International Journal of Mathematics and
Computer Science, **16**(2021), no. 3, 963–976

Optimal Performance Evaluation Metrics For Satisfiability Logic Representation In Discrete Hopfield Neural Network

Mohd. Asyraf Mansor[1], Saratha Sathasivam[2]

[1]School of Distance Education,
Universiti Sains Malaysia,
11800 USM, Penang, Malaysia

[2]School of Mathematical Sciences,
Universiti Sains Malaysia,
11800 USM, Penang, Malaysia

email: asyrafman@usm.my

(Received February 8, 2021, Revised March 5, 2021,
Accepted May 14, 2021)

Abstract

The performance measures and the quality assessment of the solutions for Satisfiability logic in Discrete Hopfield Neural Network (DHNN) are significantly dependent on the selection of the optimal performance evaluation metrics. The current performance measures were mostly leveraging the computational time, absolute error, mean squared error, and goodness of fit measures. To assess the learning capability of a neural network model, the optimal performance metrics are adopted in measuring the quality of the solutions and interpretations obtained by the network especially when dealing with the different number of clauses of Satisfiability logic. The core impetus of this study is to investigate the effects of various performance evaluations metrics towards the models performance analysis based on the learning error, similarity analysis, and energy analysis. Overall, the simulation results have revealed the significant impact of various performance

Key words and phrases: Performance Evaluation Metrics, Satisfiability Logic, Discrete Hopfield Neural Network, Similarity Index.
AMS (MOS) Subject Classifications: 68T27, 68T07.
ISSN 1814-0432, 2021, http://ijmcs.future-in-tech.net

evaluation metrics in terms of learning error, energy evaluation, and similarity analysis for k Satisfiability logic in Discrete Hopfield Neural Network, when $k = 3$ with different complexities. This finding will reveal the ideal performance metrics that comply with Satisfiability logic and neural network model evaluation.

1 Introduction

The study on performance evaluation measures in computational mathematics has rapidly broadened due to the performance assessment of the artificial neural network in solving various complex problems or data sets. The importance of learning error measures can be acknowledged by various applications such as in assessing the accuracy of neural network approach for employment analysis as shown by Zamri et al. [1]. The error evaluation in neural network learning process refers the degree of correctness of the fitness as compared to the expected fitness obtained after the convergence. The Mean Absolute Error (MAE) has been adopted in numerous works such as in [1] and [2], to quantify the accuracy of the solutions. Additionally, the Sum of Squared Error (SSE) has emerged to be employed as a performance measure as used in [3] due to the capability in the sensitivity analysis. The forecast metrics such as Mean Absolute Percentage Error (MAPE) [4] and Symmetric Mean Absolute Percentage Error (SMAPE) [5] are reliable metrics in predicting the accuracy of the model. The investigation on the aforementioned metrics towards the neural network learning phase is required.

The similarity analysis depicts the closeness of the solution toward the benchmark solutions. The well-known similarity analysis metrics such as the Jaccard index [6], Sokal Sneath 2 index [7], Dice index [9] and Kulczynski index as proposed in [8] are being adopted in comparing the quality of the solutions by any computational model. The notable work of Singh and Kumar [9] has proved the ability of Dice similarity measure in assessing the pattern and medical screening data. Rodionov and Sozontov [10] further applied the Kulczynski similarity metric in forecasting the correctness between the quantitative similarity coefficient exhibited by a network. However, there is still limited work in bridging the analysis of various similarities with the quality assessment of the final states obtained by logic programming in ANN.

Energy analysis will determine the convergence of the logic programming in ANN. The energy analysis such as Global Minima ratio has been applied in logic programming in Hopfield Neural Network [11, 12]. The performance evaluation metrics are being implemented to improve the current performance indicator for the 3 Satisfiability logic analysis [13] in Discrete Hopfield Neural Network. Discrete Hopfield Neural Network is a variant of recurrent artificial neural network and a powerful computational model proposed by [14] and applied in the pattern analysis [15] and logic programming [11, 12].

