International Journal of Mathematics and
Computer Science, **16**(2021), no. 3, 871–884

Dynamics of Bank's Balance Sheet: A System of Deterministic and Stochastic Differential Equations Approach

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(Received February 1, 2021, Revised March 28, 2021,
Accepted May 9, 2021)

Abstract

We study a system of deterministic and stochastic differential equations of a bank's balance sheet variables: deposits, loans, equity and liquid assets. The deposits and loans model are assumed to follow the logistic harvesting model. The growth rate of equity is proportional to the bank's profit or loss, and it can be reduced by the non-performing loans factor. The liquid assets act as the balancing variables of the balance sheet. The stochastic model is presented with randomness that appeared in the deposit withdrawals and the non-performing loans parameters. The parameters in the deterministic model are estimated using spiral optimization algorithm employing the Indonesian banking data. The estimation results are quite well as shown by the errors between the data and the estimated model are really small. Using

Key words and phrases: Banking dynamics, balance sheet, logistic harvesting model, stochastic differential equations.
AMS (MOS) Subject Classifications: 37N40, 34F05.
ISSN 1814-0432, 2021, http://ijmcs.future-in-tech.net

Euler-Maruyama method, the simulation of the stochastic model is also given.

1 Introduction

There have been considerable studies on the banking model. Some of those that attract attention are mathematical models build upon the composition of bank's balance sheet. Based on the approach used, these models can be divided into two groups. First, the banking model with an industrial organization approach. Bank is seen as a company having main goal to maximize its profit given several constraints. The optimization problem is usually solved using the Lagrange multiplier method. From the optimal solution, some interpretations are drawn for scrutinize bank's portfolio strategy or evaluate banking regulations. This model was first introduced by Klein [1] and Monti [2]. Therefore, this model known as the Monti-Klein model.

Because of its simplicity and applicability, the Monti-Klein model developed quite rapidly. Some investigations on the structure of the model abstraction can be found in [3] and [4]. The developments of the model with the addition of various assumptions, variables, and constraints, which to observe their impact on bank's portfolio decision making, are presented in [5] and [6]. The Monti-Klein model, which was previously a monopoly model, then developed into duopoly and oligopoly model for covering the competition between banks in banking system. Some notable works are [7], [6] and [8]. The implementation of the model for survey the effect of some existing banking policies are appeared in [9], [10] and [8].

The second approach in the banking model is to employ a dynamic model. The observation in this model is focused on how the amount of bank's balance sheet variables change over time. The time can be continuous or discrete. The banking dynamic model with continuous time observes changes in variables aggregately, where each factors that cause dynamics is assumed to occur uniformly at every small time intervals. Several literatures that examine this topic are [11] and [12]. Opposite the continuous model, the banking dynamic model with discrete time treats the changes that arise in the bank's balance sheet differently in each period of time. Sometimes, this model is combined with the banking industrial model. For examples, see [13], [14] and [15].

This paper studies on the continuous banking dynamic model of Sumarti et al. (2018) [11] given below:

$$\begin{cases} \frac{\mathrm{d}D(t)}{\mathrm{d}t} = a_D D(t)\Big(1 - \frac{D(t)}{K_D}\Big) - wD(t) + \epsilon_D D(t)A(t) + \xi_D D(t)E(t), \\ \frac{\mathrm{d}L(t)}{\mathrm{d}t} = a_L L(t)\Big(1 - \frac{L(t)}{K_L}\Big) - r(1-\eta)L(t) + \epsilon_L L(t)A(t) + \xi_L L(t)E(t), \\ \frac{\mathrm{d}E(t)}{\mathrm{d}t} = (1-s)(r_B \kappa_2 D(t) + r_{I_P} I_P(t) + r_L(1-\eta)L(t) - r_D D(t) - r_{I_L} I_L(t)), \end{cases} \tag{1.1}$$

where $A(t) = (D(t) + I_L(t) + E(t)) - (L(t) + \kappa_1 D(t) + \kappa_2 D(t) + I_P(t))$. The bank's balance sheet variables $D(t)$, $L(t)$, $E(t)$, $A(t)$, $I_P(t)$ and $I_L(t)$ stand for deposits, loans, equity, liquidity tool, interbank placement and interbank loan at time t, respectively.

