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On Q-Fuzzy Hyperideals of Semihyperrings

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Abstract

The concept of Q-fuzzy left (resp. right) hyperideals of semihyperrings is introduced and studied. We show that the set of all Q-fuzzy left (resp. right) hyperideals of semihyperrings forms a distributive lattice and show that a normal Q-fuzzy left (resp. right) hyperideal, which is a maximal element in the set of all normal Q-fuzzy left (resp. right) hyperideals of semihyperrings, has only two values 0 and 1.

1 Introduction

A fuzzy set was introduced by Zadeh [24] in 1965, as a generalization of abstract set theory. Many classical mathematics is extended to fuzzy mathematics, and various properties of them in the context of fuzzy set are established. In 2011, Majumder [16] introduced and studied the concept of a Q-fuzzification of ideals of Γ-semigroups. Akram and Sathakathulla [1], Lekkoksung [12, 13], studied this concept in case of Γ-semigroups and ordered semigroups [11] and investigated some important properties. Then, Mandel [17, 18] extended the concept of Q-fuzzification in Γ-semigroups to Γ-semirings.

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The concept of a hyperstructure was first introduced by Marty [19] in 1934, as a generalization of ordinary algebraic structures. This theory was studied in the following decades and nowadays by many mathematicians (see, e.g., [2, 3, 7, 21, 20]). Vougiouklis [20] introduced the concept of a semihyperring, as a generalization of a semiring, where both the addition and the multiplication are hyperoperations. Many fuzzy theorems in hyperstructures have been discussed by several authors, for example, Corsini, Cristea, Davvaz, Kazanci, Leoreanu, Yin and Zhan (see, e.g., [4, 5, 6, 7, 8, 10, 14, 15, 22, 23, 25, 26]).

In this paper, the concept of Q-fuzzy left (resp. right) hyperideals of semihyperrings is introduced and investigated. Moreover, we show that the set of all Q-fuzzy left (resp. right) hyperideals of semihyperrings forms a distributive lattice and show that a normal Q-fuzzy left (resp. right) hyperideals, which is a maximal element in the set of all normal Q-fuzzy left (resp. right) hyperideals of semihyperrings, has only two values 0 and 1.

2 Preliminaries

Let H be a nonempty set. A mapping $\circ : H \times H \to \mathcal{P}^*(H)$, where $\mathcal{P}^*(H)$ denotes the set of all nonempty subsets of H, is called a *hyperoperation* on H (see, e.g., [2, 3, 7, 21]). The hyperstructure $(H, \circ)$ is said to be a *hypergroupoid*. Let A and B be any two nonempty subsets of H and $x \in H$. Then, we denote

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, A \circ x = A \circ \{x\} \text{ and } x \circ B = \{x\} \circ B.$$

A hypergroupoid $(H, \circ)$ is called a *semihypergroup* if $(x \circ y) \circ z = x \circ (y \circ z)$ for all $x, y, z \in H$.

A hyperstructure $(S, +, \cdot)$ is called a *semihyperring* [20] if it satisfies the following conditions:

(i) $(S, +)$ is a semihypergroup;

(ii) $(S, \cdot)$ is a semihypergroup;

(iii) $x \cdot (y + z) = x \cdot y + x \cdot z$ and $(y + z) \cdot x = y \cdot x + z \cdot x$ for all $x, y, z \in S$.

A nonempty subset T of a semihyperring $(S, +, \cdot)$ is said to be a *subsemihyperring* of S if for all $x, y \in T$, $x + y \subseteq T$ and $x \cdot y \subseteq T$. An element 0 of a semihyperring $(S, +, \cdot)$ is called *zero* if $0 \cdot a = a \cdot 0 = \{0\}$ and $a + 0 = 0 + a = \{a\}$

for all $a \in S$. A nonempty subset I of a semihyperring $(S, +, \cdot)$ is called a *left* (resp. *right*) *hyperideal* of S if for every $x, y \in I$, $x + y \subseteq I$ and for every $s \in S$, $s \cdot x \subseteq I$ (resp. $x \cdot s \subseteq I$). We call I a *hyperideal* of S if it is both a left and a right hyperideal of S. For more convenient, we write S instead of a semihyperring $(S, +, \cdot)$ and xy instead of $x \cdot y$ for all $x, y \in S$.