The main contributions of this paper are: (i) To propose the optimal performance evaluation metrics, particularly in assessing the effectiveness of the learning for 3 Satisfiability logic in Discrete Hopfield Neural Network (DHNN-3SAT). (ii) To measure the compatibility of the model during retrieval phase via the energy analysis metric. (iii) To analyze the quality of the final states generated by DHNN-3SAT via similarity measures. In addition, this work will reveal the profound insight on the metrics.

This article is organized as follows. Section 2 discusses the theory of 3SAT in Discrete Hopfield Neural Network. Section 3 and Section 4 explain the performance metric and the experimental setup. Section 5 and Section 6 encompass the results, discussion, and concluding remarks.

2 Satisfiability Logic Representation in Discrete Hopfield Neural Network

The 3 Satisfiability logic in Discrete Hopfield Neural Network (DHNN-3SAT) is formulated and assessed by using the optimal performance metrics. The 3 Satisfiability (3SAT) logical representation is a discrete logic representation, containing strictly 3 literals per clause [13] as shown in Eq. (2.1).

$$\omega_{3SAT} = (K \vee L \vee M) \wedge (\neg E \vee F \vee \neg G) \wedge (P \vee Q \vee \neg R), \tag{2.1}$$

From the aforementioned ω_{3SAT}, the cost function is formulated as follows:

$$E_{\omega_{3SAT}} = \frac{1}{2^3} \sum_{i=1}^{NC} (\prod_{j=1}^{3} C_i)_j, \tag{2.2}$$

given

$$C_i = \begin{cases} (1 - S_C) & , if \neg C_i \\ (1 + S_C) & , otherwise \end{cases} \tag{2.3}$$

where NC refers to the number of clauses, C_i denotes the clause in the ω_{3SAT} and S_C is the state corresponds to the literal in the clause. The learning process will be completed if achieve the satisfied states leads to $E_{\omega_{3SAT}} = 0$.

Then, the fitness of ω_{3SAT} can be further computed as in Eq. (2.4), whereby the maximum fitness, f_{max} correspond to the maximum values of f_i.

$$f_i = \sum_{i=1}^{n} C_i^{(3)}, \tag{2.4}$$

where

$$C_i^{(3)} = \begin{cases} 1 & , if\ E_{\omega_{3SAT}} = 0 \\ 0 & , otherwise \end{cases} \tag{2.5}$$

The final neuron updating process can be obtained via:

$$h_i = \sum_{k=1, i \neq j \neq k}^{N} W_{ijk}^{(3)} S_j S_k + \sum_{j=1, i \neq j \neq k}^{N} W_{ij}^{(2)} S_j + W_i^{(1)} \quad , \tag{2.6}$$

where $W_i^{(1)}$, $W_{ij}^{(2)}$ and $W_{ijk}^{(3)}$ are the synaptic weight corresponded to neurons N and S_j refers to the neuron state corresponds to literal j. These local field will establish the adaptability of the final states attained by DHNN after being squashed via Hyperbolic tangent activation function as applied in [15]. The final energy function of ω_{3SAT} is formulated as in [13].

The synaptic weights of the neuron are calculated via Wan Abdullah method [16]. Note that the final energy function in Energy equation serves as the filtering benchmark, whether the final states will converge towards local minimum energy or global minimum energy [17].

3 Performance Evaluation Metrics

The model evaluation will utilize the learning error measures, energy analysis, and similarity metrics of the retrieved neurons in DHNN-3SAT.

3.1 Performance Metric for Learning Phase

The learning process of the DHNN-3SAT is an intensive phase that contributes to the effectiveness of the final states obtained during retrieval phase.

3.1.1 Mean Absolute Error (MAE)

Mean Absolute Error (MAE) is the standard error based on the average difference from the computed fitness values and expected fitness of the model.

