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Interval Valued Fuzzy Ideal Extensions of Ternary Semigroups

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Abstract

The notion of fuzzy sets was introduced by Zadeh in 1965. Applications of the fuzzy set theory appear in various fields. The theory of fuzzy sets was studied in various kinds of algebraic systems. In 1975, Zadeh made an extension of the concept of a fuzzy subset by an interval valued fuzzy subset. A ternary semigroup is a nonempty set together with a ternary multiplication which is associative. In this paper, we study interval valued fuzzy ideal extensions of ternary semigroups.

1 Introduction

A ternary semigroup is a nonempty set together with a ternary multiplication which is associative. Every semigroup can be reduced to a ternary semigroup but Banach showed that a ternary semigroup does not necessarily reduce to a semigroup. Lehmer [7] investigated certain triple systems called triplexes

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which turn out to be commutative ternary groups. He has also defined regular ternary semigroups. Vagner [14] studied semiheaps which are ternary systems with a different type of associativity. Los [8] showed that every ternary semigroup can be imbedded in a semigroup. Sioson [13] studied ternary semigroups with special reference to ideals and radicals. He extended to ternary semigroups various well known concepts concerning ideals such as primality, semiprimality etc. Santiago and Sri Bala [12] studied regularity conditions in a ternary semigroup.

The concept of fuzzy subsets was given by Zadeh [17] in 1965. Since its inception, the theory has developed in many directions and found applications in a wide variety of fields. Rosenfeld [10] applied the concept of Zadeh to define fuzzy subgroups and fuzzy ideals. Kuroki ([4],[5] and [6]) defined a fuzzy semigroup and various kinds of fuzzy ideals in semigroups and characterized them. Fuzzy ideals in ternary semirings were studied in [2] and [3]. In [9] and [11], fuzzy ideal of ternary semigroups were studied. Zadeh [18] made an extension of the concept of fuzzy subsets by an interval valued fuzzy subset. Interval valued fuzzy subsets have many application in several areas. In [16], interval valued fuzzy ideal in LA-semigroups were studied by Yaqoob, Chinram, Ghareeb and Aslam.

Ahsan, Saifullah and Khan [1] have shown that a semigroup S is semisimple if and only if each proper fuzzy ideal of S is the intersection of prime ideal of S. Xie [15] introduced the concepts of the extension of a fuzzy ideal of S and 3-prime fuzzy ideal of S. He studied some properties of the extension of a fuzzy ideal of S and the relationship between prime ideal and 3-prime fuzzy ideal of S by means of the extensions of fuzzy ideals of S.

In this paper, we introduce the concepts of the extension of an interval value fuzzy ideal of a ternary semigroup T.

2 Preliminaries

A nonempty set T is called a *ternary semigroup* if there exists a ternary operation $T \times T \times T \to T$, written as $(x_1, x_2, x_3) \mapsto x_1x_2x_3$ satisfying the following identity for any $x_1, x_2, x_3, x_4, x_5 \in T$,

$$(x_1x_2x_3)x_4x_5 = x_1(x_2x_3x_4)x_5 = x_1x_2(x_3x_4x_5).$$

Any semigroup can be reduced to a ternary semigroup but a ternary semigroup does not necessarily reduce to a semigroup, for example, $\mathbb{Z}^-$ is a ternary semigroup while $\mathbb{Z}^-$ is not a semigroup under the multiplication over integers. Let T be a ternary semigroup. For nonempty subsets A, B and C of T, let $ABC := \{abc|a \in A, b \in B \text{ and } c \in C\}$. A nonempty subset S of T is called a *ternary subsemigroup* if $SSS \subseteq S$. A nonempty subset A of a ternary semigroup T is called a *left ideal* of T if $TTA \subseteq A$, a *right ideal* of T if $ATT \subseteq A$ and a *middle ideal* or a *lateral ideal* of T if $TAT \subseteq A$. If A is a left, right and middle ideal of T, A is called an *ideal* of T. A nonempty subset A of T is called *prime* if for all $x, y, z \in T, xyz \in A \to x \in A$ or $y \in A$ or $z \in A$. A nonempty subset A of T is called *semiprime* if for all $x \in T, x^3 \in A \to x \in A$. A prime subset of T is a semiprime subset of T but a semiprime subset need not a prime subset.

