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A note on angular geometric graphs

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Abstract

In this short note we first define angular geometric graphs, angular degrees and geometric degrees in graph theory as follows. An angular geometric graph denoted as AGG is a graph in which given angles between vertices and edges can not be changed. If the angles are not given specifically in an angular geometric graph, all the angles are considered to be equal. The sum of the sines of the all angles of a vertex v is called the angular degree of v and denoted as $ang(v)$. The sum of the degree of the vertex v and the angle degree of the vertex v is called the geometric degree of v and denoted as $geom(v)$. The aim of this study is to investigate the geometric degrees of the Cartesian product of two paths and a path with a cycle.

1 Introduction

Graph theory is one of the important branches of applied mathematics. Graphs are used to model many real world problems in physics, chemistry, engineering and biological sciences. In chemistry, models of the molecules are exactly corresponding to simple graphs in graph theory. The angles between the atoms (vertices) and bonds (edges) are important in chemistry but are not important in graph theory. In this respect, we define a special

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graph class angular geometric graphs in which the angles within this graph are important and unalterable.

We consider only connected graphs throughout this paper. For undefined terminology, we referred to the reference [1]. Let G be a graph with the vertex set $V(G)$, the edge set $E(G)$ and $v \in V(G)$. The degree of a vertex $v \in V(G)$, $deg(v)$, equals the number of edges incident to v that is the cardinality of the set $N(v) = \{u|uv \in E(G)\}$. P_n and C_n showed the path and cycle, respectively. Let A and B be two non-empty sets. Then the Cartesian product of these sets is the set $A\square B = \{(a,b)|a \in A \text{ and } b \in B\}$. The Cartesian product of two graphs G and H have the vertex set $V(G)\square V(H)$. The edge set of the Cartesian product of G and H is $E(G\square H) = \{(a,b)(c,d)|a = c$ and $bd \in E(H)$ or $b = d$ and $ac \in E(G)\}$. The following Lemma 1.1 gives the degrees in Cartesian product of two graphs. For more details about product graphs, we referred to the references [2, 3].

Lemma 1.1. *Let G and H be two graphs. Then; $deg_{G\square H}(a,b) = deg(a) + deg(b)$.*

In the literature geometric graphs are defined as follows [4, 5].

Definition 1.2. *A geometric graph is a graph whose vertices are points in the plane and whose edges are straight-line segments between the points.*

For more information about geometric graphs see in [4, 5] and references therein. The following definition, we expand this definition as follows.

Definition 1.3. *An angular geometric graph denoted as AGG is a graph in which given angles between vertices and edges can not be changed. If the angles are not given spesifically in an angular geometric graph, all the angles are considered to be equal.*

The difference between geometric and angular geometric graphs is: There is no need to be the line segments of the edges of an angular geometric graph. Since we do not find the concept of geometric degree in the literature, we now define the concept of geometric degree.

Definition 1.4. *Let AGG be an angular geometric graph and $v \in GG$. The sum of the sines of the all angles of the vertex v is called the angular degree of v and denoted as $ang(v)$.*

Definition 1.5. *Let AGG be an angular geometric graph and $v \in AGG$. The sum of the degree of the vertex v and the angular degree of the vertex v is called the geometric degree of v and denoted as $geom(v)$. That is $geom(v) = deg(v) + ang(v)$.*

2 The geometric degrees in the Cartesian product of two paths and a path with a cycle

Proposition 2.1. *Let P_m and P_n be two paths with the vertex sets $V(P_m) = \{v_1, v_2, ..., v_m\}$ and $V(P_n) = \{u_1, u_2, ..., u_n\}$ for $m, n \geq 4$. Then the geometric degrees of the vertices of $P_m \square P_n$ are;*

$$geom(v_i, u_j) = \begin{cases} 2, & \text{for } i = 1, m \text{ and } j = 1, n, \\ 3 + 3\sqrt{3}/2, & \text{for } i = 1 \text{ and } 2 \leq j \leq n-1 \text{ or } j = 1 \text{ and } 2 \leq i \leq m-1, \\ 8, & \text{for } 2 \leq i \leq m-1 \text{ and } 2 \leq j \leq n-1. \end{cases}$$

Proof. *Case 1:* Let $i = 1, m$ and $j = 1, n$. In this case $deg(v_i, u_j) = 2$ and there is only one angle between the vertex and adjacent edges. Therefore we must consider the degree of the angle 180 degree. Hence $ang(v_i, u_j) = 0$. And then; $geom(v_i, u_j) = deg(v_i, u_j) + ang(v_i, u_j) = 2 + 0 = 2$.
Case 2: Let $i = 1$ and $2 \leq j \leq n-1$ or $j = 1$ and $2 \leq i \leq m-1$. In this case $deg(v_i, u_j) = 3$ and there are three equal angles between the vertex and adjacent edges. Therefore we must consider the degrees of the angles 120 degree. Hence $ang(v_i, u_j) = 3\sqrt{3}/2$. And then; $geom(v_i, u_j) = deg(v_i, u_j) + ang(v_i, u_j) = 3 + 3\sqrt{3}/2$.
Case 3: Let $2 \leq i \leq m-1$ and $2 \leq j \leq n-1$. In this case $deg(v_i, u_j) = 4$ and there are four angles between the vertex and adjacent edges. Therefore we must consider the degrees of the angles 90 degree. Hence $ang(v_i, u_j) = 4$. And then; $geom(v_i, u_j) = deg(v_i, u_j) + ang(v_i, u_j) = 4 + 4 = 8$. □

Proposition 2.2. *Let P_m a path and C_n be a cycle with the vertex sets $V(P_m) = \{v_1, v_2, ..., v_m\}$ and $V(C_n) = \{u_1, u_2, ..., u_n\}$ for $m, n \geq 4$. Then the geometric degrees of the vertices of $P_m \square C_n$ are;*

$$geom(v_i, u_j) = \begin{cases} 3 + 3\sqrt{3}/2, & \text{if } i = 1, m \text{ and } 1 \leq j \leq n, \\ 8, & \text{if } 2 \leq i \leq m-1 \text{ and } 1 \leq j \leq n\ . \end{cases}$$

Proof. *Case 1:* Let $i = 1, m$ and $1 \leq j \leq n$. In this case $deg(v_i, u_j) = 3$ and there are three equal angles between the vertex and adjacent edges. Therefore we must consider the degrees of the angles 120 degree. Hence $ang(v_i, u_j) = 3\sqrt{3}/2$. And then; $geom(v_i, u_j) = deg(v_i, u_j) + ang(v_i, u_j) = 3 + 3\sqrt{3}/2$.
Case 2: Let $2 \leq i \leq m-1$ and $1 \leq j \leq n$. In this case $deg(v_i, u_j) = 4$ and there are four angles between the vertex and adjacent edges. Therefore we must consider the degrees of the angles 90 degree. Hence $ang(v_i, u_j) = 4$. And then; $geom(v_i, u_j) = deg(v_i, u_j) + ang(v_i, u_j) = 4 + 4 = 8$. □

3 Conclusion

To investigate the geometric degrees in other type graph operations are interesting problems for the future studies.

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