According to [1], MAE is apparently a reliable metric in assessing the accuracy of the learning phase of the DHNN model. Thus, MAE for DHNN-3SAT learning phase is recrafted as follows:

$$MAE = \sum_{i=1}^{n} \frac{1}{n} |f_{max} - f_i| \tag{3.7}$$

where f_i refers to the obtained fitness values, n denotes the number of iterations and f_{max} is the maximum number of fitness corresponds to the number of clauses of 3SAT logic.

3.1.2 Sum of Squared Error (SSE)

Sum of Squared Error (SSE) is a statistical metric adopted to measure the dispersion of the data from the expected values [3]. The SSE can be formulated as:

$$SSE = \sum_{i=1}^{n} (f_i - f_{max})^2 \tag{3.8}$$

where f_i refers to the obtained fitness values and f_{max} is the maximum number of fitness corresponds to the number of clauses of 3SAT logic.

3.1.3 Mean Absolute Percentage Error (MAPE)

Mean Absolute Percentage Error (MAPE) is an extended version of MAE, where the values are normalized into percentage [4]. The formulation of MAPE in DHNN is given as:

$$MAPE = \sum_{i=1}^{n} \frac{100}{n} \frac{|(f_{max} - f_i)|}{|f_{max}|} \tag{3.9}$$

where f_i denotes the fitness value during learning phase and f_{max} is the expected maximum fitness value. Theoretically, the MAPE values are ranging $MAPE = [0, 100]$, where the optimal value is zero.

3.1.4 Symmetric Mean Absolute Percentage Error (SMAPE)

SMAPE is the normalized version of MAPE with more valuable symmetry and less biasness [5]. The learning SMAPE for DHNN-3SAT is given as:

$$SMAPE = \sum_{i=1}^{n} \frac{2}{n} \frac{|(f_{max} - f_i)|}{|(f_{max} + f_i)|} \tag{3.10}$$

where n refers to the number of iterations, f_i is the fitness values and f_{max} refers to the maximum number of fitness obtained by the model. The range of this metric is $SMAPE = [0, 2]$.

3.2 Performance Metric For Retrieval Phase

To effectively analyze the energy analysis, we adopted global minima and local minima ratio for simplicity as being utilized in the work of [1]. The Global Minima Ratio [11] can be calculated as follows:

$$\Phi_{\omega_{3SAT}} = \frac{1}{pq}\sum_{i=1}^{NN} N_{H_{\omega 3SAT}} \tag{3.11}$$

In addition, the Local Minima Ratio equation is give as follows:

$$\xi_{\omega_{3SAT}} = 1 - (\frac{1}{pq}\sum_{i=1}^{NN} N_{H_{\omega 3SAT}}) \tag{3.12}$$

where NN denotes the number of neurons, p refers to the number of trials and q depicts the neuron combinations. Based on [17], the energy analysis is an indicator of the quality of the solutions obtained by the network. The range value, $\Phi_{\omega_{3SAT}} = [0, 1]$, where the optimal value is approaching to one.

3.3 Similarity Analysis

The similarity metrics is an indicator of DHNN capability in generating the optimal solutions. By taking inspiration from work of [18], the similarity metrics will be further employed to assess the final states retrieved by the DHNN. The comparison will be done by taking the benchmark states S_i^{max} with the states attained by the network S_i. The formula of the general comparison of the benchmark state and the final state is given as follows:

$$C_{S_i^{max}, S_i} = \{(S_i^{max}, S_i) | i = 1, 2, 3, ..., n\}. \tag{3.13}$$

The standard specification variables can be defined as:
a refers the total number of occurrences for $(S_i^{max} = 1, S_i = 1)$ in $C_{S_i^{max}, S_i}$
b denotes the total number of occurrences for $(S_i^{max} = 1, S_i = -1)$ in $C_{S_i^{max}, S_i}$
c is the total number of occurrences for $(S_i^{max} = -1, S_i = 1)$ in $C_{S_i^{max}, S_i}$