The straighforward utilization of the model is that we can easily apply it to the banking data. From model (1.1), we made some simplifications and also added other factors that were not listed in the model (this will be explained in section 2). We also examined the stochastic form of the model where the randomness appeared in deposits withdrawal and non-performing loans parameters. The deterministic model is applied to one of the Indonesian commercial conventional bank group data. The estimation of the model's parameters uses the spiral optimization (SPO) algorithm.

2 Model

Suppose the funding side of a bank's balance sheet consist of deposits and equity, and financing side consists of liquid assets and loans. Liquid assets contain cash, minimum reserves requirements, securities and other assets that are easy to convert into cash or have short maturities.

Let $D(t)$, $E(t)$, $L(t)$ and $A(t)$ denote the amount of deposits, equity, loans and liquid assets at time t, respectively. Our modified model of (1.1) is written below:

$$\begin{cases} \frac{\mathrm{d}D(t)}{\mathrm{d}t} = a_D D(t)\Big(1 - \frac{D(t)}{K_D}\Big) - wD(t), \\ \frac{\mathrm{d}L(t)}{\mathrm{d}t} = a_L L(t)\Big(1 - \frac{L(t)}{K_L}\Big) - nL(t) - r(1-\eta)L(t), \\ \frac{\mathrm{d}E(t)}{\mathrm{d}t} = a_P(1-d)(1-\tau)P(t) - nL(t), \\ A(t) = D(t) + E(t) - L(t), \end{cases} \tag{2.2}$$

with initial values that satisfy $D_0 + E_0 = L_0 + A_0$, and $P(t)$ is the profit or loss at time t given by

$$P(t) = (1-\eta)L(t)r_L(t) - (1-\omega)D(t)r_D(t) - C(t). \tag{2.3}$$

The explanation of parameters and variables in system (2.2) and Eq. (2.3) are presented in Table 1. Both of the differential equations of the deposits and loans are assumed to follow the logistic harvesting model. In deposits growth, there is harvesting in term of deposit withdrawals. And in loans growth, there are two kinds of harvesting: non-performing loans and loan repayments. The growth rate of equity is proportional to the net profit or loss. The bank's profit is calculated as the income from interest on loans minus the expense on deposits and bank's operational costs. The net profit means the profit after taxes and dividend distributions. In [16], the loans losses (or non-performing loans) reduce the net worth (or equity). Using this argument, the average of non-performing loans ($-nL(t)$) does not only appears in the differential equation of loans but also in the differential equation of equity. The value of w are the same as ω, they only have difference in dimension. Thus, when it comes in estimating parameters, w and ω must be treated as one parameter. The same for n and η.

Table 1: Description parameters of model (2.2) and Eq. (2.3). The abbreviation p.u.t. means per unit time

	Description	Dimension
a_D	The growth rate of deposits	time^{-1}
K_D	The carrying capacity of deposits	Rp
w	The average portion of deposit withdrawals p.u.t.	time^{-1}
a_L	The growth rate of loans	time^{-1}
K_L	The carrying capacity of loans	Rp
n	The average portion of non-performing loans p.u.t.	time^{-1}
r	The average portion of loan repayments p.u.t.	time^{-1}
η	The average portion of non-performing loans	Dimensionless
a_P	The growth rate of equity due to profit or loss	time^{-1}
d	The portion of dividend	Dimensionless
τ	The portion of tax	Dimensionless
$r_L(t)$	The interest rate of loans at time t	Dimensionless
ω	The average portion of deposit withdrawals	Dimensionless
$r_D(t)$	The interest rate of deposits at time t	Dimensionless
$C(t)$	The operational cost at time t	Rp

The differences between our model (2.2) and Sumarti et al. model (1.1) are explained as follows. First, we simplify the bank's balance sheet only consist of deposits, equity, loan and liquid assets. We combine the other balance sheet components from Sumarti et al., like primary reserve ($\kappa_1 D(t)$), secondary reserve ($\kappa_2 D(t)$), interbank placement ($I_P(t)$) and interbank loan