Let $D[0,1]$ denote the family of all closed subintervals of $[0,1]$, that is

$$D[0,1] = \{[a^-, a^+]|a^-, a^+ \in [0,1] \text{ and } a^- \leq a^+\},$$

where the elements in $D[0,1]$ are called the *interval numbers* on $[0,1], 0 = [0,0]$ and $1 = [1,1]$. We consider two elements $a = [a^-, a^+]$ and $b = [b^-, b^+]$ in $D[0,1]$. Define the operations $\leq, =, <$, min and max in case of two elements in $D[0,1]$ as follow

(1) $a \leq b$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$,

(2) $a = b$ if and only if $a^- = b^-$ and $a^+ = b^+$,

(3) $a < b$ if and only if $a \leq b$ and $a \neq b$,

(4) $\min\{a, b\} = [\min\{a^-, b^-\}, \min\{a^+, b^+\}]$,

(5) $\max\{a, b\} = [\max\{a^-, b^-\}, \max\{a^+, b^+\}]$.

Let T be a set. A function $\tilde{f} : T \longrightarrow D[0,1]$ is called an *interval valued fuzzy subset* (for short, i-v fuzzy subset) on T. Let $\tilde{f}, \tilde{g}$ and $\tilde{h}$ be three i-v fuzzy subsets of a ternary semigroup T then $\tilde{f} \circ \tilde{g} \circ \tilde{h}$ is defined as

$$(\tilde{f} \circ \tilde{g} \circ \tilde{h})(x) = \begin{cases} \sup\limits_{x=abc} \min\{\tilde{f}(a), \tilde{g}(b), \tilde{h}(c)\} & \text{if } x = abc\ \exists a, b, c \in T \\ 0 & \text{otherwise.} \end{cases}$$

Let $IVF(T)$ denote the set of all i-v fuzzy subsets of S.

Proposition 2.1. *Let T be a ternary semigroup, then the set $(IVF(T), \circ)$ is a ternary semigroup.*

Let T be a ternary semigroup. A function $\tilde{f} : T \longrightarrow D[0,1]$ is an i-v fuzzy subset of T. The set of all i-v fuzzy subset of T with the relation

$$\tilde{f} \subseteq \tilde{g} \Longleftrightarrow \tilde{f}(x) \leq \tilde{g}(x), \forall x \in T$$

is a complete lattice where, for a nonempty family $\{\tilde{f}_i | i \in I\}$ of fuzzy subsets of T, the $\inf\{\tilde{f}_i | i \in I\}$ and the $\sup\{\tilde{f}_i | i \in I\}$ are the fuzzy subsets of T defined by

$$\inf\{\tilde{f}_i | i \in I\} : T \longrightarrow D[0,1], \quad x \longrightarrow \inf\{\tilde{f}_i(x) | i \in I\}.$$
$$\sup\{\tilde{f}_i | i \in I\} : T \longrightarrow D[0,1], \text{and} \quad x \longrightarrow \sup\{\tilde{f}_i(x) | i \in I\}.$$

Let T be a ternary semigroup and $A \subseteq T$. The characteristic function $\tilde{f}_A : T \to D[0,1]$ defined

$$\tilde{f}_A(x) := \begin{cases} [1,1] & \text{if } x \in A, \\ [0,0] & \text{if } x \notin A. \end{cases}$$

3 Interval valued fuzzy ideals

Definition 3.1. Let T be a ternary semigroup and $\tilde{f}$ be an i-v fuzzy subset of T.

(1) $\tilde{f}$ is called an *i-v fuzzy left ideal* of T if $\tilde{f}(xyz) \geq \tilde{f}(z), \forall x, y, z \in T$.

(2) $\tilde{f}$ is called an *i-v fuzzy right ideal* of T if $\tilde{f}(xyz) \geq \tilde{f}(x), \forall x, y, z \in T$.

(3) $\tilde{f}$ is called an *i-v fuzzy middle ideal* of T if $\tilde{f}(xyz) \geq \tilde{f}(y), \forall x, y, z \in T$.

(4) $\tilde{f}$ is called an *i-v fuzzy ideal* of T if it is an i-v fuzzy left, fuzzy right and fuzzy middle ideal of T, i.e. $\tilde{f}(xyz) \geq \max\{\tilde{f}(x), \tilde{f}(y), \tilde{f}(z)\}, \forall x, y, z \in T$.