3.3.1 Jaccard Index

Jaccard similarity index is ratio of the similarity between two distinct data and widely used in global evaluation as discussed in [6]. In this work, we extend the Jaccard index proposed in [18], in gauging the quality of the final states retrieved by DHNN, S_i as compared to the benchmark neuron states, S_i^{max}. The Jaccard index for DHNN is recrafted as follows:

$$J(S_i^{max}, S_i) = \frac{a}{a+b+c}. \tag{3.14}$$

3.3.2 Sokal Sneath 2 Index

Sokal Sneath 2 similarity index has been applied in [7] in finding the similarity in hybrid clustering method. In this research, the Sokal-Sneath 2 equation is formulated based on [18] to measure the existence of the non-repeating final states retreived by DHNN. The Sokal Sneath 2 index for DHNN is as follows:

$$SS(S_i^{max}, S_i) = \frac{a}{a+2(b+c)}. \tag{3.15}$$

3.3.3 Dice Index

Dice similarity measure involves the inner product, with more weight given to the positive data in order to enhance the possible limitations in the existing similarity index [9]. In this work, the positive neuron states retrieved by DHNN will be the main emphasis and the equation is shown as follows:

$$DI(S_i^{max}, S_i) = \frac{2a}{2a+b+c}. \tag{3.16}$$

3.3.4 Kuszyuski Index

Kuszyuski Index is measured based on the geometric and arithmetic mean component of the two binary states [8, 10]. The formula of Kuszyuski Index:

$$KZ(S_i^{max}, S_i) = \frac{1}{2}(\frac{a}{a+b} + \frac{a}{a+c}). \tag{3.17}$$

4 Experimental Setup

The DHNN-3SAT model is implemented in Dev C++ Version 5.11 via Windows 10 Intel Core i5 with 2.2 GHz processor. The simulations are conducted with different number of neurons (NN) ranging from $15 \leq NN \leq 120$. The parameters involved in DHNN-3SAT is set according to Table 1.

Table 1: Experimental Setup and Parameter

Parameter	Definition (Reference)
Neuron Combinations (q)	100 [12]
Tolerance Value (Tol)	0.001 [11]
Number of Trials (p)	100 [15]
CPU Time Threshold	24 Hours [1]