$(I_L(t))$, implicitly into liquid assets component. Second, we neglect the appearance of interactions between deposits and loans with liquidity tool and equity in (1.1) by setting $\epsilon_D = \xi_D = \epsilon_L = \xi_L = 0$. Third, we add the rate of decline due to non-performing loans $(-nL(t))$ into loans and equity model. Fourth, we include dividend and tax portion in the calculation of bank's net profit. Fifth, the interest rate of deposits and loans in Sumarti et al. model (1.1), r_D and r_L, are constant, meanwhile in our model they will be approximated by the Fourier series below:

$$r_i(t) = p_{i,0} + \sum_{k=1}^{N_i} p_{i,k} \cos(2k\pi t/T_i) + q_{i,k} \sin(2k\pi t/T_i), \tag{2.4}$$

where $p_{i,0}$, $p_{i,k}$, $q_{i,k}$, with $k = 1, \ldots, N_i$, are the parameter of the Fourier series, T_i is the period and $i \in \{D, L\}$ is an index.

There are several ways to assume the operational cost function $C(t)$. Sumarti et al. in [11] define it as $C(t) = cD(t)L(t) + \frac{c}{2}(D(t)^2 + L(t)^2)$, with $0 < c < 1$. Dalla and Varelas in [8] define it as $C(t) = c_1D(t) + c_2L(t) + c_3D(t)L(t)$, with $0 < c_1, c_2 < 1$ and $-1 < c_3 < 1$. Operational cost usually only serves as a calibration of the model. Thus, we chose the formulation of the operational cost as a simple linear function of the amount of deposits and loans, that is

$$C(t) = c_1D(t) + c_2L(t),$$

where $0 < c_1, c_2 < 1$.

As part of the financial system that contains uncertainty, bank will surely face a variety of uncertainties that may arise from the internal or external of the industry. Some concrete examples are the uncertainty in customers' deposits withdrawals and non-performing loans. The customers have an irregular withdrawing behavior because of unpredictable consumptions that urgently need cash. Borrowers are also faced the possibility of default in business due to the risk of unsmooth economy.

Consider the average portion of deposit withdrawals ω in system (2.2) is replaced by ω_s, where $\omega_s = \omega + \varepsilon_\omega(t)$, $\varepsilon_\omega(t) \sim N(0, \sigma_\omega^2)$, and the average portion of non-performing loans η is replaced by η_s, where $\eta_s = \eta + \varepsilon_\eta(t)$, $\varepsilon_\eta(t) \sim N(0, \sigma_\eta^2)$. Then we can write

$$\omega_s \mathrm{d}t = \omega \mathrm{d}t + \sigma_\omega \mathrm{d}B_\omega(t) \text{ and } \eta_s \mathrm{d}t = \eta \mathrm{d}t + \sigma_\eta \mathrm{d}B_\eta(t),$$

where $B_\omega(t)$ and $B_\eta(t)$ are independent standard Wiener process.

It has been said that $w = \omega$ and $n = \eta$, they only have difference in dimension. Thus, both w and n are also stochastic, and must be replaced by w_s and n_s, respectively, where $w_s = w + \varepsilon_w(t)$, $\varepsilon_w(t) \sim N(0, \sigma_w^2)$ and $n_s = n + \varepsilon_n(t)$, $\varepsilon_n(t) \sim N(0, \sigma_n^2)$. Here $\sigma_w = \sigma_\omega$ and $\sigma_n = \sigma_\eta$, they only have difference in dimension.

$$w_s \mathrm{d}t = w\mathrm{d}t + \sigma_w \mathrm{d}B_\omega(t) \text{ and } n_s\mathrm{d}t = n\mathrm{d}t + \sigma_n \mathrm{d}B_\eta(t).$$