Theorem 3.2. *Let T be a ternary semigroup and A be a nonempty subset of T. Then*

(1) A is a left ideal of T if and only if $\tilde{f}_A$ is an i-v fuzzy left ideal of T.

(2) *A is a right ideal of T if and only if $\tilde{f}_A$ is an i-v fuzzy right ideal of T.*

(3) *A is a middle ideal of T if and only if $\tilde{f}_A$ is an i-v fuzzy middle ideal of T.*

(4) *A is an ideal of T if and only if $\tilde{f}_A$ is an i-v fuzzy ideal of T.*

Definition 3.3. Let $\tilde{f}$ be an i-v fuzzy subset of T. $\tilde{f}$ is called *prime* if

$$\tilde{f}(xyz) \leq \max\{\tilde{f}(x), \tilde{f}(y), \tilde{f}(z)\}, \forall x, y, z \in T.$$

Theorem 3.4. *Let T be a ternary semigroup and A be a nonempty subset of T . Then I is a prime subset of T if and only if $\tilde{f}_I$ is a prime i-v fuzzy subset of T.*

Corollary 3.5. *Let T be a ternary semigroup and A be a nonempty subset of T. Then*

(1) *A is a prime left ideal of T if and only if $\tilde{f}_A$ is a prime i-v fuzzy left ideal of T.*

(2) *A is a prime right ideal of T if and only if $\tilde{f}_A$ is a prime i-v fuzzy right ideal of T.*

(3) *A is a prime middle ideal of T if and only if $\tilde{f}_A$ is a prime i-v fuzzy middle ideal of T.*

(4) *A is a prime ideal of T if and only if $\tilde{f}_A$ is a prime i-v fuzzy ideal of T.*

Definition 3.6. An i-v fuzzy subset $\tilde{f}$ of T is called *semiprime* if $\tilde{f}(a) \geq \tilde{f}(a^3), \forall a \in T$.

Theorem 3.7. *Let T be a ternary semigroup and A be a nonempty subset of T. Then A is semiprime of T if and only if $\tilde{f}_A$ is a semiprime i-v fuzzy subset of T.*

Corollary 3.8. *Let T be a ternary semigroup and A be a nonempty subset of T. Then*

(1) *A is a semiprime left ideal of T if and only if $\tilde{f}_A$ is a semiprime i-v fuzzy left ideal of T.*

(2) *A is a semiprime right ideal of T if and only if $\tilde{f}_A$ is a semiprime i-v fuzzy right ideal of T.*

(3) *A is a semiprime middle ideal of T if and only if $\tilde{f}_A$ is a semiprime i-v fuzzy middle ideal of T.*

(4) *A is a semiprime ideal of T if and only if $\tilde{f}_A$ is a semiprime i-v fuzzy ideal of T.*

4 Interval valued fuzzy ideal extensions

Definition 4.1. Let T be a ternary semigroup, $\tilde{f}$ an i-v fuzzy subset of T and $x, y \in T$. The i-v fuzzy subset $< x, y, \tilde{f} >: T \longrightarrow D[0,1]$ defined by

$$< x, y, \tilde{f} > (z) = \tilde{f}(xyz) \text{ for all } z \in T$$

is called the *extension* of $\tilde{f}$ by (x, y).

Proposition 4.2. *Let T be a commutative ternary semigroup. If $\tilde{f}$ is an i-v fuzzy ideal of T and $x, y \in T$, then the extension of $\tilde{f}$ by (x, y) is an i-v fuzzy ideal of T.*

Proof. Let $a, b, c \in T$. Since T is commutative and $\tilde{f}$ is an i-v fuzzy left ideal of T, $< x, y, \tilde{f} > (abc) = \tilde{f}(xy(abc)) = \tilde{f}(ab(xyc)) \geq \tilde{f}(xyc) = < x, y, \tilde{f} > (c)$. Thus $< x, y, \tilde{f} >$ is an i-v fuzzy left ideal of T. Since $\tilde{f}$ is an i-v fuzzy right ideal of T, $< x, y, \tilde{f} > (abc) = \tilde{f}(xy(abc)) = \tilde{f}((xya)bc) \geq \tilde{f}(xya) =< x, y, \tilde{f} > (a)$. Thus $< x, y, \tilde{f} >$ is an i-v fuzzy right ideal of T. Since T is commutative and $\tilde{f}$ is an i-v fuzzy middle ideal of T, $< x, y, \tilde{f} > (abc) = \tilde{f}(xy(abc)) = \tilde{f}(a(xyb)c) \geq \tilde{f}(xyb) =< x, y, \tilde{f} > (b)$. Thus $< x, y, \tilde{f} >$ is an i-v fuzzy middle ideal of T. Hence $< x, y, \tilde{f} >$ is an i-v fuzzy ideal of T. $\square$

