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Existence of Solutions of Fuzzy Fractional panto-graph Equations

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Abstract

This article studies the existence of solutions of the non-linear fuzzy fractional panto-graph equations. Strongly anomalous media problems are modeled by using panto-graph equations. Fixed point technique used to derive the results. An example is given to demonstrate the consequences.

1 Introduction

Recently, the significance of Fractional Differential Equations (FDEs) in mathematics has been tremendous. Like differential equations with integer order, fractional calculus has gained importance since it better depicts the physical system and dynamical system [10] and the fractional operators are non local. Differential and integral calculus for fuzzy valued mapping was studied by Prade and Dubois [8]. Allahviranloo et. al [7] introduced the existence of solution of fractional calculus with uncertainty and also proved the uniqueness result. It is widely realized that within the deterministic scenario there may be extremely exceptional delay differential equations referred to panto-graph equations. Panto-graph equations emerge in very extraordinary fields of pure and applied science; for example, electro elements, quantum mechanics, probability, number theory and dynamical frameworks. Panto-graph

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equation conditions have been studied by numerous analysts and understood by a few numerical strategies. Fractional panto-graph equations were established by Balachandran et. al [3] who explored the existence and uniqueness results. In the last decade, many works have been done using concepts of fuzzy fractional calculus [2, 7, 5] in variety of fields. Agarwal et. al [1] introduced the concepts R-L FDEs. Based on the Hukuhara differentiability, they considered the concepts of R-L H-differentiability to illuminate uncertain fractional differential equations. In this article, with the help of fuzzy Caputo derivative, Fuzzy Fractional Panto-graph Equations (FFPEs) were introduced. Also the problem of existence and uniqueness of the results for this set of equations was studied. These equations are used to model emphatically strange media. In this paper, we build up the presence of dynamical FFPEs. A Fixed point method is utilized to derive the results.
The structure of this article is as follows: In section 2, we introduce the FF-PEs. The existence and uniqueness results of FFPEs are given in section 3. In section 4, the method is illustrated by a example..

2 Fuzzy Fractional panto-graph Equations

Consider non linear fuzzy fractional panto-graph equation

$$\begin{cases} ({}_{gH}D^{q}_{0+})u(x) = f(x, u(x), u(\lambda x)) \quad x \in J \\ u(0) = u_0 \end{cases} \tag{2.1}$$

where $0 < q \leq 1$ is a real number and the operator $(gH^{D^q}_{0+})$ indicated the Caputo fractional generalized derivative of order q, $f : J \times \mathbb{R}_f \times \mathbb{R}_f \to \mathbb{R}_f$ is continuous function. We study the solutions of problem (2.1) to demonstrate the principal result. we need accompanying hypotheses and a lemma.

Theorem 2.1 Let $f : [a, b] \to \mathbb{R}_f$ be a fuzzified function on[a,b].

(i) f is $[(i) - gH]$ -differentiable at $c \in [a, b]$ iff $f is \quad {}^{cf}[(i) - gH]$ -diff at c

(ii) f is $[(ii) - gH]$-differentiable at $c \in [a, b]$ iff $f is \quad {}^{cf}[(ii) - gH]$ -diff at c,

Lemma(2.1) Equation (2.1) is the same as one of the following equations

$$u(x) = u_0 \oplus \frac{1}{\Gamma(q)} \int_{x_0}^{x} (x - s)^{(q-1)} \ f(s, u(s), u(\lambda s))ds$$

$$u(x) = u_0 \ominus \frac{-1}{\Gamma(q)} \int_{x_0}^{x} (x-s)^{(q-1)} \ f(s, u(s), u(\lambda s))ds$$

There exists a point $c \in [a, b]$ such that $u(x)$is $cf_{[(i)-gH]}$ -differentiable on [a,c] and $cf_{[(ii)-gH]}$ -differentiable on [c,b] and $(c, u(c), u(\lambda c)) \in \mathbb{R}$.
Further, We need the following hypothesis:

$(Hf) f : J \times \mathbb{R}_f \times \mathbb{R}_f \to \mathbb{R}_f$ is continuous and there exists a positive constant $L \geq 0$ such that,

$$||f(x, u, g) - f(x, v, h)|| \leq L(||u - v|| + ||x - y||)$$

$t \in J,\ u, v, g, h \in \mathbb{R}_f$
Using the Banach contraction rule, uniqueness of solution is obtained.

Theorem 2.2 Assume that $f : J \times \mathbb{R}_f \times \mathbb{R}_f \to \mathbb{R}_f$ is a bounded continuous function and satisfies Hf and if $4\beta L < 1$. Then the (FFPEs) (2.1) solution is unique and is $cf_{[(i-gH)]}$- differentiable on J.
Proof: Let us take $u(x)$ is $cf_{[(i-gH)]}$-differentiability and $u_0 \in \mathbb{R}_f$ is fixed.

