About 240 data instances are generated randomly. Assume that the demand for manufacturing is normally distributed with mean, $\mu = 100$ and standard deviation, $\sigma = 50$. Demand for remanufacturing is also drawn from a normal distribution with three different scenarios which are low returns ($\mu = 10$, $\sigma = 5$), medium returns ($\mu = 50$, $\sigma = 25$) and high returns ($\mu = 90$, $\sigma = 45$). Each of the possible scenarios is simulated about 10 times, generating 30 different data sets. This study obtains the original data sets of demands for new and remanufactured parts from [7]. All cost and time parameters are assumed to be time-invariant and both demands are nonnegative. If the cost parameter is non-stationary, the problem is not difficult to solve as compared to the time-invariant case [7]. The remaining parameters are as follows. The setup costs for both processes that are equal which are 125, 250, 500 and 1000. There are no production costs for both processes. This assumption is valid as Teunter et al. [10] stated that the inclusion of variable production costs in the formulation may affect the balance between setups and inventory costs substantially. The disassembly cost for returned-used products is also set to zero. The inventory costs for both parts and returned-used products are 1. The setup cost for disassembling returned-used products is 50. The acquisition cost for returned-used products is 0. The production time for manufacturing and remanufacturing are 100 and 80, respectively. The setup times for manufacturing and remanufacturing are 50 and 30, respectively. The average rate of recovered part from returned-used products is 0.5. Lastly, the available production time for both manufacturing and remanufacturing is extremely large which is 1,000,000 so as to ensure that the production takes place.

In addition, the data about the amount of parts included in the returned-used products, stated in [2] are used in these computational tests. Both data sets consider the same number of products, J and the number of items, I which are 3 and 6, respectively.

4.2 Numerical tests

All formulations are run on a PC with an Intel®Core™ i5 1.60GHz processor and 8 GB of RAM. They are implemented using Xpress Mosel version 8.5 with default time of 10 minutes (600 seconds) for each test instance and without any solver cuts. The binary setup variables, y, $\bar{y}$ and $\hat{y}$ are the integer values of either 0 or 1. If the integrality constraint of these binary variables y, $\bar{y}$ and $\hat{y}$ are free to take any value from the interval $[0, 1]$, this situation is called as LP relaxation.

First, the performance analysis of the existing method and our method using the Type 1 data set of [2] are initially compared and analyzed in terms of Linear Programming (LP) relaxation gap (%). This LP gap (%) measures the quality of LP relaxation. According to the results presented in Table 1, in comparison to the heuristic method, our exact method, (ℓ, S) inequalities yield smallest LP gap (the best LP relaxation) which is less than 1%. The formulation has significantly closed the gap about 99%. Specifically, the cuts generated by the first two valid inequalities (3.15) and (3.16) are very effective in tightening the lower bounds. However, the third family of valid inequalities that is (3.17) is not considered in the discussion as it has been found to be insignificant in improving the lower bounds. One of the reasons is that this type of valid inequalities is unable to make any cuts due to 'big M' constraints which often produce bad LP relaxation.

Note that the best performance of the formulations is highlighted in boldface. In short, the exact method produces better results than the heuristic approach. Even though the heuristic method offers a quick solution yet may not result in the most optimal and ideal solution.

Table 1: A computational comparison of existing and proposed methods (Type 1 data set)

Results	Heuristic method [2]	Exact method - (ℓ, S) inequalities
LP relaxation	212087	**191937**
Best feasible solution	220101	**192708**
LP gap (%)	3.64%	**0.40%**

Furthermore, the effectiveness of original and proposed methods are examined using our simulated data sets, that is the Type 2 data set. Table 2 represents the performance analysis of the original formulation and our proposed method for a short period of 25 and a large period of 100. This table indicates the average percentage of LP gaps, the average MIP solution times. The solution time is counted as 600 seconds if an instance could not be solved to optimality within the given time limit. Lastly, these tables also report the number of times the LP relaxation of a formulation found the integer optimal solution.