If T is a ternary semigroup and $\tilde{f}$ is an i-v fuzzy subset of T, the support of $\tilde{f}$ is defined by

$$supp\tilde{f} := \{x \in T | \tilde{f}(x) > [0,0]\}.$$

Proposition 4.3. *Let T be a ternary semigroup, $\tilde{f}$ be an i-v fuzzy ideal of T, $x, y \in T$. Then*

1. *$\tilde{f} \subseteq < x, y, \tilde{f} >$.*
2. *If T is commutative, then*

$$< x^n, y^n, \tilde{f} > \subseteq < x^{n+2}, y^{n+2}, \tilde{f} >$$

   for all odd positive integers n.
3. *If $\tilde{f}(x) > [0, 0]$, then supp $< x, y, \tilde{f} > = T$.*

Proof. (1) Let $z \in T$. Since $\tilde{f}$ is an i-v fuzzy ideal of T,

$$< x, y, \tilde{f} > (z) = \tilde{f}(xyz) \geq \tilde{f}(z).$$

Thus $\tilde{f} \subseteq < x, y, \tilde{f} >$.
(2) Assume T is commutative. Let n be an odd positive integer and $z \in T$. Since $\tilde{f}$ is an i-v fuzzy ideal of T,

$$\begin{aligned} < x^{n+2}, y^{n+2}, \tilde{f} > (z) = \tilde{f}(x^{n+2}y^{n+2}z) = \tilde{f}(x(xy^2)(x^ny^nz)) \geq \tilde{f}(x^ny^nz) \\ = < x^n, y^n, \tilde{f} > (z). \end{aligned}$$

Thus $< x^n, y^n, \tilde{f} > \subseteq < x^{n+2}, y^{n+2}, \tilde{f} >$.
(3) Assume $\tilde{f} > [0, 0]$. Clearly, *supp* $< x, y, \tilde{f} > \subseteq T$. Let $z \in T$. Since $\tilde{f}$ is an i-v fuzzy ideal of T,

$$< x, y, \tilde{f} > (z) = \tilde{f}(xyz) \geq \tilde{f}(x) > [0, 0].$$

So $z \in$ *supp* $< x, y, \tilde{f} >$. Thus *supp* $< x, y, \tilde{f} > = T$. □

Proposition 4.4. *If $\tilde{f}$ is a prime i-v fuzzy ideal of a ternary semigroup T, then*

$$< x, y, \tilde{f} > = < x^3, y^3, \tilde{f} > \text{ for all } x, y \in T.$$

Proof. Let $x, y, z \in T$. Since $\tilde{f}$ is an i-v fuzzy ideal of T,

$$< x, y, \tilde{f} > (z) = \tilde{f}(xyz) \geq \max\{\tilde{f}(x), \tilde{f}(y), \tilde{f}(z)\}$$

and

$$< x^3, y^3, \tilde{f} > (z) = \tilde{f}(x^3y^3z) \geq \max\{\tilde{f}(x^3), \tilde{f}(y^3), \tilde{f}(z)\}.$$

Since $\tilde{f}$ is prime,

$$< x, y, \tilde{f} > (z) = \tilde{f}(xyz) \leq \max\{\tilde{f}(x), \tilde{f}(y), \tilde{f}(z)\}$$

and

$$< x^3, y^3, \tilde{f} > (z) = \tilde{f}(x^3y^3z) \leq \max\{\tilde{f}(x^3), \tilde{f}(y^3), \tilde{f}(z)\}.$$