Let R and S be semihyperrings. A mapping $f : R \to S$ is said to be *homomorphism* [2] if for every $x, y \in R$, $f(x + y) \subseteq f(x) + f(y)$ and $f(x \cdot y) \subseteq f(x) \cdot f(y)$.

A *fuzzy subset* [24] of a nonempty set X is a mapping $f : X \to [0, 1]$. A *Q-fuzzy subset* [16] of a nonempty set X is a mapping $\mu : X \times Q \to [0, 1]$, where Q is a nonempty set.

In this paper, we assume that Q is a nonempty set and S is a semihyperring with a zero element 0.

The set $\mu_t = \{(x, q) \in X \times Q \mid \mu(x, q) \geq t\}$ is called a *level subset* of μ, where $t \in [0, 1]$. Let μ and ν be any two Q-fuzzy subsets of a nonempty set X. Then $\mu \subseteq \nu$ if and only if $\mu(x, q) \leq \nu(x, q)$ for all $x \in X, q \in Q$.

The intersection and the union of two Q-fuzzy subsets μ and ν of a nonempty set X, denoted by $\mu \cap \nu$ and $\mu \cup \nu$, respectively, are defined by letting $x \in X$ and $q \in Q$,

$$(\mu \cap \nu)(x, q) = \min\{\mu(x, q), \nu(x, q)\},$$
$$(\mu \cup \nu)(x, q) = \max\{\mu(x, q), \nu(x, q)\}.$$

3 Q-fuzzy hyperideals

In this section, we introduce the concept of Q-fuzzy left (resp. right) hyperideals of semihyperrings and study some important properties.

Definition 3.1. *Let μ be a nonempty Q-fuzzy subset of a semihyperring S (i.e., $\mu(x, q) \neq 0$ for some $x \in S, q \in Q$). Then μ is called a Q-fuzzy left (resp. right) hyperideal of S if it satisfies the following axioms:*

(i) $\inf_{z \in x+y} \mu(z, q) \geq \min\{\mu(x, q), \mu(y, q)\}$;

(ii) $\inf_{z \in xy} \mu(z, q) \geq \mu(y, q)$ *(resp.* $\inf_{z \in xy} \mu(z, q) \geq \mu(x, q)$*);*

for all $x, y \in S$ and $q \in Q$.

We call that μ is a *Q-fuzzy hyperideal* of S if it is both a Q-fuzzy left hyperideal and a Q-fuzzy right hyperideal of S.

Example 3.2. Let Q be a nonempty set and $S = \{0, a, b, c\}$. Define two hyperoperations $+$ and $\cdot$ on S by the following equations:

$+$	0	a	b	c
0	$\{0\}$	$\{a\}$	$\{b\}$	$\{c\}$
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{b\}$	$\{a\}$	$\{0, b\}$	$\{0, b, c\}$
c	$\{c\}$	$\{a\}$	$\{0, b, c\}$	$\{0, c\}$

$+$	0	a	b	c
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{0\}$	$\{a\}$	$\{0, b\}$	$\{0\}$
b	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
c	$\{0\}$	$\{0, c\}$	$\{0\}$	$\{0\}$

Then, $(S, +, \cdot)$ is a semihyperring [9]. We define a Q-fuzzy subset μ of S by for every $x \in S$ and $q \in Q$,

$$\mu(x, q) = \begin{cases} 0.7 & \text{if } x \in \{0, b\}, \\ 0.2 & \text{otherwise.} \end{cases}$$

By routine calculations, we have that μ is a Q-fuzzy hyperideal of S.

Theorem 3.3. *Let S be a semihyperring and μ be a Q-fuzzy subset of S. Then μ is a Q-fuzzy left (resp. right) hyperideal of S if and only if the set μ_t is a left (resp. right) hyperideal of S for all $t \in [0, 1]$, where it is nonempty.*

Proof. Assume that μ is a Q-fuzzy left hyperideal of S. Let $a, b \in \mu_t, s \in S$ and $q \in Q$. Then $\mu(a, q) \geq t$ and $\mu(b, q) \geq t$. For every $z \in a + b$, $\inf\limits_{z \in a+b} \mu(z, q) \geq \min\{\mu(a, q), \mu(b, q)\} \geq t$; that is, $z \in \mu_t$ for all $z \in a + b$. This implies that $a + b \subseteq \mu_t$. Next, for any $z \in sa$, $\inf\limits_{z \in sa} \mu(z, q) \geq \mu(a, q) \geq t$. It follows that $sa \subseteq \mu_t$. Hence, μ_t is a left hyperideal of S.