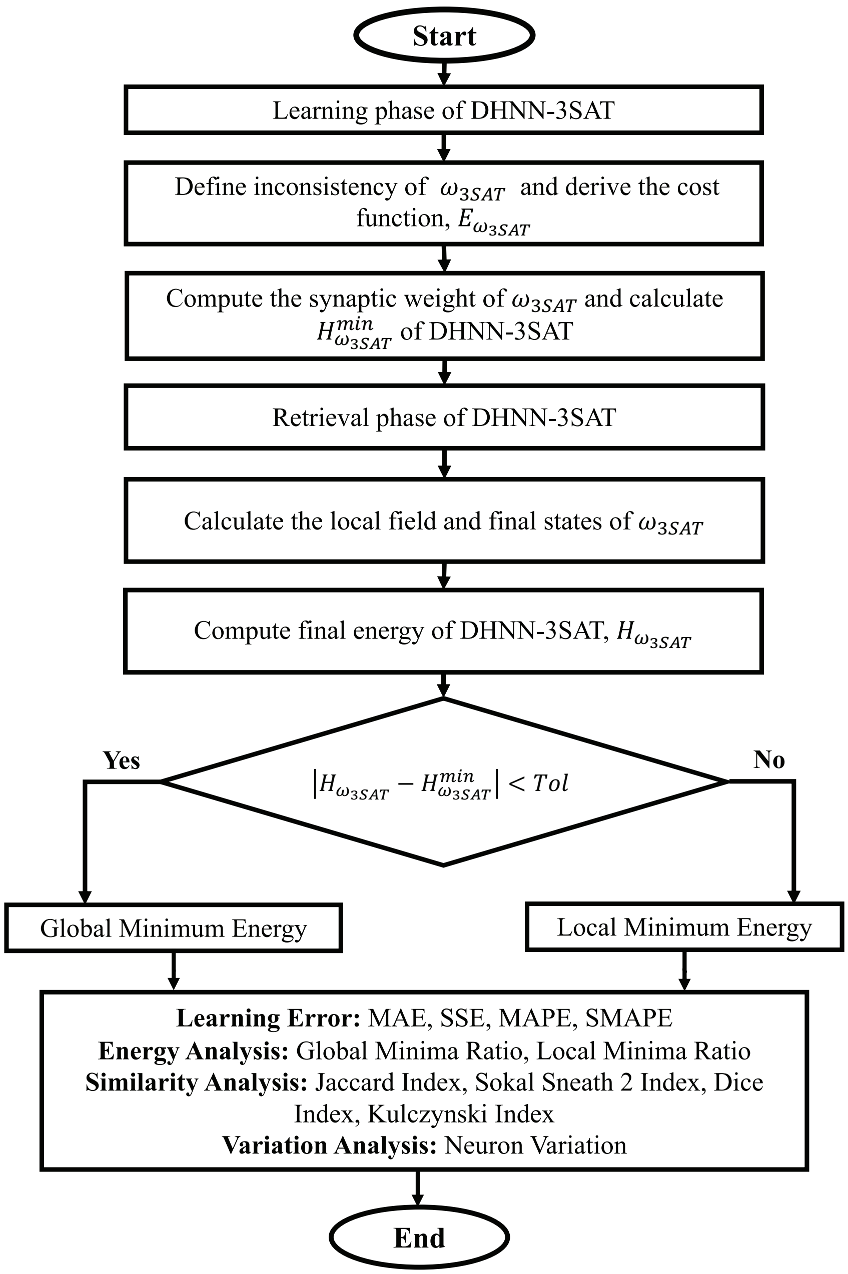

Figure 1: Algorithm for implementation of DHNN-3SAT.

According to Figure 1, the DHNN-3SAT can be divided into learning and retrieval phase, with the aim of attaining the global final states of ω_{3SAT}. Based on Figure 1, the learning error, energy metric, similarity index and variation will be recorded after each of simulation for further analysis.

5 Results and Discussion

Table 2: Performance Metrics For Learning Phase of DHNN-3SAT model

NN	MAE	SSE	MAPE	SMAPE
15	1.151309	27.750000	23.026190	0.460524
30	4.232100	318.000000	42.321007	0.846420
45	7.786492	1012.500000	51.909946	1.038198
60	14.761854	6348.000000	73.809250	1.476185
75	20.883018	16268.750000	83.532074	1.670641
90	26.381559	38529.000000	87.938515	1.758770
105	32.482967	76513.500000	92.808380	1.856168
120	38.879711	135008.000000	97.199219	1.943985

Table 3: Performance Metric for Retrieval Phase of DHNN-3SAT model

NN	Global Minima Ratio	Local Minima Ratio
15	1.0000	0.0000
30	1.0000	0.0000
45	1.0000	0.0000
60	0.9900	0.0100
75	0.9800	0.0200
90	0.9000	0.1000
105	0.5800	0.4200
120	0.4001	0.5999

Table 2 manifests the training errors for DHNN-3SAT model under different number of neurons. Generally, the trend for MAE, SSE, MAPE and SMAPE are apparently increasing with the increase in the complexity. MAE values reflect the correctness of the fitness as compared with maximum fitness obtained by DHNN-3SAT during learning phase. Based on Table 2, the SSE is increasing exponentially as the number of neuron increase. SSE shows the most significant differences as it demonstrates the sensitivity of

Table 4: Similarity Analysis of DHNN-3SAT model

NN	Jaccard	Sokal Sneath 2	Dice	Kuszyuski
15	0.562566	0.391368	0.720054	0.764695
30	0.550633	0.379913	0.710204	0.756218
45	0.543700	0.373343	0.704412	0.751294
60	0.545778	0.375305	0.706153	0.752477
75	0.547723	0.377148	0.707779	0.754336
90	0.550858	0.380127	0.710391	0.756275
105	0.546368	0.375864	0.706647	0.753770
120	0.555819	0.384868	0.714503	0.760258

our model towards errors and iterations [1]. In addition, the values of MAE and SMAPE were slightly higher at $NN = 120$, indicating less forecasting capability of attaining the maximum fitness when the complexity is getting higher. The role of each learning error can be vary based on the analysis. From this experiment, MAE can be utilized as a reliable metric to describe accuracy of the learning phase and SSE can be adopted to investigate the sensitivity of the DHHNN-3SAT model.