The previous deterministic model will become

$$\begin{cases} \mathrm{d}D(t) = & \big[a_D D(t)\big(1 - \frac{D(t)}{K_D}\big) - wD(t)\big]\mathrm{d}t - \sigma_w D(t)\mathrm{d}B_\omega(t), \\ \mathrm{d}L(t) = & \big[a_L L(t)\big(1 - \frac{L(t)}{K_L}\big) - nL(t) - r(1-\eta)L(t)\big]\mathrm{d}t \\ & +(r\sigma_\eta - \sigma_n)L(t)\mathrm{d}B_\eta(t), \\ \mathrm{d}E(t) = & \big[a_P(1-d)(1-\tau)\big[(1-\eta)L(t)r_L(t) - (1-\omega)D(t)r_D(t) \\ & -C(t)\big] - nL(t)\big]\mathrm{d}t + \sigma_\omega a_P(1-d)(1-\tau)D(t)r_D(t)\mathrm{d}B_\omega(t) \\ & -[\sigma_\eta a_P(1-d)(1-\tau)r_L(t) + \sigma_n]L(t)\mathrm{d}B_\eta(t), \\ A(t) = & D(t) + E(t) - L(t). \end{cases} \tag{2.5}$$

3 Data and Method

This paper uses the data of Indonesian commercial conventional banks group type 3 which in Indonesian language called as Bank Umum Kelompok Usaha 3 or abbreviated as BUKU 3. It is a monthly data of deposits, loans, equity, average interest rates of deposits and average interest rates of loans from January 2010 (used as $t = 0$) until February 2015 ($t = 61/12$), the increment of time t for each month is $\Delta t = 1/12$. BUKU 3 is a collection of banks that have capital between 5 and 30 trillion rupiah.

Employing the data, we will estimate the deterministic model's parameters applying spiral optimization (SPO) algorithm. SPO is a metaheuristic algorithm which inspired by the spiral events in real life such as spiral galaxy, nautilius shell and whirling currents of water. The algorithm was first proposed by Tamura and Yasuda [17]. The SPO describes a collection of points that rotated towards the global optimum point which the center of the rotation.

The SPO algorithm for a minimization problem $\min_{\mathbf{x}\in\mathbb{R}^n} f(\mathbf{x})$ is written as follows.

- Initialization:
  $I \subset \mathbb{R}^n$ as the search space; m as the number of points; $\mathbf{x}_i(0) \in I$ as the initial points i, $i = 1, 2, \ldots, m$; θ as the angle of the rotation,

$0 < \theta \leq \pi$; δ as the scalling radius of the rotation, $0 < \delta < 1$; $k_{\max}$ as the iteration maximum number.

- Process:
  Set $\mathbf{x}^{\text{gbest}}(0) = \mathbf{x}^{\text{g}0}$, where $f(\mathbf{x}^{\text{g}0}) = \min_i f(\mathbf{x}_i(0))$, as the initial center of the rotation;
  Set $k = 0$.
  1. For each i, $i = 1, 2, \ldots, m$, update point's position:
  $$\mathbf{x}_i(k+1) = S_n(\delta, \theta)\mathbf{x}_i(k) - (S_n(\delta, \theta) - I_n)\mathbf{x}^{\text{gbest}}(k),$$
  where $S_n(\delta, \theta) = \delta \Pi_{i=1}^{n-1}\big(\Pi_{j=1}^{i} R_{n-i,n+1-j}^{(n)}(\theta)\big)$, and $R_{i,j}^{(n)}(\theta)$ is $n \times n$ identity matrix with the ii and jj entries are $\cos\theta$, the ji entry is $\sin\theta$, and the ij entry is $-\sin\theta$.
  2. Checking whether the updated position is still in the search space or not: If $\mathbf{x}_i(k+1) \in I$, then calculate $f(\mathbf{x}_i(k+1))$. If $\mathbf{x}_i(k+1) \notin I$, then set $f(\mathbf{x}_i(k+1)) = \text{NaN}$.
  3. Update the center of rotation: $\mathbf{x}^{\text{gbest}}(k+1) = \mathbf{x}^{\text{g}}$, where $f(\mathbf{x}^{\text{g}}) = \min_i f(\mathbf{x}_i(k+1))$.
  4. If $k < k_{\max}$, then redo Process 1 until 3 with setting $k = k+1$. If $k = k_{\max}$, then stop the process.
- Output:
  $\mathbf{x}^{\text{gbest}}(k_{\max})$ as the minimum point of $f(\mathbf{x})$.