Conversely, assume that all nonempty level set μ_t is a left hyperideal of S. Let $x, y \in S$ and $q \in Q$. Choose $\min\{\mu(x, q), \mu(y, q)\} = t_0$ for some $t_0 \in [0, 1]$. Then $\mu(x, q) \geq t_0$ and $\mu(y, q) \geq t_0$, so $x, y \in \mu_{t_0}$. Thus, $x+y \subseteq \mu_{t_0}$, that is, for every $z \in x + y$, $\inf\limits_{z \in x+y} \mu(z, q) \geq t_0 = \min\{\mu(x, q), \mu(y, q)\}$. Let $\mu(x, q) = s_0$ for some $s_0 \in [0, 1]$. Then $x \in \mu_{s_0}$. It follows that $sx \subseteq \mu_{s_0}$. Hence, $\inf\limits_{z \in sx} \mu(z, q) \geq s_0 = \mu(x, q)$ for all $z \in sx$. Therefore, μ is a Q-fuzzy left hyperideal of S. $\square$

Theorem 3.4. *If μ and ν are Q-fuzzy left (resp. right) hyperideals of a semihyperring S, then $\mu \cap \nu$ is also a Q-fuzzy left (resp. right) hyperideal of S.*

Proof. Assume that μ and ν are Q-fuzzy left (resp. right) hyperideals of a semihyperring S. Let $x, y \in S$ and $q \in Q$. Then

$$\begin{aligned}\min\{(\mu\cap\nu)(x,q),(\mu\cap\nu)(y,q)\} &= \min\{\min\{\mu(x,q),\nu(x,q)\},\min\{\mu(y,q),\nu(y,q)\}\}\\ &= \min\{\min\{\mu(x,q),\mu(y,q)\},\min\{\nu(x,q),\nu(y,q)\}\}\\ &\leq \min\{\inf_{z\in x+y}\mu(z,q),\ \inf_{z\in x+y}\nu(z,q)\}\\ &= \inf_{z\in x+y}(\mu\cap\nu)(z,q),\end{aligned}$$

$$\begin{aligned}(\mu\cap\nu)(y,q) &= \min\{\mu(y,q),\nu(y,q)\}\\ &\leq \min\{\inf_{z\in xy}\mu(z,q),\ \inf_{z\in xy}\nu(z,q)\} = \inf_{z\in xy}(\mu\cap\nu)(z,q).\end{aligned}$$

Hence, $\mu\cap\nu$ is a Q-fuzzy left hyperideal of S. □

Theorem 3.5. *Let μ and ν be Q-fuzzy left (resp. right) hyperideals of a semihyperring S such that $\mu \subseteq \nu$ or $\nu \subseteq \mu$. Then $\mu\cup\nu$ is a Q-fuzzy left (resp. right) hyperideal of S.*

Proof. Assume that $\nu \subseteq \mu$. Let $x, y \in S$ and $q \in Q$. Then

$$\begin{aligned}\inf_{z\in x+y}(\mu\cup\nu)(z,q) &= \inf_{z\in x+y}\{\max\{\mu(z,q),\nu(z,q)\}\}\\ &= \max\{\inf_{z\in x+y}\mu(z,q),\ \inf_{z\in x+y}\nu(z,q)\}\\ &\geq \max\{\min\{\mu(x,q),\mu(y,q)\},\min\{\nu(x,q),\nu(y,q)\}\}\\ &= \min\{\max\{\mu(x,q),\nu(x,q)\},\max\{\mu(y,q),\nu(y,q)\}\}\\ &= \min\{(\mu\cup\nu)(x,q),(\mu\cup\nu)(y,q)\}.\end{aligned}$$