Based on the energy analysis during retrieval phase as shown in Table 3, we can conclude that the global minima ratio is recorded as the highest during $15 \leq NN \leq 45$. The local minima ratio can be observed clearly within the range of $60 \leq NN \leq 120$ because the final neuron states are facing minor neuron oscillation due to the higher complexity. The solutions are trapped at sub-optimal states, causing the changes of energy do not comply with our selected threshold value [13]. The energy analysis will be a good compatibility measures of the final state obtained by the network during retrieval phase. Additionally, the presence of global minimum solution (optimal states) and local minimum solution (sub-optimal states) indicating the dynamics of our model. Therefore, the best model will record global minima ratio equal to 1 due to the effectiveness in synaptic weight management.

Table 4 shows the similarity analysis of the final neuron states retrieved by DHNN-3SAT as compared to the benchmark state. Generally, a higher similarity index indicates the major deviation and bias in the final states generated when compared to the benchmark states of the model [18]. Based on Table 4, the Sokal Snealth 2 and Jaccard Index are significantly lower than the Dice and Kuszyuski index, especially during $45 \leq NN \leq 75$. Hence, a higher similarity coefficient shows the model achieving the overfitting as the DHNN-3SAT model failed to generate variety in the final states of the neuron. The lower value similarity index was typically supported by a higher number of neuron variations. From the experiment, the trend of learning error correlates well with the energy analysis. The similarity analysis index is obviously independent towards the learning error and energy analysis. The simulations have proved that higher global energy analysis does not guarantee lower similarity index. Thus, the various performance measures will guarantee the model being assessed from a different perspective. Note that the failure in adopting the proposed performance evaluations will allow the practitioner to lose important insight into the actual performance of the DHNN model. Thus, the holistic evaluation of the compatibility and the behaviour of ω_{3SAT} in DHNN will be less accurate. The performance evaluation in the learning phase is essential in assessing the fitness of the solution obtained by the DHNN-3SAT model, before undergoing the retrieval phase. Henceforth, the performance evaluation metrics for the retrieval phase such as energy analysis and similarity analysis will measure the quality of the final states that lead to global minimum energy.

6 Conclusion

The optimal performance evaluation measures for the DHNN-3SAT model have been successfully proposed for various domains of model assessment under different complexities such as learning error, similarity analysis, and energy evaluation. Overall, MAE is seen to be a reliable metric to measure the accuracy and effectiveness of the learning phase of the network. It was found that the learning SSE can be utilized in sensitivity analysis, whereas MAPE and SMAPE for the forecast of the model. Based on the energy analysis, the global minima ratio and local minima ratio was successfully employed to assess the quality of the solutions produced by the model. Based on the similarity analysis, the results vary for different types of similarity index. We can conclude that lower similarity index has generated the solution with higher variability. This work has provided profound insight into the role of different evaluation measures in model evaluation.

Acknowledgment. This research is supported by Short Term Grant by Universiti Sains Malaysia (304/PJJAUH/6315263).

References

[1] Nur Ezlin Zamri, Mohd Asyraf Mansor, Mohd Shareduwan Mohd Kasihmuddin, Alyaa Alway, Siti Zulaikha Mohd Jamaludin, Alyaa Alway, Shehab Abdulhabib Alzaeemi, Amazon Employees Resources Access Data Extraction via Clonal Selection Algorithm and Logic Mining Approach, Entropy, **22,** no. 6, (2020), 596.