In Process 1, $R_{i,j}^{(n)}(\theta)$ is a matrix that rotates vector $\mathbf{x} = (\ldots, x_i, \ldots, x_j, \ldots)^T$ counterclockwise in x_ix_j-plane, while $S_n(\delta, \theta)$ is a composition of rotation matrices $R_{i,j}^{(n)}(\theta)$. Different in choose θ and δ will bring different behavior in the rotation. If θ is too small (near zero) then the rotation will be too slow but the search space can be more explored, and if θ is too big (near π) then the rotation will be too fast and the search space will be less explored. Meanwhile, in choosing δ, if δ is too small ($\delta < 0.9$) then $\mathbf{x}_i$ will be rotated towards $\mathbf{x}^{\text{gbest}}$ too fast, so the search space can not be more explored. Therefore, to gain the rotation scheme not too slow or too fast, it is common to set θ and δ which in the range $\pi/4 \leq \theta \leq \pi/2$ and $0.9 \leq \delta \leq 0.99$. The checking condition in Process 2 assures that the center of rotation will always be in the search space. This is important because we do not want to update the points out of the search space due to the center that placed out of the search space.

Compared to some metaheuristic algorithms, this algorithm is quite more powerful and faster because the random elements only appear at the preamble and the updates do not require random numbers. Even, random elements in the preamble can be replaced with Sobol sequences which are deterministic and more uniform, so the algorithm execution become faster, see [18].

In estimating parameters of Eq. (2.4), we use the SPO algorithm with $m = 1500$, $\theta = \pi/4$, $\delta = 0.95$, $k_{\max} = 100$, and 5 runs. The objective function is the Mean Average Percentage Error (MAPE) between the data of deposits and loans interest rates and the model in Eq. (2.4).

For estimating parameters in system (2.2), we use the SPO algorithm with $m = 1000$, $\theta = \pi/4$, $\delta = 0.95$, $k_{\max} = 200$, and 5 runs. The objective function is the MAPE between the data of deposits, loans, and equity and the numerical solution of the system (2.2) using Runge-Kutta method. For further explanation, see [19] and [20].

4 Result

The estimated parameters for the interest rates model using the SPO algorithm are presented in Table 2. Although only using the Fourier series approximation, the interest rates data can be well approached, as seen in the MAPE value which is quite small. For deposit interest rates, MAPE = 5.17% and for loans interest rates, MAPE = 0.63%. A comparison between the plot of the data and the interest rates model can be seen in Fig. 1. From the figure, the deposits interest rates have larger and irregular fluctuations compared to the loans interest rates.

Table 2: Estimated parameters for interest rates model.

$r_D(t)$	$p_{D,0}$	$p_{D,1}$	$p_{D,2}$	$q_{D,1}$	$q_{D,2}$	T_D	MAPE
	0.089620	0.001203	0.001175	0.004302	-0.000544	4	5.17%
$r_L(t)$	$p_{L,0}$	$p_{L,1}$	$p_{L,2}$	$q_{L,1}$	$q_{L,2}$	T_L	MAPE
	0.240549	0.008908	0.001138	0.001106	-0.001304	6	0.63%

Table 3 presents the results of parameter estimation using the SPO algorithm for the differential equation system in (2.2). From the table, the MAPE value is quite small, MAPE = 1.9875%. This MAPE value is the average MAPE of deposits, loans, and equity. The obtained carrying capacity value for deposits is greater than for loans. This indicates, in long time, the amount of loans of BUKU 3 will not exceed the amount of deposits. Although, of course, these carrying capacity value may change if more training data is used. Some important things are also worth to be noted: the average portion of deposit withdrawals $\omega = 0.1714$ or $\omega = 17.14\%$, the average portion

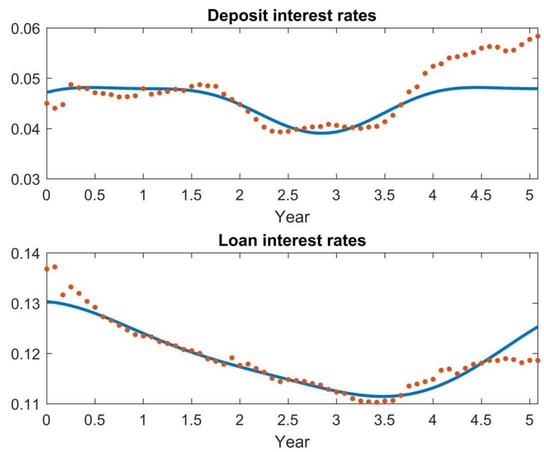

Figure 1: Plots of the interest rates data (marked by red dots) versus the model (marked by blue line).