In general, $\max\{\min\{\ \}\} \leq \min\{\max\{\ \}\}$. Suppose for this case

$$\begin{aligned}&\max\{\min\{\mu(x,q),\mu(y,q)\},\min\{\nu(x,q),\nu(y,q)\}\}\\ &\quad\neq \min\{\max\{\mu(x,q),\mu(y,q)\},\max\{\nu(x,q),\nu(y,q)\}\}.\end{aligned}$$

Then there exists $r \in [0,1]$ such that

$$\begin{aligned}&\max\{\min\{\mu(x,q),\mu(y,q)\},\min\{\nu(x,q),\nu(y,q)\}\}\\ &\quad< r < \min\{\max\{\mu(x,q),\mu(y,q)\},\max\{\nu(x,q),\nu(y,q)\}\}.\end{aligned}$$

Thus, $r < \min\{\mu(x,q),\mu(y,q)\}$. On the other hand, $\min\{\mu(x,q),\mu(y,q)\} < r$, which is a contradiction. Similarly, we have

$$\begin{aligned}\inf_{z\in xy}(\mu\cup\nu)(z,q) &= \inf_{z\in xy}\{\max\{\mu(z,q),\nu(z,q)\}\} = \max\{\inf_{z\in xy}\mu(z,q),\ \inf_{z\in xy}\nu(z,q)\}\\ &\geq \max\{\mu(y,q),\nu(y,q)\} = (\mu\cup\nu)(y,q).\end{aligned}$$

Therefore, $\mu\cup\nu$ is a Q-fuzzy left hyperideal of S. □

Let X and Y be any two nonempty sets and μ be a Q-fuzzy subset of X. Define a Q-fuzzy subset ν of Y by letting $y \in Y$,

$$\nu(y,q) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x,q) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise.} \end{cases}$$

We call ν the *image* of μ under f denoted by $f(\mu)$. Conversely, for a Q-fuzzy subset ν of $f(X)$, define a Q-fuzzy subset μ of X by $\mu(x,q) = \nu(f(x),q)$ for all $x \in X$, and we call μ the *preimage* of ν under f denoted by $f^{-1}(\nu)$.

Theorem 3.6. *Let R and S be semihyperrings and $f : R \to S$ be a homomorphism. If ν is a Q-fuzzy left (resp. right) hyperideal of S, then $f^{-1}(\nu)$ is a Q-fuzzy left (resp. right) hyperideal of R.*

Proof. Assume that ν is a Q-fuzzy left hyperideal of S. Let $x, y \in R$ and $q \in Q$. Then

$$\begin{aligned} \inf_{z \in x+y} f^{-1}(\nu)(z,q) &= \inf_{f(z) \in f(x)+f(y)} \nu(f(z),q) \geq \min\{\nu(f(x),q), \nu(f(y),q)\} \\ &= \min\{f^{-1}(\nu)(x,q), f^{-1}(\nu)(y,q)\}, \end{aligned}$$

$$\inf_{z \in xy} f^{-1}(\nu)(z,q) = \inf_{f(z) \in f(x)f(y)} \nu(f(z),q) \geq \nu(f(y),q) = f^{-1}(\nu)(y,q).$$

Hence, $f^{-1}(\nu)$ is a Q-fuzzy left hyperideal of S. □

Let μ be a Q-fuzzy subset of a nonempty set X, $\alpha \in [0, 1 - \sup\limits_{x \in X} \mu(x,q)]$ and $\beta \in [0,1]$. The mapping $\mu_\alpha^T : X \times Q \to [0,1]$, $\mu_\beta^M : X \times Q \to [0,1]$ and $\mu_{\beta,\alpha}^{MT} : X \times Q \to [0,1]$, where Q is a nonempty set, are called a *Q-fuzzy translation*, a *Q-fuzzy multiplication* and a *Q-fuzzy magnified translation* of μ, respectively, if $\mu_\alpha^T(x,q) = \mu(x,q) + \alpha$, $\mu_\beta^M = \beta\mu(x,q)$ and $\mu_{\beta,\alpha}^{MT} = \beta\mu(x,q) + \alpha$ for all $x \in X$ and $q \in Q$.

