International Journal of Mathematics and Computer Science, **15**(2020), no. 4, 1207–1211

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Prime Labeling of some Star Related Mirror Graphs

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(Received July 7, 2020, Accepted September 9, 2020)

Abstract

Let $G = (V, E)$ be a graph. A bijection $f : V \to \{1, 2, \ldots, |V|\}$ is called a prime labeling if, for each $e = (u, v)$ belonging to E, we have $GCD[f(u), f(v)] = 1$. A graph that admits a prime labeling is called a prime graph [1]. In this paper, we prove that the mirror graphs of some star related graphs such as $K_{1,n}$, $B(n, n)$ and $S(K_{1,n})$ admit prime labeling.

1 Introduction

In this paper, we consider only finite, simple, connected and undirected graphs. We denote the vertex set and the edge set by $V(G)$ and $E(G)$ of the graph G and the corresponding cardinality by $|V(G)|$ and $|E(G)|$ respectively. A graph labeling is an assignment of integers to the vertices or edges, or both subject to certain conditions. Graph labelings were first introduced in the mid sixties [6]. One of the important areas in graph theory is graph labeling which is useful in many applications like coding theory, x-ray, crystallography, radar, astronomy, circuit design, communication network addressing,

Key words and phrases: Prime labeling, Mirror Graph, Star graph $K_{1,n}$, Bistar $B(n, n)$, Subdivision of a Star graph $S(K_{1,n})$.
AMS Subject Classifications: 05C76, 05C78
ISSN 1814-0432, 2020, http://ijmcs.future-in-tech.net

and data base management [7]. The notion of Prime labeling was introduced by Entringer and was discussed in the paper by Tout, Dabboucy and Howalla [10]. Let $G = (V, E)$ be a graph. A bijection $f : V \to \{1, 2, \ldots, |V|\}$ is called a prime labeling if, for each $e = (u, v)$ belonging to E, we have $GCD[f(u), f(v)] = 1$. A graph that admits a prime labeling is called a prime graph [1]. In 2004, the concept of Mirror graphs was introduced by Bresar et. al as an intriguing class of graphs.

Let G be a bipartite graph with partite sets V_1 and V_2 and G' be the copy of G with corresponding partite sets $V_1^{'}$ and $V_2^{'}$. The mirror graph $M(G)$ of G is obtained from G and G' by joining each vertex of V_2 to its corresponding vertex in $V_2^{'}$ by an edge (9). In this paper, we prove that the Mirror graphs of some star related graphs such as star graph $K_{1,n}$, Bistar graph $B(n, n)$ and Subdivision of a star graph $S(K_{1,n})$ admit prime labeling.

2 Mirror graph $M(K_{1,n})$ of star graph $K_{1,n}$

Definition 2.1. [5] *A star graph $K_{1,n}$ is a complete bipartite graph with a single vertex belonging to one set and all the remaining vertices belonging to the other set. The single central vertex is called the apex vertex.*

Theorem 2.2. *Mirror graph $M(K_{1,n})$ of a star graph $K_{1,n}$ is a prime graph.*

Proof. Let G be a Star graph $K_{1,n}$. $V(G) = \{v, v_i/1 \leq i \leq n\}$ be the vertex set of G. $E(G) = \{vv_i/1 \leq i \leq n\}$ be the edge set of G. G is a bipartite graph with partite sets $V_1 = \{v\}$ and $V_2 = \{v_i/1 \leq i \leq n\}$. Let G' be the copy of G with the corresponding partite sets $V_1' = \{u\}$ and $V_2' = \{u_i/1 \leq i \leq n\}$, where V_1' and V_2' are copies of V_1 and V_2 respectively. $E(G') = \{uu_i/1 \leq i \leq n\}$ be the edge set of G'. Let $M(G)$ be the mirror graph of G. The mirror graph $M(G)$ of G is obtained from G and G' by joining each vertex of V_2 to its corresponding vertex in V_2' by additional edges $\{v_iu_i/1 \leq i \leq n\}$. $V[M(G)] = \{v, u, v_i, u_i/1 \leq i \leq n\}$ is the vertex set of $M(G)$. $E[M(G)] = \{vv_i/1 \leq i \leq n\} \cup \{uu_i/1 \leq i \leq n\} \cup \{v_iu_i/1 \leq i \leq n\}$ is the edge set of $M(G)$. $|V[M(G)]| = 2n + 2$. We define a bijective function, $f : V[M(G)] \to \{1, 2, 3, \ldots, 2n + 2\}$ given by $f(u) = 1$, $f(v) = 2$, $f(v_i) = 2i + 1$, $1 \leq i \leq n$, $f(u_i) = 2i + 2$,$1 \leq i \leq n$.
For the edges vv_i, $gcd(f(v), f(v_i)) = gcd(2, 2i + 1) = 1, 1 \leq i \leq n$
For the edges uu_i, $gcd(f(u), f(u_i)) = gcd(1, 2i + 2) = 1, 1 \leq i \leq n$
For the edges v_iu_i, $gcd(f(v_i), f(u_i)) = gcd(2i + 1, 2i + 2) = 1, 1 \leq i \leq n$
Therefore Mirror graph $M(K_{1,n})$ of star graph $K_{1,n}$ is a prime graph. □