of non-performing loans $\eta = 0.0239$ or $\eta = 2.39\%$, and the average portion of loan repayments $r = 0.375$ or $r = 3.75\%$. The plot of deposits, loans, and equity data alongside with the solution of system (2.2) is presented in Fig. 2. The figure shows that the deposits and loans data are well estimated. The equity model is not quite enough for following many fluctuations that appear in the equity data.

Table 3: Estimated parameters for system (2.2), where the tax $\tau = 0.15$.

a_D	K_D	$w = \omega$	a_L	K_L	$n = \eta$
0.5379	2502	0.1714	0.4747	1850	0.0239
r	a_P	d	c_1	c_2	MAPE
0.0375	3.0701	0.5151	0.0002	0.0276	1.9875%

We use the Euler-Maruyama method [21] for solving the system of stochastic differential equations in (2.5) as presented below:

$$\begin{cases} D_{k+1} = & D_k + \left[a_D D_k\left(1 - \frac{D_k}{K_D}\right) - wD_k\right]\Delta t - \sigma_w D_k W_{k,\omega}(\Delta t), \\ L_{k+1} = & L_k + \left[a_L L_k\left(1 - \frac{L_k}{K_L}\right) - nL_k - r(1-\eta)L_k\right]\Delta t \\ & +(r\sigma_\eta - \sigma_n)L_k W_{k,\eta}(\Delta t), \\ E_{k+1} = & E_k + \Big(- nL_k + a_P(1-d)(1-\tau)\big[(1-\eta)L_k r_{k,L} - (1-\omega)D_k r_{k,D} \\ & -C_k\big]\Big)\Delta t + \sigma_\omega a_P(1-d)(1-\tau)D_k r_{k,D} W_{k,\eta}(\Delta t) \\ & -[\sigma_\eta a_P(1-d)(1-\tau)r_{k,L} + \sigma_n]L_k W_{k,\eta}(\Delta t) \\ A_{k+1} = & D_{k+1} + E_{k+1} - L_{k+1}, \end{cases}$$

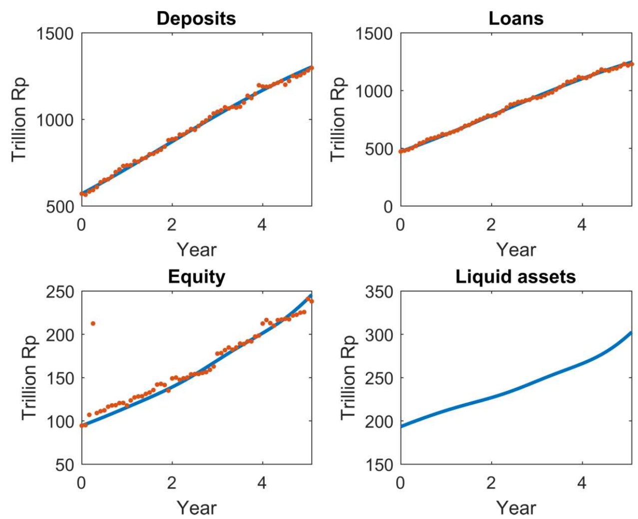

Figure 2: Plots of the deterministic model (marked by blue lines) versus the deposit, loans and equity data (marked by red dots).

where $W_{k,\omega}(\Delta t) \sim N(0, \Delta t)$ and $W_{k,\eta}(\Delta t) \sim N(0, \Delta t)$ are independent.

Using values $\Delta t = 1/12$, $\sigma_w = \sigma_\omega = 0.07$, $\sigma_n = \sigma_\eta = 0.05$, Fig. 3 presents simulations of three stochastic models versus the deterministic model.