Theorem 3.7. *Let S be a semihyperring, μ be a Q-fuzzy subset of S, $\alpha \in [0, 1 - \sup\limits_{x \in X} \mu(x,q)]$ and $\beta \in (0,1]$. Suppose that $\mu_{\beta,\alpha}^{MT}$ is a Q-fuzzy magnified translation of μ, with respect to α and β. Then μ is a Q-fuzzy left (resp. right) hyperideal of S if and only if $\mu_{\beta,\alpha}^{MT}$ is a Q-fuzzy left (resp. right) hyperideal of S.*

Proof. Assume that μ is a Q-fuzzy left hyperideal of S. Let $x, y \in S$ and $q \in Q$. Then

$$\begin{aligned}\inf_{z\in x+y}\mu^{MT}_{\beta,\alpha}(z,q) &= \inf_{z\in x+y}(\beta\mu(z,q)+\alpha) = \beta(\inf_{z\in x+y}\mu(z,q))+\alpha \\ &\geq \beta\min\{\mu(x,q),\mu(y,q)\}+\alpha = \min\{\beta\mu(x,q)+\alpha, \beta\mu(y,q)+\alpha\} \\ &= \min\{\mu^{MT}_{\beta,\alpha}(x,q), \mu^{MT}_{\beta,\alpha}(y,q)\},\end{aligned}$$

$$\begin{aligned}\inf_{z\in xy}\mu^{MT}_{\beta,\alpha}(z,q) &= \inf_{z\in xy}(\beta\mu(z,q)+\alpha) = \beta(\inf_{z\in xy}\mu(z,q))+\alpha \\ &\geq \beta\mu(y,q)+\alpha = \mu^{MT}_{\beta,\alpha}(y,q).\end{aligned}$$

Hence, $\mu^{MT}_{\beta,\alpha}$ is a Q-fuzzy left hyperideal of S.

Conversely, assume that $\mu^{MT}_{\beta,\alpha}$ is a Q-fuzzy left (resp. right) hyperideal of S. Let $x, y \in S$ and $q \in Q$. Then

$$\begin{aligned}\beta(\inf_{z\in x+y}\mu(z,q))+\alpha &= \inf_{z\in x+y}(\beta\mu(z,q)+\alpha) = \inf_{z\in x+y}\mu^{MT}_{\beta,\alpha}(z,q) \\ &\geq \min\{\mu^{MT}_{\beta,\alpha}(x,q), \mu^{MT}_{\beta,\alpha}(y,q)\} \\ &= \min\{\beta\mu(x,q)+\alpha, \beta\mu(y,q)+\alpha\} \\ &= \beta\min\{\mu(x,q),\mu(y,q)\}+\alpha,\end{aligned}$$

$$\beta(\inf_{z\in xy}\mu(z,q))+\alpha = \inf_{z\in xy}\mu^{MT}_{\beta,\alpha}(z,q) \geq \mu^{MT}_{\beta,\alpha}(y,q) = \beta\mu(y,q)+\alpha.$$

Since $\alpha \geq 0$ and $\beta > 0$, we have $\inf_{z\in x+y}\mu(z,q) \geq \min\{\mu(x,q),\mu(y,q)\}$ and $\inf_{z\in xy}\mu(z,q) \geq \mu(y,q)$. Therefore, μ is a Q-fuzzy left hyperideal of S. □

Corollary 3.8. *Let S be a semihyperring, μ be a Q-fuzzy subset of S, $\alpha \in [0, 1 - \sup_{x\in X}\mu(x,q)]$ and $\beta \in (0,1]$. Suppose that μ^T_α is a Q-fuzzy translation and μ^M_β is a Q-fuzzy multiplication of μ, with respect to α and β, respectively. Then the following are equivalent:*

(i) *μ is a Q-fuzzy left (resp. right) hyperideal of S;*

(ii) *μ^T_α is a Q-fuzzy left (resp. right) hyperideal of S;*

(i) *μ^M_β is a Q-fuzzy left (resp. right) hyperideal of S;*

Let S be a semihyperring. We denote by $\mathcal{FH}_L(S)$ (resp. $\mathcal{FH}_R(S)$) the set of all Q-fuzzy left (resp. right) hyperideals of S, and we assume that $(\mathcal{FH}_L(S), \subseteq)$ (resp. $(\mathcal{FH}_R(S), \subseteq)$) is a totally ordered set by the set inclusion.