Thus $< x, y, \tilde{f} > (z) = \max\{\tilde{f}(x), \tilde{f}(y), \tilde{f}(z)\}$ and $< x^3, y^3, \tilde{f} > (z) = \max\{\tilde{f}(x^3), \tilde{f}(y^3), \tilde{f}(z)\}$. Since $\tilde{f}$ is a prime i-v fuzzy ideal of T, $\tilde{f}(x) = \tilde{f}(x^3), \forall x \in T$. So

$$\begin{aligned} < x, y, \tilde{f} > (z) &= \max\{\tilde{f}(x), \tilde{f}(y), \tilde{f}(z)\} = \max\{\tilde{f}(x^3), \tilde{f}(y^3), \tilde{f}(z)\} \\ &=< x^3, y^3, \tilde{f} > (z). \end{aligned}$$

□

Let T be a ternary semigroup, $A \subseteq T$ and $x, y \in T$. We define

$$< x, y, A >:= \{z \in T | xyz \in A\}.$$

Proposition 4.5. *Let $\tilde{f}$ be an i-v fuzzy subset of a ternary semigroup T and A be a nonempty subset of T. Then*

$$< x, y, \tilde{f}_A >= \tilde{f}_{<x,y,A>}$$

for all $x, y \in T$.

Proof. Let $x, y, z \in T$.
Case 1. $xyz \in A$. Then $\tilde{f}_A(xyz) = 1$. Since $xyz \in A, z \in< x, y, A >$. So $\tilde{f}_{<x,y,A>}(z) = 1$. Thus

$$< x, y, \tilde{f}_A > (z) = \tilde{f}_A(xyz) = 1 = \tilde{f}_{<x,y,A>}(z).$$

Case 2. $xyz \notin A$. Then $\tilde{f}_A(xyz) = 0$. Since $xyz \notin A, z \notin< x, y, A >$. So $\tilde{f}_{<x,y,A>}(z) = 0$. Thus

$$< x, y, \tilde{f}_A > (z) = \tilde{f}_A(xyz) = 0 = \tilde{f}_{<x,y,A>}(z).$$

Hence $< x, y, \tilde{f}_A >= \tilde{f}_{<x,y,A>}$. □

Theorem 4.6. *Let T be a ternary semigroup. If $\tilde{f}$ is a prime i-v fuzzy ideal of T and $x, y \in T$ such that $\tilde{f}(x) = \tilde{f}(y) = \inf_{a \in T} \tilde{f}(a)$, then $< x, y, \tilde{f} >= \tilde{f}$.*

Proof. Let $\tilde{f}$ be a prime i-v fuzzy ideal of T and $x, y \in T$ such that $\tilde{f}(x) = \tilde{f}(y) = \inf_{a\in T} \tilde{f}(a)$. Suppose that $< x, y, \tilde{f} > (z) \neq \tilde{f}(z)$ for some $z \in T$. That is $\tilde{f}(xyz) \neq \tilde{f}(z)$. Since $\tilde{f}$ is a prime i-v fuzzy ideal of T, $< x, y, \tilde{f} > (z) = \tilde{f}(xyz) = \max\{\tilde{f}(x), \tilde{f}(y), \tilde{f}(z)\}$. Then $\tilde{f}(xyz) = \tilde{f}(x)$ or $\tilde{f}(xyz) = \tilde{f}(y)$ or $\tilde{f}(xyz) = \tilde{f}(z)$. Since $\tilde{f}(xyz) \neq \tilde{f}(z)$, $\tilde{f}(xyz) = \tilde{f}(x)$ or $\tilde{f}(xyz) = \tilde{f}(y)$.

Case 1. $\tilde{f}(xyz) = \tilde{f}(x)$.By proposition 4.3(1), we know$\tilde{f} \subseteq < x, y, \tilde{f} >$. Then $\tilde{f}(z) \leq < x, y, \tilde{f} > (z) = \tilde{f}(xyz) = \tilde{f}(x)$. Thus $\tilde{f}(z) \leq \tilde{f}(x)$. Since $\tilde{f}(x) = \inf_{a\in T} \tilde{f}(a)$, $\tilde{f}(x) \leq \tilde{f}(z)$. So $\tilde{f}(x) = \tilde{f}(z)$. That is $\tilde{f}(xyz) = \tilde{f}(x) = \tilde{f}(z)$, a contradiction.