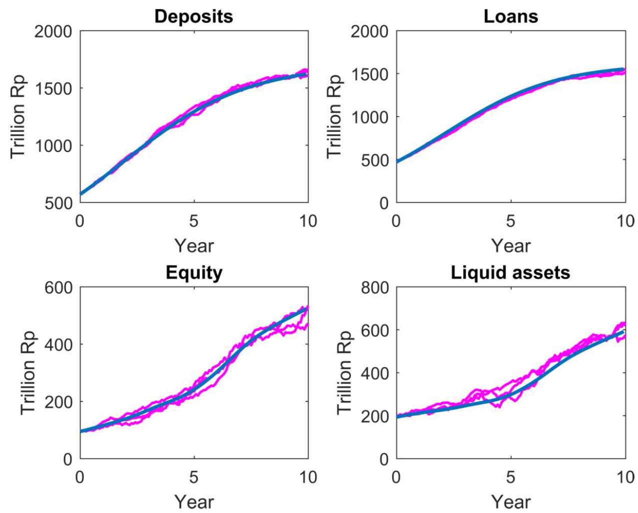

Figure 3: Simulations of the stochastic model (marked by pink lines) versus the deterministic model (marked by blue lines).

5 Conclusion

In this paper we present a system of deterministic and stochastic differential equations of a bank's balance sheet variables: deposits, loans, equity, and liquid assets. The deterministic model refers to the model presented in [11]. The deposits and loans model follow the logistic growth model with harvesting. The growth rate of equity is proportional with bank's profit or loss, and it can be reduced by the non-performing loans factor. And the liquid assets acts as the balancing variable of the balance sheet. The interest rates of deposits and loans are included in the equity model, and they are approximated by Fourier series. The stochastic model is presented with randomness appeared in the average portion of deposit withdrawals and the average portion of non-performing loans parameters.

The parameters of the Fourier series for interest rates model and the parameters in the system of deterministic differential equations are estimated using spiral optimization algorithm employing the data of Indonesian commercial conventional banks group type 3. The data is the monthly data of deposits, loans, equity, deposits interest rates and loans interest rates from January 2010 until February 2015. The estimation results are quite well as shown by the MAPE values between the data and the estimated model are small. Using Euler-Maruyama method, the simulation of the stochastic models are also presented.

The presented models are so simple, and not yet to be perfect. Some considerations for further studies are to appear the other balance sheet variables and to include some banking regulations into the models. By adding those factors, the models are expected to become richer and be able to analyze banking regulation in overcoming future crises such as the bank run shown by Fig. 4 and the bubble burst shown by Fig. 5. The figures are obtained by applying the stochastic model with making the values of deposit withdrawals and non-performing loans parameters big enough in some interval time.

Figure 4: The dynamics of deposits when bank run happens

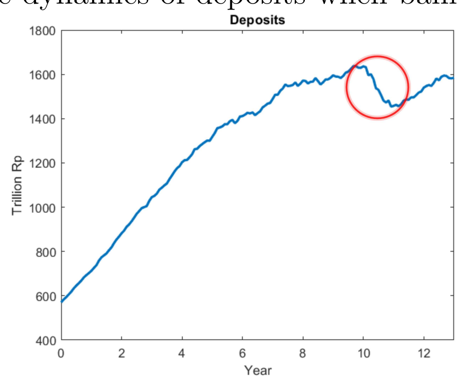

Figure 5: The dynamics of loans when bubble burst happens

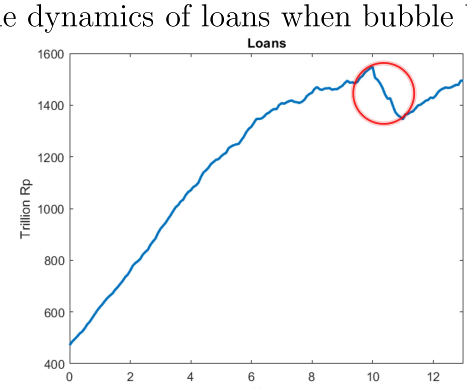

Figure 6: Examples of crises that may occur at a bank.

Acknowledgment. This research is fully financed by the Indonesian Directorate General for Higher Education over the PMDSU Batch III scholarship program.