Let $Z = C(J, \mathbb{R}_f)$. Define a mapping $\psi : Z \to Z$ by

Let us take $\beta = \frac{T^q}{\Gamma(q+1)}$ and $G = max_{t \in J} ||f(x, 0, 0)||$.

$$\psi u(x) = u_0 + \frac{1}{\Gamma(q)} \int_{x_0}^{x} (x-s)^{(q-1)} \ f(s, u(s), u(\lambda s))ds \tag{2.2}$$

We have to show thatψ has a fixed point.
Choose $2(||u_0|| + G\beta) \leq m$. Then we can show that $\psi B_m \subset B_m$, where $B_m := \{t \in Z : ||t|| \leq m\}$. From the assumption we have,

$||\psi u(x)|| \leq ||u_0|| + \frac{1}{\Gamma(q)} \int_{x_0}^{x} (x-s)^{(q-1)} \ ||f(s, u(s), u(\lambda s))||ds$

$$\leq ||u_0|| + \frac{1}{\Gamma(q)} \int_{x_0}^{x} (x-s)^{(q-1)} L[||u(s) + u(s)||]ds + \frac{1}{\Gamma(q)} \max_{x \in J} ||f(x, 0, 0)|| \int_{x_0}^{x} (x-s)^{(q-1)} ds$$

$\leq ||u_0|| + \frac{T^q}{\Gamma(1-q)}(2Lm + G) \leq m.$

Thus, ψ maps $B_m$into itself. Now, for $u_1, u_2 \in Z$, we have,

$$||\psi u_1(x) - \psi u_2(x)|| \leq \frac{1}{\Gamma(q)} \int_{x_0}^{x} (x-s)^{(q-1)} ||f(s, u_1(s) + u_1(\lambda s) - f(s, u_2(s) + u_2(\lambda s)||] ds$$
$$\leq \frac{T^q}{\Gamma(1-q)} 2L[||u_1(x) - u_2(x)||]$$

$$||\psi u_1(x) - \psi u_2(x)|| \leq ||u_1(x) - u_2(x)||.$$

Therefore, as $2L\beta < \frac{1}{2}$, the mapping ψ is a contraction on J. $\psi(u(x)) = u(x)$. Hence, problem (2.1) has a unique solution. Next, we have to prove existence of a solution of equation (2.1) with non local condition.

$$u(0) + h(u) = u_0 \tag{2.3}$$

Let the mapping $h : Z \to R_f$ satisfy the following condition:
(Hh) h:Z$\to$ $R_f$is continuous and there exists a constant $N > 0$, such that $||h(u) - h(v)|| \leq N||u, v|| \quad u, v \in C(J, R_f)$.

Theorem 2.3 If $2(N + 2L\beta) < 1$ and if the hypotheses Hf, Hh are satisfied, then the equation (FFPEs) (2.1) with nonlocal condition (2.3) has a unique result.

Proof: We need to show that the operator $\chi : Z \to Z$ characterized by

$$\chi u(x) = u_0 - h(u) + \frac{1}{\Gamma(q)} \int_{x_0}^{x} (x-s)^{(q-1)} \ f(s, u(s), u(\lambda s)) ds \tag{2.4}$$

has a fixed point which is solution of (2.1),(2.3).

If $2 \leq (||u_0|| + ||h(0)|| + G\beta) \leq m$, then we can show that $\chi B_m \subset B_m$.

$$||\chi u(x)|| \leq ||u_0|| + ||h(u) - h(0)|| + ||h(0)|| + \frac{1}{\Gamma(q)} \int_{x_0}^{x} (x-s)^{(q-1)} [||f(s, u(s), u(\lambda s)) - f(s, 0, 0)| + ||f(s, 0, 0)||] ds$$

$$\leq ||u_0|| + Nm + ||h(0)|| + 2Lm\gamma + G\beta \quad \leq m$$

Now, for $u_1, u_2 \in Z$, we have

$$||\chi u_1(x) - \chi u_2(x)|| \leq ||h(u_1) - h(u_2)|| + \frac{1}{\Gamma(q)} \int_{x_0}^{x} (x-s)^{q-1} ||f(s, u_1(s), u_1(\lambda s)) - f(s, u_2(s), u_2(\lambda s))|| ds$$

$$\leq (N + 2\beta L)||u_1(x) - u_2(x)||$$

$||\chi u_1(x) - \chi u_2(x)|| \leq \frac{1}{2}||u_1(x) - u_2(x)||$

3 Example:

In this section, we give an illustration to demonstrate the results.
Consider the following nonlinear fuzzy fractional differential equation

$$({}_{gH}D^q_{0+}u)(x) = au(x) + f(x, u(\lambda x)), \qquad 0 \leq x \leq T, \quad u(0) = 1, \qquad (3.5)$$

where $0 < \lambda < 1.0 < q < 1$ and $f : [0, T] \times \mathbb{R}_f \to \mathbb{R}_f$ is a scalar continuous function. The equation (3.5) is given in [12]

$$u(x) = E_q(ax)^q + \int_{x_0}^{x} (x - s)^{q-1} E_{q,q}(a(x - s)^q) f(s, u(x), u(\lambda x)) ds,$$

where $E_q(ax)^q = \sum_{k=0}^{\infty} \frac{(ax)^{qk}}{\Gamma(1+kq)}$ and $E_{q,q}(a(t - s)^q) = \sum_{k=0}^{\infty} \frac{a^k(x-s)^{qk}}{\Gamma[(1+k)q]}$.

Let us take $f(x, u(\lambda x)) = vcos(\lambda x), v \geq 0$. Then the right hand side of the initial condition (3.5) satisfies the Lipschitz condition of the form $eT^q \sum_{k=0}^{\infty} \frac{a^k T^{kq}}{[(1+k)q]\Gamma[(1+k)q]}|u_1(x) - u_2(x)|$. Choose a, e and T in a way that the hypothesis is fulfilled. It is clear that equation (3.5) with non local condition has a unique result on $[0, T]$.

4 Conclusion

In this paper, we presented the ideas of fuzzy fractional panto-graph equations under the caputo gH-derivative. These type of equations are helpful for strongly anomalous media. We studied the fuzzy FFPEs using the fixed point theorem. Finally, we provided an example to justify our results.

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