[2] Bilal Khan, Rashid Naseem, Fazal Muhammad, Ghulam Abbas, Sunghwan Kim, An Empirical Evaluation of Machine Learning Techniques for Chronic Kidney Disease Prophecy, IEEE Access, **8,** (2020), 55012–55022.

[3] Muhammad Bilal, Shahid Masud, Shahrukh Athar, FPGA design for statistics-inspired approximate sum-of-squared-error computation in multimedia applications, IEEE Transactions on Circuits and Systems II: Express Briefs, **59,** no. 8, (2012), 506–510.

[4] Arnaud De Myttenaere, Boris Golden, Bénédicte Le Grand, Fabrice Rossi, Mean absolute percentage error for regression models, Neurocomputing, **192,** (2016), 38–48.

[5] Baraka Jacob Maiseli, Optimum design of chamfer masks using symmetric mean absolute percentage error, EURASIP Journal on Image and Video Processing, no. 74, (2019), 1–15.

[6] Sujoy Bag, Sri Krishna Kumar, Manoj Kumar Tiwari, An efficient recommendation generation using relevant Jaccard similarity, Information Sciences, **483,** (2019), 53–64.

[7] Murugaiyan Pachayappan, Ramasamy Panneerselvam, An efficient recommendation generation using relevant Jaccard similarity, Materials Today, **5,** no. 5, (2018), 12285–12302.

[8] Marie Jeanne Lesot, Maria Rifqi, Hamid Benhadda, Similarity measures for binary and numerical data: a survey, International Journal of Knowledge Engineering and Soft Data Paradigms, **1,** no. 1, (2009), 63–84.

[9] Akanksha Singh, Sanjay Kumar, A novel dice similarity measure for IFSs and its applications in pattern and face recognition, Expert Systems with Applications, **149,** (2020), 113245.

[10] Igor Vladimirovich Rodionov, Artem Nikolaevich Sozontov, Similarity measures for binary and numerical data: a survey, Automation and Remote Control, **81,** no. 2, (2020), 320–332.

[11] Saratha Sathasivam, Upgrading logic programming in Hopfield network, Sains Malaysiana, **39,** no. 1, (2010), 115–118.

[12] Mohd Shareduwan Mohd Kasihmuddin, Mohd Asyraf Mansor, Saratha Sathasivam, Hybrid Genetic Algorithm in the Hopfield Network for Logic Satisfiability Problem, Pertanika Journal of Science & Technology, **25,** no. 1, (2017), 139–152.

[13] Mohd Asyraf Mansor, Saratha Sathasivam, Accelerating activation function for 3-satisfiability logic programming, International Journal of Intelligent Systems and Applications, **8,** no. 10, (2016), 44–50.

[14] John J. Hopfield, David W. Tank, Neural computation of decisions in optimization problems, Biological Cybernetics, **52,** no. 3, (1985), 141–152.

[15] Mohd Asyraf Mansor, Mohd Shareduwan Mohd Kasihmuddin, Saratha Sathasivam, Enhanced Hopfield network for pattern satisfiability optimization, International Journal of Intelligent Systems and Applications, **8,** no. 11, (2016), 27–33.

[16] Wan Ahmad Tajuddin Wan Abdullah, Logic programming on a neural network,International journal of intelligent systems, **7,** no. 6, (1992), 513–519.

[17] Mohd Shareduwan Mohd Kasihmuddin, Mohd Asyraf Mansor, Saratha Sathasivam, Discrete Hopfield neural network in restricted maximum $k-$satisfiability logic programming, Sains Malaysiana, **47,** (2018), 1327-1335.

[18] Mohd Shareduwan Mohd Kasihmuddin, Mohd Asyraf Mansor, Shehab Abdulhabib Alzaeemi, Md Faisal Md Basir, Saratha Sathasivam, Quality Solution of Logic Programming in Hopfield Neural Network, Journal of Physics: Conference Series, **1366,** (2019), 012094.