3 Mirror graph $M(B(n,n))$ of Bistar graph $B(n,n)$

Definition 3.1. [8] *Bistar graph $B(n,n)$ is a graph obtained by joining the apex vertices of two copies of star graph $K_{1,n}$.*

Theorem 3.2. *Mirror graph $M(B(n,n))$ of bistar graph $B(n,n)$ is a prime graph.*

Proof. Let G be Bistar graph $B(n,n)$. $V(G) = \{v, w, v_i/1 \leq i \leq 2n\}$ be the vertex set of G. $E(G) = \{vv_i/1 \leq i \leq n\} \cup \{vw\} \cup \{wv_{n+i}/1 \leq i \leq n\}$ be the edge set of G. G is a bipartite graph with partite sets $V_1 = \{v, v_{n+i}/1 \leq i \leq n\}$ and $V_2 = \{w, v_i/1 \leq i \leq n\}$. Let G' be the copy of G with the corresponding partite sets $V_1' = \{u, u_{n+i}/1 \leq i \leq n\}$ and $V_2' = \{x, u_i/1 \leq i \leq n\}$ where V_1' and V_2' are copies of V_1 and V_2 respectively. $E(G') = \{uu_i/1 \leq i \leq n\} \cup \{ux\} \cup \{xu_{n+i}/1 \leq i \leq n\}$ be the edge set of G' . Let $M(G)$ be the mirror graph of G. The mirror graph $M(G)$ of G is obtained from G and G' by joining each vertex of V_2 to its corresponding vertex in V_2' by additional edges $\{v_iu_i, /1 \leq i \leq n\} \cup \{wx\}$. $V[M(G)] = \{v, u, w, x, v_i, u_i, /1 \leq i \leq 2n\}$, $E[M(G)] = \{vv_i/1 \leq i \leq n\} \cup \{vw\} \cup \{wv_{n+i}/1 \leq i \leq n\} \cup \{uu_i/1 \leq i \leq n\} \cup \{ux\} \cup \{xu_{n+i})/1 \leq i \leq n\} \cup \{v_iu_i/1 \leq i \leq n\} \cup \{wx\}, |V[M(G)]| = 4n + 4$. We define a bijective function $f : V[M(G)] \rightarrow \{1, 2, 3, \ldots, 4n + 4\}$ given by $f(v) = 1$, $f(u) = 2$, $f(w) = p$, where p is the highest prime number less than the number $4n+4$, $f(x) = q$, where q is the second highest prime number less than the number $4n + 4$, $q < p$, $f(v_i) = 2i + 2$, $1 \leq i \leq n$,$f(u_i) = 2i + 1$, $1 \leq i \leq n$, $f(v_{n+i}) = 2n + 2 + i$,$1 \leq i \leq n$, $f(u_{n+i}) = 3n + 2 + i$,$1 \leq i \leq n$, where $f(u_{n+i}) \neq p$ and $f(u_{n+i}) \neq q$.

For the edges vv_i, $gcd(f(v), f(v_i)) = gcd(1, 2i + 1) = 1, 1 \leq i \leq n$
For the edge vw, $gcd(f(v), f(w)) = gcd(1, p) = 1$
For the edges wv_{n+i}, $gcd(f(w), f(v_{n+i})) = gcd(p, 2n + 2 + i) = 1, 1 \leq i \leq n$
For the edges uu_i, $gcd(f(u), f(u_i)) = gcd(2, 2i + 1) = 1, 1 \leq i \leq n$
For the edge ux,$gcd(f(u), f(x)) = gcd(2, q) = 1$,
For the edges xu_{n+i}, $gcd(f(x), f(u_{n+i}) = gcd(q, 3n + 2 + i) = 1, 1 \leq i \leq n$
For the edges v_i, u_i, $gcd(f(v_i), f(u_i)) = gcd(2i + 2, 2i + 1) = 1, 1 \leq i \leq n$
For the edge wx, $gcd(f(w), f(x)) = gcd(p, q) = 1$
Therefore Mirror graph $M(B(n,n))$ of Bistar graph $B(n,n)$ is a prime graph.