Theorem 3.9. *Let S be a semihyperring. Then $(\mathcal{FH}_L(S), \subseteq, \cap, \cup)$ forms a lattice.*

Proof. Let μ and ν be any two elements in $\mathcal{FH}_L(S)$. By Theorems 3.4 and 3.5, we have that $\mu \cap \nu$ and $\mu \cup \nu$ are Q-fuzzy left hyperideals of S. Thus, $\mu \cap \nu$ is the greatest lower bound and $\mu \cup \nu$ is the least upper bound of μ and ν. Hence, $(\mathcal{FH}_L(S), \subseteq, \cap, \cup)$ is a lattice. □

Similarly one can prove the following

Theorem 3.10. *Let S be a semihyperring. Then $(\mathcal{FH}_R(S), \subseteq, \cap, \cup)$ forms a lattice.*

Theorem 3.11. *Let S be a semihyperring and $\mu, \nu, \lambda \in \mathcal{FH}_L(S)$ (resp. $\mathcal{FH}_R(S)$). Then $\mu \cap (\nu \cup \lambda) = (\mu \cap \nu) \cup (\mu \cap \lambda)$ and $\mu \cup (\nu \cap \lambda) = (\mu \cup \nu) \cap (\mu \cup \lambda)$.*

Proof. Let $x \in S$ and $q \in Q$. We have

$$\begin{aligned}(\mu \cap (\nu \cup \lambda))(x, q) &= \min\{\mu(x, q), (\nu \cup \lambda)(x, q)\} \\ &= \min\{\mu(x, q), \max\{\nu(x, q), \lambda(x, q)\}\} \\ &= \max\{\min\{\mu(x, q), \nu(x, q)\}, \min\{\mu(x, q), \lambda(x, q)\}\} \\ &= \max\{(\mu \cap \nu)(x, q), (\mu \cap \lambda)(x, q)\} \\ &= ((\mu \cap \nu) \cup (\mu \cap \lambda))(x, q).\end{aligned}$$

Hence, $\mu \cap (\nu \cup \lambda) = (\mu \cap \nu) \cup (\mu \cap \lambda)$. Similarly, we can show that $\mu \cup (\nu \cap \lambda) = (\mu \cup \nu) \cap (\mu \cup \lambda)$. □

Corollary 3.12. *Let S be a semihyperring. Then $(\mathcal{FH}_L(S), \subseteq, \cap, \cup)$ and $(\mathcal{FH}_R(S), \subseteq, \cap, \cup)$ form distributive lattices.*

A Q-fuzzy subset μ of a nonempty set X is called *normal* if there exist $x \in X$ and $q \in Q$ such that $\mu(x, q) = 1$. Let μ be a Q-fuzzy subset of a nonempty set X. Define a Q-fuzzy subset μ^+ of X by $\mu^+(x, q) = \mu(x, q) + 1 - \mu(0, q)$ for all $x \in X$ and $q \in Q$.

Theorem 3.13. *Let S be a semihyperring. If μ is a Q-fuzzy left (resp. right) hyperideal of S, then μ^+ is also a Q-fuzzy left (resp. right) hyperideal of S containing μ.*

Proof. Assume that μ is a Q-fuzzy left hyperideal of S. Let $x, y \in S$ and $q \in Q$. Then

$$\begin{aligned}\inf_{z\in x+y} \mu^+(z,q) &= \inf_{z\in x+y}(\mu(z,q)+1-\mu(0,q)) = \inf_{z\in x+y}\mu(z,q)+1-\mu(0,q)\\ &\geq \min\{\mu(x,q),\mu(y,q)\}+1-\mu(0,q)\\ &= \min\{\mu(x,q)+1-\mu(0,q),\mu(y,q)+1-\mu(0,q)\}\\ &= \min\{\mu^+(x,q),\mu^+(y,q)\},\end{aligned}$$

$$\begin{aligned}\inf_{z\in xy} \mu^+(z,q) &= \inf_{z\in xy}(\mu(z,q)+1-\mu(0,q)) = \inf_{z\in xy}\mu(z,q)+1-\mu(0,q)\\ &\geq \mu(y,q)+1-\mu(0,q) = \mu^+(y,q).\end{aligned}$$