In general, (ℓ, S) inequalities formulation has provided the best LP relaxation, in the sense that they have smallest average of LP gaps (%) than the original formulation, in each ten replications at both periods of 25 and 100. When there are 25 periods, the findings show that the original formulation of the capacitated lot-sizing problem in HMRS is fully strengthened with all violated (ℓ, S) inequalities generated at the root node of the Branch-and-Bound tree. The integer optimal solutions for all test instances are found in very short computational times.

Meanwhile, the test instances with a large horizon (100) are generally able to quickly find the optimal solutions. Unlike the single-item uncapacitated case, this capacitated lot-sizing problem in a HMRS does not provide a complete linear description of convex hull of solutions as some test instances are unable to find the integer solutions especially when the horizon is long (100 periods) and the setup costs for both manufacturing and remanufacturing are large. It can however be concluded that these valid inequalities have improved the original formulation significantly in terms of LP gaps, solution times, the frequency of integer optimal solution and when the cost parameters are time-invariant.

Table 2: Performance analysis of original and (ℓ, S) formulation (T = 25 and 100) (Type 2 data set)

Setup costs		T =25						T =100					
		Low		Medium		High		Low		Medium		High	
		Original	(ℓ, S)	Original	(ℓ, S)	Original	(ℓ, S)	Original	(ℓ, S)	Original	(ℓ, S)	Original	(ℓ, S)
125	Avg. LP gap (%)	79.26	**0**	78.68	**0**	78.39	**0**	93.19	**0**	93.00	**0**	93.64	**0**
	Avg. solution times MIP (s)	600	**0**	600	**0**	600	**0**	600	**0.21**	600	**0.23**	600	**0.20**
	No. of integer solutions LP	0	**10**	0	**10**	0	**10**	0	**10**	0	**10**	0	**10**
250	Avg. LP gap (%)	75.91	**0**	75.53	**0**	75.16	**0**	92.36	**0**	92.35	**0**	92.21	**0**
	Avg. solution times MIP (s)	600	**0**	600	**0**	600	**0**	600	**0.20**	600	**0.22**	600	**0.21**
	No. of integer solutions LP	0	**10**	0	**10**	0	**10**	0	**10**	0	**10**	0	**10**
500	Avg. LP gap (%)	69.91	**0**	70.51	**0**	70.57	**0**	90.98	**0**	90.84	**0**	90.99	**0**
	Avg. solution times MIP (s)	600	**0**	600	**0**	600	**0**	600	**0.20**	600	**0.22**	600	**0.20**
	No. of integer solutions LP	0	**10**	0	**10**	0	**10**	0	**10**	0	**10**	0	**10**
1000	Avg. LP gap (%)	61.92	**0**	63.19	**0**	63.87	**0**	88.85	**0**	88.95	**0.007**	89.09	**0**
	Avg. solution times MIP (s)	600	**0**	600	**0**	600	**0**	600	**0.28**	600	**0.52**	600	**0.29**
	No. of integer solutions LP	0	**10**	0	**10**	0	**10**	0	**10**	0	**8**	0	**10**

5 Conclusion

This paper studies the capacitated lot-sizing problem with simple setup structure in a hybrid manufacturing and remanufacturing system (HMRS), which is NP-hard. Three well-known families of valid inequalities are introduced to improve the basic HMRS formulation. The performance of these inequalities are tested using existing and new simulated data sets. The findings show that these valid inequalities strengthen the lower bounds significantly as compared to the existing method as well as outperform the original formulation in the case where time-invariant cost parameters are considered. As for future research, it would be interesting to study this problem if the variable r, that is the amount of returned product acquired is deterministic and stochastic. Another extension involves joint demands for new and remanufactured parts, where both parts can serve the same market demands.

Acknowledgment. We would like to thank the Ministry of Higher Education Malaysia for financially supporting this study under the Fundamental Research Grants Scheme (FRGS/1/2017/STG06/UPNM/03/2) as well as the authors, Retel Helmrich and Chen for providing the data sets.

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