References

[1] Michael A. Klein, A theory of the banking firm, Journal of Money, Credit, and Banking, **3**, no. 2, (1971), 205–218.

[2] Mario Monti, Deposit, credit and interest rates determination under alternative objective functions, in Mathematical Methods in Investment and Finance, edited by G. P. Szego, K. Shell, North-Holland, Amsterdam, (1972), 431–454.

[3] Ernst Baltensperger, Alternative approaches to the theory of the banking firm, Journal of Monetary Economics, **6**, no. 1, (1980), 1–37.

[4] Anthony M. Santomero, Modeling the banking firm: A survey, Journal of Money, Credit and Banking, **16**, no. 4, (1984), 576–602.

[5] Jean Dermine, Deposit rates, credit rates and bank capital: The Klein-Monti model revisited, Journal of Banking and Finance, **10**, no. 1, (1986), 99–114.

[6] Xavier Freixas, Jean-Charles Rochet, Microeconomics of Banking, 2nd edition, MIT Press, Massachusetts, 2008.

[7] Linda A. Toolsema, Lambert Schoonbeek, Bank behavior and the interbank rate in an oligopolistic market, CCSO Working Papers, no. 199909, (1999).

[8] Eleni Dalla, Erotokritos Varelas, Regulation & oligopoly in banking: The role of banking cost structure, Journal of Economics and Business, **104**, (2019), 105836.

[9] Iman Gunadi, Cicilia A. Harun, Revitalising Reserve Requirement in Banking Model: An Industrial Organisation Approach, SEACEN Occasional Paper, no. 51, (2011).

[10] Luciano Fanti, The dynamics of a banking duopoly with capital regulations, Economic Modelling, **37**, (2014), 340–349.

[11] Novriana Sumarti, Akmal Fadhlurrahman, Hanifah R. Widyani, Iman Gunadi, The Dynamical System of the Deposit and Loan Volumes of a Commercial Bank Containing Interbank Lending and Saving Factors, Southeast Asian Bulletin of Mathematics, **42**, (2018), 757–772.

[12] Victor Selyutin, Simplified model of bank balance sheet management, CEUR Workshop Proceedings, **2393**, (2019), 422–433.

[13] Jochen Mankart, Alexander Michaelides, Spyros Pagratis, Bank capital buffers in a dynamic model, Deutsche Bundesbank Discussion Paper, no. 51, (2018).

[14] Markus Behn, Claudio Daminato, Carmelo Salleo, A dynamic model of bank behavior under multiple regulatory constraints, ECB Working Paper Series, no. 2233, (2019).

[15] Roman Goncharenko, Asad Rauf, Still Too Much, Too Late: Provisioning for Expected Loan Losses, SSRN, http://dx.doi.org/10,2139/ssrn.3247631 (2020).

[16] Robert M. May, Nimalan Arinaminpathy, Systemic risk: the dynamics of model banking systems, J. R. Soc. Interface, **7** , (2010), 823–838.

[17] Kenichi Tamura, Keiichiro Yasuda, Spiral dynamics inspired optimization, J. of Advanced Computational Intelligence and Intelligence Informatics (JACIII), **42**, no. 5, (2011), 757–772.

[18] Kuntjoro Adji Sidarto, Adhe Kania, Finding all solutions of systems of nonlinear equations using spiral dynamics optimization with clustering, J. of Advanced Computational Intelligence and Intelligence Informatics (JACIII), **19**, no. 5, (2015), 697–707.

[19] Moch Fandi Ansori, Kuntjoro Adji Sidarto, Novriana Sumarti, Model of deposit and loan of a bank using spiral optimization algorithm, J. Indones. Math. Soc., **25**, no. 3, (2019), 292–301.

[20] Moch Fandi Ansori, Kuntjoro Adji Sidarto, Novriana Sumarti, Logistic models of deposit and loan between two banks with saving and debt transfer factors, AIP Conference Proceedings, **2192**, (2019), 060002.

[21] Desmond J. Higham, An Algorithmic introduction to numerical simulation of stochastic differential equation, SIAM review, **43**, no. 3, (2001), 525–546.