Hence, μ^+ is a Q-fuzzy left hyperideal of S. Now, $\mu^+(0,q) = \mu(0,q) + 1 - \nu(0,q) = 1$. Since $1 - \mu(0,q) \geq 0$, $\mu(x,q) \leq \mu(x,q) + 1 - \mu(0,q) = \mu^+(x,q)$ for all $x \in S, q \in Q$. So, $\mu \subseteq \mu^+$. Therefore, μ^+ is a normal Q-fuzzy left hyperideal of S containing μ. □

Let $\mathcal{FHN}_L(S)$ (resp. $\mathcal{FHN}_R(S)$) denotes the set of all normal Q-fuzzy left (resp. right) hyperideals of a semihyperring S. Then $\mathcal{FHN}_L(S)$ and $\mathcal{FHN}_R(S)$ are posets under inclusion.

Theorem 3.14. *Let μ be a normal Q-fuzzy left (resp. right) hyperideal of a semihyperring S such that nonconstant. If μ is a maximal element of $\mathcal{FHN}_L(S)$ (resp. $\mathcal{FHN}_R(S)$), then μ takes only two values* 0 *and* 1.

Proof. Assume that μ is a maximal element of $\mathcal{FHN}_L(S)$. Since μ is normal, $\mu(0,q) = 1$. Let $x_0 \in S$ such that $x_0 \neq 0$ and $\mu(x_0,q) \neq 1$. We want to show that $\mu(x_0,q) = 0$. Suppose that $\mu(x_0,q) \neq 0$. Then $0 < \mu(x_0,q) < 1$. Define a Q-fuzzy subset ν of S by

$$\nu(x,q) = \frac{1}{2}[\mu(x,q) + \mu(x_0,q)] \text{ for all } x \in S, q \in Q.$$

Thus, ν is well-defined. Let $x, y \in S$ and $q \in Q$. We have

$$\begin{aligned}\inf_{z\in x+y} \nu(z,q) &= \inf_{z\in x+y}\{\frac{1}{2}[\mu(z,q)+\mu(x_0,q)]\}\\ &\geq \frac{1}{2}[\min\{\mu(x,q),\mu(y,q)\}+\mu(x_0,q)]\\ &= \min\{\frac{1}{2}[\mu(x,q)+\mu(x_0,q)],\frac{1}{2}[\mu(y,q)+\mu(x_0,q)]\}\\ &= \min\{\nu(x,q),\nu(y,q)\},\end{aligned}$$

$$\inf_{z\in xy}\nu(z,q)=\inf_{z\in xy}\{\frac{1}{2}[\mu(z,q)+\mu(x_0,q)]\}\geq\frac{1}{2}[\mu(y,q)\}+\mu(x_0,q)]=\nu(y,q).$$

Hence, ν is a Q-fuzzy left hyperideal of S. By Theorem 3.13, ν^+ is a normal Q-fuzzy left hyperideal of S. Then, for every $x\in S, q\in Q$,

$$\begin{aligned}\nu^+(x,q)=\nu(x,q)+1-\nu(0,q)&=\frac{1}{2}[\mu(x,q)+\mu(x_0,q)]+1-\frac{1}{2}[\mu(0,q)+\mu(x_0,q)]\\&=\frac{1}{2}[\mu(x,q)+1].\end{aligned}\tag{3.1}$$

In particular,

$$\nu^+(0,q)=\frac{1}{2}[\mu(0,q)+1]=1 \text{ and } \nu^+(x_0,q)=\frac{1}{2}[\mu(x_0,q)+1].\tag{3.2}$$

From (3.1), we have that ν^+ is nonconstant, since μ is nonconstant. From (3.2), we see that $\mu(x_0,q)<\mu^+(x_0,q)$. By the maximality of μ, we get a contradiction. □

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References

[1] M. Akram, A. A. Sathakathulla, On Q-fuzzy prime bi-Γ-ideals of Γ-semigroups, International Journal of Algebra and Statistics, **1**, no. 2, (2012), 123-129.

[2] P. Corsini, Prolegomena of hypergroup theory, Aviani Editore, 1993.