Case 2. $\tilde{f}(xyz) = \tilde{f}(y)$.By proposition 4.3(1), we have$\tilde{f} \subseteq < x, y, \tilde{f} >$. Then $\tilde{f}(z) \leq < x, y, \tilde{f} > (z) = \tilde{f}(xyz) = \tilde{f}(y)$. Thus $\tilde{f}(z) \leq \tilde{f}(y)$. Since $\tilde{f}(y) = \inf_{a\in T} \tilde{f}(a)$, $\tilde{f}(y) \leq \tilde{f}(z)$. So $\tilde{f}(y) = \tilde{f}(z)$. That is $\tilde{f}(xyz) = \tilde{f}(y) = \tilde{f}(z)$, a contradiction.

Hence $< x, y, \tilde{f} >= \tilde{f}$. □

Theorem 4.7. *Let $\tilde{f}$ be an i-v fuzzy ideal of a ternary semigroup T. Suppose that for any $x, y \in T$ such that $\tilde{f}(x)$ and $\tilde{f}(y)$ is not maximal in $\tilde{f}(T)$, we have $< x, y, \tilde{f} >= \tilde{f}$. Then $\tilde{f}$ is prime.*

Proof. Let $x, y, z \in T$. Since $\tilde{f}$ is an i-v fuzzy ideal,

$$\tilde{f}(xyz) \geq \tilde{f}(x), \tilde{f}(xyz) \geq \tilde{f}(y) \text{ and } \tilde{f}(xyz) \geq \tilde{f}(z).$$

Case 1. $\tilde{f}(xyz) = \tilde{f}(x)$ or $\tilde{f}(xyz) = \tilde{f}(y)$. Then

$$\tilde{f}(xyz) = \max\{\tilde{f}(x), \tilde{f}(y), \tilde{f}(z)\}.$$

Case 2. $\tilde{f}(xyz) \neq \tilde{f}(x)$ and $\tilde{f}(xyz) \neq \tilde{f}(y)$. So $\tilde{f}(x)$ and $\tilde{f}(y)$ is not maximal in $\tilde{f}(T)$. Then $< x, y, \tilde{f} >= \tilde{f}$. Thus $< x, y, \tilde{f} > (z) = \tilde{f}(z)$, this implies that $\tilde{f}(xyz) = \tilde{f}(z)$. Hence $\tilde{f}(xyz) = \max\{\tilde{f}(x), \tilde{f}(y), \tilde{f}(z)\}$.

In both cases, therefore $\tilde{f}$ is prime. □

Corollary 4.8. *Let T be a ternary semigroup and I be an ideal of T. Then I is prime if and only if for $x, y \in T$ such that $x, y \notin I$, $< x, y, \tilde{f}_I >= \tilde{f}_I$.*

Proof. Let I be a prime ideal of T. Then $\tilde{f}_I$ is a prime i-v fuzzy ideal of T. For $x, y \in T$ such that $x, y \notin I$, we have $\tilde{f}_I(x) = 0 = \inf_{z\in T} \tilde{f}_I(z)$ and $\tilde{f}_I(y) = 0 = \inf_{z\in T} \tilde{f}_I(z)$. By Theorem 4.6, $< x, y, \tilde{f}_I >= \tilde{f}_I$. Conversely, let

I be an ideal of T. Then $\tilde{f}_I$ is an i-v fuzzy ideal of T. Let $x, y \in T$ such that $\tilde{f}_I(x)$ and $\tilde{f}_I(y)$ is not maximal in $\tilde{f}_I(T)$. Thus $\tilde{f}_I(x) = \tilde{f}_I(y) = 0$, this implies $x, y \notin I$. Then $< x, y, \tilde{f}_I >= \tilde{f}_I$. By Theorem 4.7 and Corollary 3.5, I is prime.

□

Theorem 4.9. *Let T be a commutative ternary semigroup and $\tilde{f}$ be an i-v fuzzy subset of T such that $< x, y, \tilde{f} >= \tilde{f}$ for all $x, y \in T$. Then $\tilde{f}$ is constant.*

Proof. Let $x, y, z \in T$. Then

$$\tilde{f}(y) =< x, z, \tilde{f} > (y) = \tilde{f}(xzy) = \tilde{f}(xyz)$$

and

$$\tilde{f}(z) =< x, y, \tilde{f} > (z) = \tilde{f}(xyz)$$

So $\tilde{f}(y) = \tilde{f}(z)$. Hence $\tilde{f}$ is constant. □

Proposition 4.10. *Let T be a ternary semigroup and $\tilde{f}$ be a prime i-v fuzzy ideal of T. Then $< x, y, \tilde{f} >$ is a prime i-v fuzzy ideal of T for all $x, y \in T$.*