□

4 Mirror graph $M(S(K_{1,n}))$ of Subdivision of a Star graph $S(K_{1,n})$

Definition 4.1. [3] *Let $G = (V(G), E(G))$ be a graph. Let uv be an edge of G and w is not a vertex of G. The edge uv is sub divided when it is replaced by the edges uw and wv.*

Theorem 4.2. *Mirror graph $M(S(K_{1,n}))$ of Subdivision of a Star graph $S(K_{1,n})$ is a prime graph.*

Proof. Let G be the subdivision of a star graph $S(K_{1,n})$. $V(G) = \{v, v_i/1 \leq i \leq 2n\}$ be the vertex set of G. $E(G) = \{vv_i/1 \leq i \leq n\} \cup \{v_iv_{n+i}/1 \leq i \leq n\}$ be the edge set of G. G is a bipartite graph with partite sets $V_1 = \{v, v_{n+i}/1 \leq i \leq n\}$ and $V_2 = \{v_i/1 \leq i \leq n\}$. Let G' be the copy of G with the corresponding partite sets $V_1' = \{u, u_{n+i}/1 \leq i \leq n\}$ and $V_2' = \{u_i/1 \leq i \leq n\}$, where V_1' and V_2' are copies of V_1 and V_2 respectively. $E(G') = \{uu_i/1 \leq i \leq n\} \cup \{u_iu_{n+i}/1 \leq i \leq n\}$ be the edge set of G'. Let $M(G)$ be the mirror graph of G. The mirror graph $M(G)$ of G is obtained from G and G' by joining each vertex of V_2 to its corresponding vertex in V_2' by additional edges $\{v_iu_i/1 \leq i \leq n\}$. $V[M(G)] = \{v, u, v_i, u_i/1 \leq i \leq 2n\}$. $E[M(G)] = \{vv_i/1 \leq i \leq n\} \cup \{v_iv_{n+i}/1 \leq i \leq n\} \cup \{v_iu_i/1 \leq i \leq n\} \cup \{uu_i/1 \leq i \leq n\} \cup \{u_iu_{n+i}/1 \leq i \leq n\}$. $|V[M(G)]| = 4n + 2$. We define a bijective function $f : V[M(G)] \to \{1, 2, 3, \ldots, 4n + 2\}$ given by $f(v) = 1$, $f(u) = 2$, $f(v_i) = 4i - 1, 1 \leq i \leq n$, $f(v_{n+i}) = 4i, 1 \leq i \leq n$, $f(u_i) = 4i + 1, 1 \leq i \leq n$, $f(u_{n+i}) = 4i + 2, 1 \leq i \leq n$.
For the edges vv_i, $gcd(f(v), f(v_i)) = gcd(1, 4i - 1) = 1, 1 \leq i \leq n$.
For the edges v_iv_{n+i}, $gcd(f(v_i), f((v_{n+i})) = gcd(4i - 1, 4i) = 1, 1 \leq i \leq n$
For the edges v_iu_i, $gcd(f(v_i), f(u_i)) = gcd(4i - 1, 4i + 1) = 1, 1 \leq i \leq n$
For the edges uu_i, $gcd(f(u), f(u_i)) = gcd(2, 4i + 1) = 1, 1 \leq i \leq n$
For the edges u_iu_{n+i}, $gcd(f(u_i), f(u_{n+i})) = gcd(4i + 1, 4i + 2) = 1, 1 \leq i \leq n$
Therefore Mirror graph $M(S(K_{1,n}))$ of subdivision of a star graph $S(K_{1,n})$ is a prime graph. □

5 Conclusion

It is very interesting to investigate the mirror graphs of different families of graphs which admit prime labeling and it still has more progress. In this paper, we have proved that the Mirror graphs of some star related graphs such as Star graph $K_{1,n}$, Bistar graph $B(n, n)$ and Subdivision of a star graph

$S(K_{1,n})$ admit prime labeling. To explore some new mirror graphs that admit prime labeling is an open area of research.

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