[3] P. Corsini, V. Leoreanu, Applications of hyperstructure theory, Kluwer Academic Publishers, Dordrecht, Hardbound, 2003.

[4] I. Cristea, Hyperstructures and fuzzy sets endowed with two membership functions, Fuzzy Sets and Systems, **160**, no. 8, (2009), 1114-1124.

[5] B. Davvaz, Fuzzy Krasner (m,n)-hyperrings, Computers and Mathematics with Applications, **59**, no. 12, (2010), 3879-3891.

[6] B. Davvaz, P. Corsini, V. Leoreanu-Fotea, Fuzzy n-ary subpolygroups, Computers and Mathematics with Applications, **57**, no. 1, (2009), 141-152.

[7] B. Davvaz, V. Leoreanu-Fotea, Hyperring theorey and applications, International Academic Press, USA, 2007.

[8] B. Davvaz, J. Zhan, K. P. Shum, Generalized fuzzy H_v-submodules endowed with interval valued membership functions, Information Sciences, **178**, no. 15, (2008), 3147-3159.

[9] X. Huang, Y. Yin, J. Zhan, Characterizations of semihyperrings by their $(\epsilon_\gamma, \epsilon_\gamma \vee q_\delta)$-fuzzy hyperideals, Journal of Applied Mathematics, **2013**, (2013), 1-13.

[10] O. Kazanc, B. Davvaz, S. Yamak, Fuzzy n-ary polygroups related to fuzzy points, Neural Computing and Applications, **19**, (2010), 649655.

[11] N. Kehayopulu, M. Tsingelis, Fuzzy ideals in ordered semigroups, Quasigroups and Related Systems, **15**, (2007), 279-289.

[12] S. Lekkoksung, On Q-fuzzy bi-Γ-ideals in Γ-semigroups, International Journal of Mathematical Analysis, **6**, no. 8, (2012), 365-370.

[13] S. Lekkoksung, On Q-fuzzy ideals in ordered semigroups, International Journal of Pure and Applied Mathematics, **92**, no. 3, (2014), 369-379.

[14] V. Leoreanu-Fotea, B. Davvaz, Fuzzy hyperrings, Fuzzy Sets and Systems, **160**, no. 16, (2009), 2366-2378.

[15] V. Leoreanu Fotea, Fuzzy hypermodules, Computers and Mathematics with Applications, **57**, no. 3, (2009), 466-475.

[16] S. K. Majumber, On Q-fuzzy ideals in Γ-semigroups, World Academy of Science, Engineering of Technology, **60**, (2011), 1443-1447.

[17] D. Mandal, Q-fuzzy ideal of ordered Γ-semiring, Journal of New Theory, **3**, (2015), 89-97.

[18] D. Mandal, Q-fuzzy ideal of regular ordered Γ-semirings, Global Journal of Pure and Applied Mathematics, **13**, no. 2, (2017), 367-376.

[19] F. Marty, Sur une generalization de la notion de group, in Proceedings of the 8th Congress des Mathematiciens Scandinave, Stockholm, Sweden, (1934), 45-49.

[20] T. Vougiouklis, On some representation of hypergroups, Ann. Sci. Univ. Clermont-Ferrand II Math, 26, (1990), 2129.

[21] T. Vougiouklis, Hyperstructures and their representations, Handronic press inc., Florida, 1994.

[22] Y. Yin, B. Davvaz, J. Zhan, A fuzzy view of Γ-hyperrings, Neural Computing and Applications, **21**, (2012), 979-992.

[23] Y. Yin, J. Zhan, New types of fuzzy filters of BL-algebras, Computers and Mathematics with Applications, **60**, no. 7, (2010), 2115-2125, 2010.

[24] L. A. Zadeh, Fuzzy sets, Information and Control, **8**, no. 3, (1965), 338-353.

[25] J. Zhan, B. Davvaz, K. P. Shum, On fuzzy isomorphism theorems of hypermodules, Soft Computing, **11**, (2007), 1053-1057.

[26] J. Zhan, B. Davvaz, K. P. Shum, Generalized fuzzy hyperideals of hyperrings, Computers and Mathematics with Applications, **56**, no. 7, (2008), 1732-1740.