Proof. Let $x, y \in T$ and $a, b, c \in T$. Since $\tilde{f}$ is a prime i-v fuzzy ideal of T,

$$\begin{aligned} < x, y, \tilde{f} > (abc) &= \tilde{f}(xyabc) \\ &= \max\{\tilde{f}(xya), \tilde{f}(b), \tilde{f}(c)\} \\ &= \max\{\max\{\tilde{f}(x), \tilde{f}(y), \tilde{f}(a)\}, \tilde{f}(b), \tilde{f}(c)\} \\ &= \max\{\tilde{f}(x), \tilde{f}(y), \tilde{f}(a), \tilde{f}(b), \tilde{f}(c)\} \\ &= \max\{\tilde{f}(xya), \tilde{f}(xyb), \tilde{f}(xyc)\} \\ &= \max\{< x, y, \tilde{f} > (a), < x, y, \tilde{f} > (b), < x, y, \tilde{f} > (c)\} \end{aligned}$$

Thus $< x, y, \tilde{f} >$ is a prime i-v fuzzy ideal of T. □

Proposition 4.11. *Let T be a commutative ternary semigroup and $\tilde{f}$ be a semiprime i-v fuzzy ideal if T. Then $< x, y, \tilde{f} >$ is a semiprime i-v fuzzy ideal of T for all $x, y \in T$.*

Proof. Let $x, y \in T$. Since $\tilde{f}$ is an i-v fuzzy ideal of T, by Proposition 4.2, $< x, y, \tilde{f} >$ is an i-v fuzzy ideal of T. Let $a \in T$. Since $\tilde{f}$ is a semiprime i-v fuzzy ideal of T, we have

$$< x, y, \tilde{f} > (a) = \tilde{f}(xya) \geq \tilde{f}((xya)^3) = \tilde{f}(x(xy^2)(xya^3)) \geq \tilde{f}(xya^3) = < x, y, \tilde{f} > (a^3).$$

Thus $< x, y, \tilde{f} >$ is a semiprime i-v fuzzy ideal of T. □

Corollary 4.12. *Let T be a commutative ternary semigroup, $\{\tilde{f}_i\}_{i \in I}$ a nonempty family of a semiprime i-v fuzzy ideal of T and $\tilde{f} = \inf\{\tilde{f}_i | i \in I\}$. Then for any $x, y \in T, < x, y, \tilde{f} >$ is a semiprime i-v fuzzy ideal of T.*

Proof. Let $x, y, z \in T$. Since $\tilde{f}_i$ is an i-v fuzzy ideal of T, we have

$$\begin{aligned}
\tilde{f}(xyz) &= \inf\{\tilde{f}_i | i \in I\}(xyz) \\
&= \inf\{\tilde{f}_i(xyz) | i \in I\} \\
&\geq \inf\{\max\{\tilde{f}_i(x), \tilde{f}_i(y), \tilde{f}_i(z)\} | i \in I\} \\
&\geq \max\{\inf\{\tilde{f}_i(x) | i \in I\}, \inf\{\tilde{f}_i(y) | i \in I\}, \inf\{\tilde{f}_i(z) | i \in I\}\} \\
&= \max\{\tilde{f}(x), \tilde{f}(y), \tilde{f}(z)\}.
\end{aligned}$$

Then $\tilde{f}$ is an i-v fuzzy ideal of T. Let $a \in T$. Since $\tilde{f}_i$ is semiprime for all $i \in I$, we have

$$\begin{aligned}
\tilde{f}(a) &= \inf\{\tilde{f}_i | i \in I\}(a) \\
&= \inf\{\tilde{f}_i(a) | i \in I\} \\
&\geq \inf\{\tilde{f}_i(a^3) | i \in I\} \\
&= \inf\{\tilde{f}_i | i \in I\}(a^3) \\
&= \tilde{f}(a^3).
\end{aligned}$$

Then $\tilde{f}$ is semiprime. Thus $\tilde{f}$ is a semiprime i-v fuzzy ideal of T. By Proposition 4.11, $< x, y, \tilde{f} >$ is a semiprime i-v fuzzy ideal of T. □

Corollary 4.13. *Let T be a commutative ternary semigroup, $\{\tilde{P}_i\}_{i \in I}$ be a nonempty family of semiprime ideals of T and let $A = \bigcap_{i \in I} P_i \neq \emptyset$. Then $< x, y, \tilde{f}_A >$ is a semiprime ideal of T for all $x, y \in T$.*

Proof. Let $x, y \in T$. Since P_i is a semiprime ideal of T for all $i \in I$, $A = \bigcap_{i\in I} P_i$ is a semiprime ideal of T. Thus $\tilde{f}_A$ is a semiprime i-v fuzzy ideal of T. Thus by Proposition 4.11, $< x, y, \tilde{f}_A >$ is a semiprime i-v fuzzy ideal of T. □

Corollary 4.14. *Let T be a commutative ternary semigroup and $\tilde{f}$ be a prime i-v fuzzy ideals of T. If $\tilde{f}$ is not constant, then $\tilde{f}$ is not a maximal prime i-v fuzzy ideal of T.*

Proof. Let $\tilde{f}$ be a prime i-v fuzzy ideals of T. Then for all $x, y \in T$, $< x, y, \tilde{f} >$ is a prime i-v fuzzy ideal of T. By Proposition 4.3, we know $\tilde{f} \subseteq < x, y, \tilde{f} >$ for all $x, y \in T$. If for all $x, y \in T, < x, y, \tilde{f} >= \tilde{f}$, this implies $\tilde{f}$ is constant. Then there exist $x, y \in T$ such that $\tilde{f} \neq < x, y, \tilde{f} >$. Hence $\tilde{f}$ is not a maximal prime i-v fuzzy ideal of T. □

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References

[1] J. Ahsan, K. Saifullah, M. Farid Khan, Semigroups characterized by their fuzzy ideals, Fuzzy Systems and Mathematics, **9,** (1995), 29-32.

[2] R. Chinram, S. Malee, L-Fuzzy ternary subsemirings and L-Fuzzy ideals in ternary semirings, IAENG International Journal of Applied Mathematics, **40,** no. 3, (2010), 124-129.

[3] J. Kavikumar, A. B. Khamis, Fuzzy ideals and fuzzy quasi-ideals in ternary semirings, IAENG International Journal of Applied Mathematics, **37,** no. 2, (2007), 102-106.

[4] N. Kuroki, Fuzzy bi-ideals in semigroups, Comment. Math. Uni. St. Pauli., **28,** (1979),17-21.

[5] N. Kuroki, On fuzzy ideals and bi-ideals in semigroups, Fuzzy Sets and System, **5,** (1981), 203-215.

[6] N. Kuroki, On fuzzy semigroups, Inform. Sci., **53,** (1991), 203-236.

[7] D. H. Lehmer, A ternary analogue of abelian groups, Amer. J. Math., **59,** (1932), 329-338.

[8] J. Los, On the extending of models I, Fundam. Math., **42,** (1955), 3854.

[9] P. Petchkhaew, R. Chinram, Fuzzy, rough and rough fuzzy ideals in ternary semigroups, Int. J. Pure. Appl. Math., **56,** (2009) pp. 21-36.

[10] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl., **35,** (1971), 512-517.

[11] S. Saelee, R. Chinram, A study on rough, fuzzy and rough fuzzy bi-ideals of ternary semigroups, IAENG International Journal of Applied Mathematics, **41,** no. 3, (2011), 172-176.

[12] M. L. Santiago and S. Sri Bala, Ternary semigroups, Semigroup Forum, **81,** (2010), 380-388.

[13] F. M. Sioson, Ideal theory in ternary semigroups, Math. Jpn., **10,** (1965), 63-84.

[14] V. Vagner, The theory of generalized heaps and generalized groups, Mat. Sb. NS, **32,** (1953), 545-632(Russian).

[15] X. -Y. Xie, Fuzzy ideals extensions of semigroups, Soochow Journal of Mathematics, **27,** no. 2, (2001), 125-138.

[16] N. Yaqoob, R. Chinram, G. Ghareeb, M. Aslam, Left almost semigroups characterized by their interval fuzzy ideals, Afrika Matematika, **24,** (2013), 231-245.

[17] L. A. Zadeh, Fuzzy sets, Inform. Control, **8,** (1965), 338-353.

[18] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning I, Inform. Sci., **8,** (1975), 199-249.